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# Probabilistic analysis of fatigue crack growth using efficient surrogate model

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**Abstract.** In this paper probabilistic analysis of a fatigue crack growth process governed by the Walker law is performed. Sensitivity of its parameters to uncertainties related to the material properties is considered. Based on statistical analysis of experimental data, they are modelled as independent normal random variables. The effect of these uncertainties on the fatigue crack growth behaviour of Centre Cracked Plate specimen is assessed using Monte-Carlo simulations and surrogate model based on polynomial chaos expansion. An efficient truncation scheme allows to discard the high order interactions having a weak effect on the mechanical response and consequently reduce the number of finite elements calls when identifying the unknown coefficients of the polynomial chaos expansion. The statistical moments namely the mean and the standard deviation in addition to the probability density function of the mechanical response are derived with a good accuracy. The obtained results show that the fatigue crack growth life is significantly affected by the uncertainties on the material properties since its coefficient of variation reaches 21% and that it follows a log-normal distribution law.

## 1. Introduction

For many years now, researchers [1] have pointed out the stochastic character of the fatigue crack growth. The variability of the fatigue crack growth life was attributed to the uncertainties observed on the material properties, the geometrical parameters and the loading conditions. It is clear that to obtain a safe design, these sources of uncertainties should be taken into account. The probabilistic approaches could address this issue, but unfortunately they suffer until now from some limitations to solve real-life engineering problems. Indeed, some of them are purely statistic [2] and subject of criticisms since they are not able to describe the physical phenomena related to the fatigue crack growth. Other approaches, which are mainly based on the *probabilization* of the fatigue crack growth rate [3], are time consuming, because their efficiency is affected when the stochastic dimension (i.e. the number of random variables representing the uncertain parameters) of the problem is high, and/or the mechanical model representing the fatigue crack growth is also time consuming. Recently, a perturbation series expansion based-method is developed [4, 5], which is only accurate when the crack growth process can be described through simple deterministic mathematical models. Long *et al* [6] have developed an innovative approach which allows, on the one hand to reduce the number of mechanical model calls by the use of efficient integration scheme and on the other hand to take into account the effect of statistical dependence [7] between crack growth uncertain parameters. As an alternative, this paper presents an efficient uncertainty propagation method based on Polynomial Chaos Expansion (PCE) to deal with fatigue crack growth problem. The aim of this method is to reduce the number of simulations required to quantify uncertainties.



## 2. Uncertainty propagation based on PCE

The uncertainty propagation aims to characterize, in a statistical manner, the variability observed on the behaviour of a physical model when some of its input parameters are random quantities. Let us consider a mechanical model having  $n$  uncertain input parameters gathered in the  $n$ -dimensional vector  $\mathbf{x} = \{x_1, x_2, \dots, x_n\}^T$ , and let  $\mathbf{X} = \{X_1, X_2, \dots, X_n\}^T$  the probabilistic model of  $\mathbf{x}$  which is fully described by its joint probability density  $f_X$ . Mathematically speaking, the response  $y$  (i.e. stress, crack length, fatigue life) of the mechanical model, which is, without any loss of generality, considered here as a scalar, can be linked to the input parameters  $\mathbf{x}$ , through the mapping  $y = \mathcal{M}(\mathbf{x})$ . Since  $\mathbf{x}$  is random, consequently the mechanical response  $y$  is also a random quantity, with associated probabilistic model  $Y$  which can simply be a random variable. According to the PCE method [8], the  $p^{\text{th}}$  order approximation of the mechanical response  $y$ , written in the standard random space where the  $n$ -dimensional random variable  $\mathbf{X} = \{X_1, X_2, \dots, X_n\}^T$  is substituted by an  $n$ -dimensional standard normal variable  $\mathbf{U} = \{U_1, U_2, \dots, U_n\}^T$  using a suitable iso-probabilistic transformation  $\mathbf{X} = T(\mathbf{U})$ , reads:

$$y \approx \hat{M}(\mathbf{X}) = \hat{M} \circ T(\mathbf{U}) = \sum_{|\alpha| \leq p} a_\alpha \Psi_\alpha(\mathbf{U}) \quad (1)$$

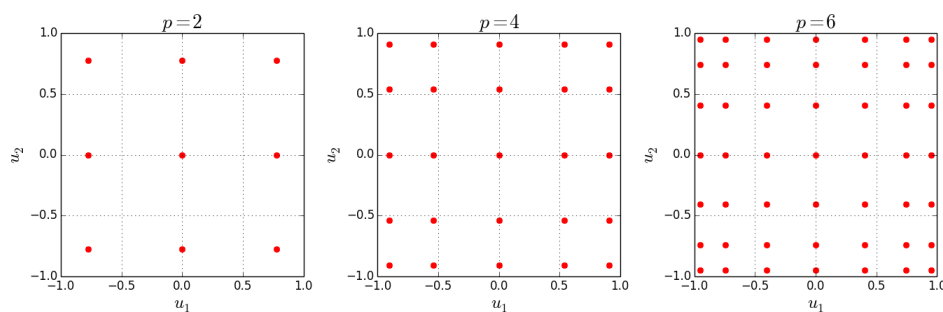
where  $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ , with  $|\alpha| = \alpha_1 + \dots + \alpha_n$ , are multi-index,  $a_\alpha$ 's are unknown deterministic coefficients and  $\Psi_\alpha$ 's are multivariate Hermite polynomials which form an orthogonal basis with respect to the normal probability density and can be obtained by tensorizing  $n$  univariate Hermite polynomials  $\{\Psi_{\alpha_k}\}_{k=1}^n$ .

The unknown coefficients  $a_\alpha$  can be obtained by performing regression analysis based on numerical plan of experiments. The estimates of the  $a_\alpha$ 's are computed through the minimization of the mean square error of the polynomial approximation  $\hat{M}$ . The regression problem reads:

$$\text{Find } \hat{\mathbf{a}} \text{ that minimizes } \mathbb{E} \left[ (\hat{\mathbf{a}}^T \Psi_\alpha(\mathbf{U}) - M(\mathbf{U}))^2 \right] \quad (2)$$

where  $\mathbb{E}[\cdot]$  denotes the mathematical expectation.

Let  $\mathcal{T}_n^p = \{\alpha \in \mathbb{N}^n \text{ such that } |\alpha| \leq p\}$  be a set of index associated with a PCE of degree  $p$ . Hence, the number of terms in the PCE is  $\text{card}(\mathcal{T}_n^p) = \binom{n+p}{p} = \frac{(n+p)!}{n!p!}$ . The convergence rate of the estimates of the unknown coefficients can be enhanced by the use of suitable plan of experiments such those based on low discrepancy random sequences, which ensure a better representation of the random space. Here, Gauss-Hermite integration points are used to construct the plan of experiments. Firstly,  $p+1$  one-dimensional Gauss-Hermite integration points are generated. Then  $n$ -tuples representing all possible combinations from the  $p+1$  one-dimensional Gauss-Hermite integration points are computed. Since there are too many combinations, only the first  $2 \times (\text{card}(\mathcal{T}_n^p) + 1)$  points are selected to construct the plan of experiments, after sorting the  $n$ -tuples in ascending order according to the distance to the origin. Figure 1 compares the obtained 2D plan of experiments for PCE order  $p \in \{2, 4, 6\}$ .

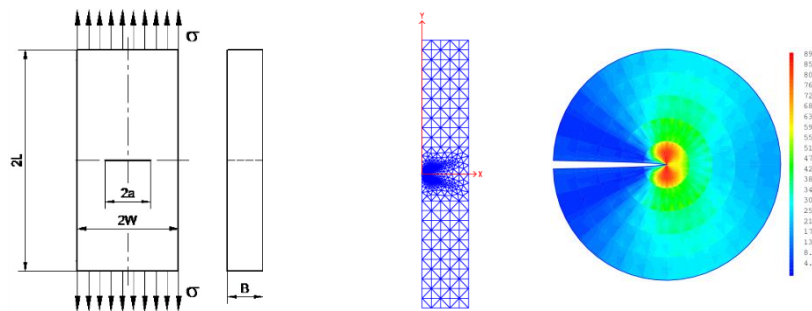


**Figure 1.** 2D plan of experiments based on Gauss-Hermite integration points

In order to reduce even more the computation efforts, an efficient truncation strategy is used, which aims to only retain the PCE coefficients associated to the low-order interactions [9]. Indeed, for engineering problems experiences have shown that high-order interactions have weak effect on the variability of the mechanical model. Let  $i$  be the maximum order of the retained interactions, according to An and Owen [10], the number of unknown coefficients of the PCE is  $card(\mathcal{T}_n^{p,i})$ , with  $\mathcal{T}_n^{p,i} = \{\alpha \in \mathbb{N}^n \text{ such that } |\alpha| \leq p \text{ and } \sum_{k=1}^n \mathbb{1}_{\{\alpha_k > 0\}} < i\}$ . It is clear that, for low values of the truncation maximum interaction order  $i$ , the *economy* defined as the ratio of  $card(\mathcal{T}_n^{p,i})$  to  $card(\mathcal{T}_n^p)$  can be very significant. For instance, for  $p = 8$ ,  $n = 7$  and  $i = 1$ , the economy is around  $10^{-2}$ . Once the unknown coefficients of the PCE are obtained, the statistical moments and the probability density function of the response of the mechanical model can be obtained by performing Monte-Carlo simulations on the polynomial approximation given by equation (1).

### 3. Application to fatigue crack growth

The uncertainty propagation presented in the previous section is applied now to conduct probabilistic analysis on Centre Cracked Plate (CCP) subjected to cyclic loads with a mean stress  $\sigma_m = 15 \text{ ksi}$  and alternating stress  $\sigma_a = 10 \text{ ksi}$ . The CCP specimen, is made of 7075-T6 aluminium alloy, measures 12 inch wide (thus,  $w = 6 \text{ in.}$ ) by 35 inch tall and 0.09 inch thick and contains an initial crack 0.1 inch (thus,  $a = 0.05 \text{ in.}$ ) long located at its centre as depicted in figure 2.



**Figure 2.** CCP specimen geometry (left), finite elements mesh (center), Von-Mises equivalent stress around the crack tip (right)

Although an analytical solution [11] is available for the Stress Intensity Factor (SIF)  $\Delta K_I$ , as given by equation (3), a Finite Elements Model (FEM) of the CCP specimen is also developed using the software Cast3m [12]. This is in order to use an implicit mechanical model within the uncertainty propagation analysis which will be conducted later. Due to the geometry symmetry, only one half of the CCP specimen is modeled as shown in the finite elements mesh in figure 2. In addition, plane stress hypothesis is assumed since the specimen thickness is small compared to the two other dimensions of the specimen.

$$\Delta K_I = 2\sigma_a \frac{1 + 0.5 \left(\frac{a}{w}\right) + 0.326 \left(\frac{a}{w}\right)^2}{\left(1 - \left(\frac{a}{w}\right)\right)^{1/2}} (\pi a)^{1/2} \quad (3)$$

The obtained results, in terms of  $\Delta K_I$ , for different loading conditions are given in table 1. As can be seen, numerical results are in good agreement with the reference solution since the relative error does not exceed 0.3 %. Furthermore, the accuracy of the finite elements model is not affected by the loading conditions. As shown in figure 2, based on the deformed mesh of the CCP specimen, the crack will propagate in opening fracture mode. In addition, linear elastic fracture behavior hypothesis is verified since the Von-Mises equivalent stress exceeds the yielding strength of the material  $\sigma_{yld} = 75.9 \text{ ksi}$ , only in the neighborhood of the crack tip. Since the accuracy of the FEM is now approved, it will be used in the sequel, firstly to fit the fatigue crack growth law and then to perform uncertainty propagation analysis based on PCE.

**Table 1.** Comparison of numerical and analytical results of the SIF range  $\Delta K_I$ 

$\sigma_m$ (ksi)	$\sigma_a$ (ksi)	Analytical $\Delta K_I$	Numerical $\Delta K_I$	Error  (%)
5	5	5.6057	5.5934	0.2192
15	10	11.211	11.187	0.2192
15	3	3.3634	3.3560	0.2192

The fatigue crack growth is considered to be governed by the Walker law [13] which allows to take into account the effect of mean stress. Based on Walker law the relationship between the Fatigue Crack Growth Rate (FCGR)  $da/dN$  and the SIF range  $\Delta K_I$  reads:

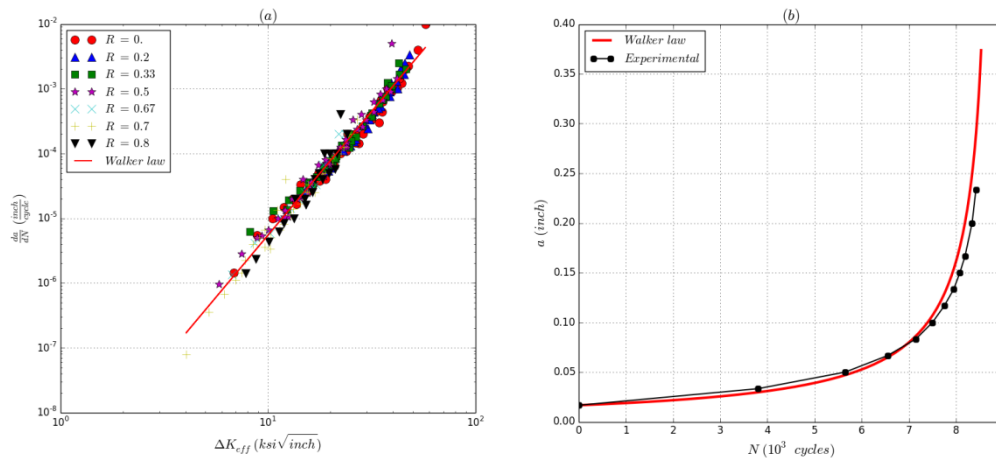
$$\frac{da}{dN} = C \frac{(\Delta K_I)^m}{(1-R)^{m(1-\gamma)}} \quad (4)$$

where  $a$  is the crack length,  $N$  is the number of loading cycles,  $R = (\sigma_m - \sigma_a)/(\sigma_m + \sigma_a)$  is the stress ratio,  $C$ ,  $m$  and  $\gamma$  are three real constants.

**Table 2.** Walker law parameters

$m$	$\gamma$	$C$	$R_{LRGF}^2$
3.82	0.56	$8.19 \cdot 10^{-10}$	0.975

The parameters  $C$ ,  $m$  and  $\gamma$  of Walker law are obtained through multiple linear regression analysis applied to  $da/dN$  and  $\Delta K_I$  after transforming them to log-log space, and based on experimental tests performed by Hudson [14]. The estimates of Walker law parameters are given in table 2 and the fitted model is shown in figure 3(a). As can be seen, Walker law fits very well the experimental data since the goodness of fit parameter  $R_{LRGF}^2$  is close to 1. In addition, the fatigue crack growth life can be easily derived by integration of the Walker law as depicted in figure 3(b).



**Figure 3.** (a) Walker law fit to CCP specimen experimental data, (b) fatigue crack growth life based on the integration of Walker law

Statistical analysis of Hudson experimental data has shown that the parameters of Walker law are not deterministic. Determination of the statistical characteristics of these parameters is done using multiple linear regression on the experimental data and based of the Walker law rewritten as:

$$y = B + A_1 x_1 + A_2 x_2 \quad (5)$$

where  $y = \log(da/dN)$ ,  $B = \log(C)$ ,  $A_1 = m$ ,  $A_2 = -m(1-\gamma)$ ,  $x_1 = \log(\Delta K_I)$ , and  $x_2 = \log(1-R)$ . The statistical moments (i.e. the mean value  $\mu$  and the variance  $\sigma^2$ ), and the distribution type of the parameters  $A_1$ ,  $A_2$  and  $B$  of the modified Walker law are given on table 3.

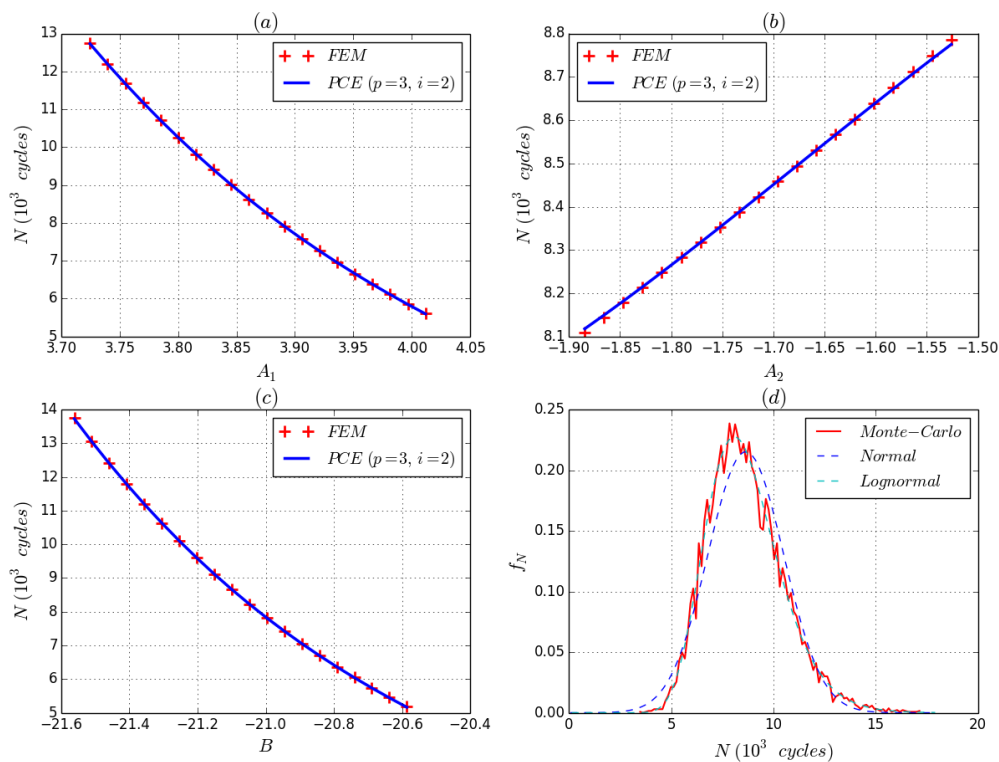
The statistical characteristics of the parameters  $C$ ,  $m$  and  $\gamma$  of the original Walker law are easily derived from the results given in table 3. Knowing this, uncertainty propagation analysis is performed



using PCE method on the previously developed finite elements model of the CCP specimen where the mechanical response is defined as the fatigue crack growth life  $N$ . First of all, the accuracy of the polynomial approximation to reproduce the real mechanical response is assessed. As can be seen from figures 4(a), 4(b) and 4(c), the polynomial approximation given by the PCE method fits very well the real mechanical behavior of the CCP specimen. Note that, 3<sup>rd</sup> order PCE is used to construct the polynomial approximation. The results show also that 3<sup>rd</sup> order interactions have no significant effect on the variability of the mechanical response since a good accuracy is achieved when only 2<sup>nd</sup> order interaction coefficients are considered in the PCE.

**Table 3.** Statistical characteristics of the parameters of the transformed Walker law

	Distribution	$\mu$	$\sigma^2$
$A_1$	Normal	3.8681	$2.2911 \cdot 10^{-3}$
$A_2$	Normal	-1.7052	$3.5716 \cdot 10^{-3}$
$B$	Normal	-21.0737	$26.4442 \cdot 10^{-3}$



**Figure 4.** PCE approximation with respect to the uncertain parameters (a)  $A_1$ , (b)  $A_2$ , (c)  $B$ , (d) PDF of the fatigue crack growth life

The two first statistical moments of the mechanical response (fatigue crack growth life  $N$ ) are derived from the coefficients of the PCE as given in equation (1) and compared to the estimates given by *crude* Monte-Carlo Simulations (MCS) directly applied to the finite elements model and taken here as the reference solutions. As can be seen from table 4, the results obtained by the two methods are in good agreement since the relative error does not exceed 0.6%. The proposed method shows eventually its efficiency since only 40 FEM calls are required against 10000 FEM calls for *crude* MCS to achieve the same accuracy level on the estimates. In addition, the PDF of the fatigue crack growth life is constructed by performing MCS on the polynomial approximation given by PCE method. As can be seen from figure 4(d), the lognormal distribution fits it very well. Compared to the statistical moments, the PDF is quite informative since for each possible event we can attribute a probability of occurrence, which is very useful for engineers.

**Table 4.** Statistical moments of the mechanical response

	PCE	Crude MCS	Error  (%)
$\mu$	8633.73	8670.88	0.428
$\sigma$	1859.09	1869.26	0.544
FEM calls	40	10000	

#### 4. Conclusion

In this paper an uncertainty propagation method based on PCE is presented to deal with fatigue crack growth problem. An efficient truncation scheme which allows to discard weak high order interaction effects is used to reduce the number of FEM calls to construct a polynomial approximation of the fatigue crack growth life. The remaining coefficients of the PCE are then computed using plan of experiments based on Gauss-Hermite integration points. The obtained results show that the variability of the fatigue crack growth life is very sensitive to the uncertainties associated to the material properties represented by the parameters of the Walker law since the coefficient of variation is around 21%. This information should be taken into account in the design of structures subjected to fatigue crack growth. Future works should focus on enhancing the proposed uncertainty propagation method and modelling uncertainties associated to geometrical parameters and loading conditions.

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