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Why health matters in the energy efficiency–consumption nexus? Some answers from a life cycle analysis

Sondès KAHOULI* and Xavier PAUTREL†

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Abstract

This paper shows that accounting for the growing interdisciplinary literature supporting the causality between energy efficiency and health and the empirical evidence re-assessing the importance of health in workforce productivity, could explain a part of the paradoxal relationship found between energy efficiency and energy consumption.

We build a 3-period overlapping generations model where we assume that residential energy inefficiency induces chronic disease for adults and bad health for elderly. We also assume that workers' health has an effect of their labor productivity. Our results suggest, in particular, that if mostly old (respectively young) people health is affected, the health impact of residential energy efficiency should have a backfire (resp. rebound) influence on residential energy consumption, by promoting precautionary saving (resp. by rising labor productivity).

In policy terms, by showing that the link between energy efficiency and energy consumption is far from being just associated with technical conditions about preferences and/or production technology, our research emphasizes how crucial and complex are for governments the discussion and policy action dealing with the connection between energy conservation policies, health insurance system and growth.

Keywords

Energy efficiency; Health; Precautionary saving; Labor productivity, Overlapping generations model.

JEL classification

D58; Q43

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1 Introduction

The economic literature dealing with energy efficiency and energy consumption is large. The analysis of the issue was usually conducted from the angle of rebound effect: why energy conservation from energy efficiency policies is not as large as expected? What type of policy pathways are more effective in mitigating the rebound effect? Through analyzing the rebound effect, a number of contributions pointed out how difficult is to estimate the magnitude of rebound effect, thus, how complex are interactions between energy efficiency and energy consumption.

In this research, we focus on analyzing the role of health in energy efficiency-consumption nexus and propose a new surrounding background to understand rebound effect persistence. The contribution of this article is to originally build upon previous insights from two different bodies of literature in order to study how health contribute to the analysis of energy efficiency-consumption nexus. The first body of literature is empirical and interdisciplinary and supports the causality assumption between energy efficiency and health. The second belongs rather to the economic growth field and focus on assessing the role of health, on particular labor productivity, in the growth process. We rely on fairly standard life-cycle model with 3-period overlapping generations under which we assume that residential energy inefficiency induces chronic disease for adults and bad health for elderly. We also assume that workers' labor productivity is dependent on their health status.

Our theoretical model relies on Kotlikoff (1989) and Wang, Zhao, and Bhattacharya (2015), the first assuming that old agents have a probability to be sick and the second assuming that this probability relies on pollution. However, it differs from these studies in several aspects. First, while Kotlikoff (1989) as well as Wang, Zhao, and Bhattacharya (2015) focus on old agents' health, our paper puts the spotlight on morbidity of both adults and elderly. This focus on adult morbidity is not simply an innocuous modeling variation. Instead, it is motivated by the observation that even empirical results support that many households living in an energy inefficient environment have low life expectancy, some other ones have high ill life expectancy especially when young and adult. Second, the morbidity-energy linkage allows us to explore a novel inter-connection between health, savings and energy policies, in particular those related to the rebound effect phenomenon. Thus, we extended Kotlikoff (1989) to allow energy inefficiency to be an important determinant of the likelihood of getting sick in the same logical than Wang, Zhao, and Bhattacharya (2015) who make it dependent from pollution. Another difference with respect to Kotlikoff (1989) and Wang, Zhao, and Bhattacharya (2015) is that we consider in our framework is that there are chronic effects of the disease meaning that the level of health does not return to its initial level, *i.e.* full health, despite the health expenses.

Our results show that health channels we introduced could significantly shape the change in energy consumption induced by an energy efficiency variation, through their impacts on the propensity to save and the disposal income. In particular, our results suggest that if mostly old (resp. young) people health is affected, the health impact of residential energy efficiency should have a backfire (resp. rebound) influence on residential energy consumption, by promoting precautionary saving (resp. by rising labor productivity). Interestingly, our results also show that if health externality on labor productivity is strong enough, as suggested by recent empirical evidence (see section 2.2), an energy efficiency improvement should rise the energy consumption not only in the residential sector but also in production sector.

Our theoretical approach based on an overlapping generation models makes it possible to consider the age effect related to energy policy conservation. As far as we know, a study of how health impacts energy efficiency-energy consumption nexus in terms of the distinction between people's ages—in particular, young versus adults—has never been carried out before, although this issue is crucial for the implementation of policies that aim to promote energy conservation in connection to health and growth issues.

The paper unfolds as follows. In Section 2, we give two brief reviews of the literature. The first review is interdisciplinary and empirical and focuses on the impact of residential energy efficiency on health outcomes. The second review deals with the relationship between health and productivity from as studied in the economic growth literature. In Sections 3 and 4, we present the basic theoretical model and we derive analytical results about the long-term equilibrium. In Section 5, we compute numerical simulations (calibrated in the U.S. economy). In Section 6 we investigate two important

extensions of the basic model: the influence of health-status on utility and the chronic dimension of bad health associated with low residential energy efficiency. Finally, in Section 7, we conclude and give some policy implications

2 Related Literatures

In this section, we present two brief literature reviews on the two frameworks within which our analysis is constructed, namely the interdisciplinary empirical literature on the impact of residential energy efficiency on health outcomes (Sub-section 2.1) and the economic growth literature on the relationship between health, in particular labor productivity, and growth (Sub-section 2.2).

2.1 Residential energy efficiency and health

Housing thermal discomfort is one of the most important proxy of indoor/residential energy (in)efficiency¹ and there has been much research on its effects on health since the pioneering assessment of the cost of indoor cold and the definition of the 10% energy-poverty indicator by Boardman (1991). For instance, Baker (2001) produced a review of the evidence on the link between living in an energy inefficient dwelling and the increased risk of illness. This study showed in particular a strong association between low indoor temperatures and increased risk of strokes, heart attacks and respiratory illness. Other evidence shows cold stress causing cardiovascular strain and increased incidence of dust mites in poorly ventilated homes – in turn affecting asthma and eczema, especially in children. Based on an epidemiological approach, the Large Analysis and Review of European Housing and Health Status (LARES) study² shows that there is a significant relationship between dwelling energy efficiency, as approximated by thermal discomfort level, and physical health (Ezratty, Duburcq, Emery, and Lambrozo, 2009). This relationship takes the form of a negative link between thermal discomfort and the risk of respiratory and cardiovascular diseases, hypertension or the presence of digestive disorders. LARES also shows the same type of negative link between thermal discomfort and mental and social well-being.

By the same, Wilkinson, Armstrong, Stevenson, Pattenden, McKee, and Fletcher (2001) showed that there is a credible chain of causation that links low indoor temperatures due to residential energy inefficiency to cold-related deaths. In particular, there is a 23% excess of deaths from heart attacks and strokes. Indoor temperatures below 16°C are a particular risk and are most likely to affect old and poorly heated housing with low-income residents. Also, Howden-Chapman, Craneb, Mathesona, Viggersa, Cunninghamc, Blakelyd, O’Deaa, Cunninghame, Woodwardf, Saville-Smithg, Bakera, and Waipara (2005) focused on analyzing the consequences of insulation measures³ on health, the well-being of the occupants, as well as on their utilisation of health care.

By considering a discursive approach, Ezratty (2010) and Ormandy and Ezratty (2012) argue that housing conditions in general and residential energy efficiency in particular considerably affect physical and mental health as well as social well-being. In the same context, based on a meta-analysis dealing with the impact of household energy efficiency measures on health, Maidment, Jones, Webb, Hathway, and Gilbertson (2014) argue that household energy efficiency interventions led to a small but significant improvement in the health of residents. Liddell and Guiney (2015), based on a literature review of nine intervention studies that outline the current prevailing framework for understanding mental well-being in the fields of psychology and psychiatry, argue that living in an energy inefficiency dwelling *i.e.* cold and damp housing, contributes to a variety of different mental health stressors, including persistent worry about debt and affordability, thermal discomfort and worry about the consequences of cold and damp for health.

More recently, an effervescent literature considering different specific country-case studies confirms previous results and argues that there is a significant link between thermal energy efficiency and health. For example, Fisk, Singer, and Chan (2020) review empirical data from evaluations of the influence of

¹The other most used proxy in the literature is damp and mould growth.

²LARES is a pan-European housing and health survey that was undertaken from 2002 to 2003 in eight European cities at the initiative of the WHO European Housing and Health task force. It was designed to improve knowledge on the impacts of existing housing conditions on health and mental and physical well-being.

³Insulation measures affect indoor temperature, humidity, energy consumption and mould growth.

thermal energy efficiency retrofits on indoor environmental quality conditions and self-reported thermal comfort and health. He shows that average indoor temperatures during winter typically increased after retrofits and that dampness and mold almost always decreased after retrofits. They add that subjectively reported thermal comfort, thermal discomfort, non-asthma respiratory symptoms, general health, and mental health nearly always improved after energy efficiency retrofit.

Interestingly, some studies point out the need to consider the negative consequences of energy efficiency measures on health. In particular, Sharpe, Machray, Fleming, Taylor, Henley, Chenor, Hutchcroft, Taylor, Heaviside, and Wheeler (2019) conduct an area-level analysis of hospital admissions in England and conclude that despite a range of limitations and some mixed and contrasting findings across the national and local analyses, there was some evidence that areas with more energy efficiency improvements resulted in higher admission rates for respiratory and cardiovascular diseases. This builds on existing evidence highlighting the complex relationships between health and housing. While energy efficiency measures can improve health outcomes (especially when targeting those with chronic respiratory illness), reduced household ventilation rates can impact indoor air quality for example and increase the risk of diseases such as asthma.

In summary, a large body of empirical interdisciplinary studies, although using different approaches, provide vast evidence that indoor environmental quality, particularly residential energy inefficiency, have negative impacts on physical and mental health outcomes. Some of these studies highlight the particular the age’s impact, i.e. young versus elderly.

2.2 Health and labor productivity

The literature recognizing the impact of health on labor productivity is growing. In particular, during the last couple years there has been an important increase in the number of academic articles that focus on estimating the economic burden of illness, in particular, chronic diseases. This literature includes not only direct but also indirect costs of the disease (Li, Gignac, and Anis, 2006; Kirsten, 2010; Anis, Zhang, Bansback, Guh, Amarsi, and Birmingham, 2010). Indirect costs are now widely referred to as productivity losses (Drummond, Sculpher, Torrance, O’Brien, and Stoddart, 2015; Gold, Siegel, Russell, and Weinstein, 2016). Zhang, Bansback, and Anis (2011) argue that there is still a lack of detailed methodological guidance on how productivity loss should be measured. They review measurement issues and valuation methods for estimating productivity loss due to poor health and assert that in some cases, i.e. risk averse workers, job involving team production or unavailability of perfect substitutes, productivity loss is likely to be underestimated.

In the environmental economics field, in the vein of empirical works showing that pollution is a driver of bad health conditions, Graff Zivin and Neidell (2012) propose one of the first empirical studies which rigorously assess what they call the environmental productivity effect. More precisely, they assess the environmental pollution impact on worker productivity by linking the exogenous daily variations in ozone with worker productivity of agricultural workers, due to health deterioration. They find a significant evidence that ozone levels well below federal air quality standards have a significant impact on productivity. They argue that the empirical estimation of this relationship is complicated because obtaining clean measures of worker productivity is a perennial challenge and because the exposure to pollution levels is typically endogenous. Recently, Aguilar-Gomez, Dwyer, Graff Zivin, and Neidell (2022) review the economic research investigating the causal effects of pollution on labor productivity, cognitive performance and multiple forms of decision making. Regarding labor productivity, they particularly show that air pollution reduces worker productivity and, in some cases, labor supply. However, they point-out that productivity estimates vary considerably and that there are several possible explanations for this divergence such as differences in occupations, setting, pollutant of interest and study design. Chang, Graff Zivin, Gross, and Neidell (2016) also empirically show that particulate pollution have a negative effect on the productivity of workers at a pear-packing factory. Gibson and Shrader (2018) argue that the role that pollution may play in “sleep” disruption and its effects for labor productivity call for further investigations. By extension, they assert that a better understanding of who bears the costs of these effects would also help in identifying the incentives for private and public efforts to invest in, both, emissions control and exposure avoidance technologies.

Beyond the specific case of pollution as a driver of health conditions, Bhattacharya, Choudhry,

and Lakdawalla (2008) studied impacts of chronic disease and severe disability among working-age populations and show that chronic conditions are recognized as an important cause of work disability. Their conclusion supports the assumption of a negative impact of health on labor productivity. By the same, Zhang, Zhao, and Harris (2009) examine the impact of several chronic diseases, i.e. diabetes, cardiovascular diseases and mental illnesses on the probability of labour force participation using data from the Australian National Health Surveys. They show that the estimated effects are significant and that they differ by gender and age groups.

Considering a more general framework, Bloom, Canning, and Sevilla (2004); Bloom, Canning, Kotschy, Prettnner, and Schunemann (2019), Dormont, Oliveira Martins, Pelgrin, and Suhrcke (2010) and Weil (2007, 2014), amongst others, emphasize the relationship between health, labor productivity and economic growth. Bloom, Canning, and Sevilla (2004) show that good health has a positive, sizable and statistically significant effect on aggregate output and argue that the life expectancy effect in growth regressions appears to be a real labor productivity effect, rather than the result of life expectancy acting as a proxy for worker experience. Dormont, Oliveira Martins, Pelgrin, and Suhrcke (2010) highlight that health could be seen as a labour-augmenting factor increasing the level of individual productivity, even if its effect could not be sufficient to generate a growth enhancing mechanism. At micro-level Weil (2007) calculates that an increase in adult survival rates of 10% raises labor productivity by about 6.7 points and at macro-level Bloom, Canning, Kotschy, Prettnner, and Schunemann (2019) estimate that an increase of 10% of the adult survival rate would lead to a 9.1% increase in labor productivity at macro level.

To summarize, there is a growing body of literature literature showing the causality between health and labor productivity.

3 The basic model

The economy consists in an infinite sequence of overlapping generations. Each generation lives for three periods (de la Croix, 1996; De La Croix and Michel, 2002). The young generation has no decision to take, just lives with their parents and therefore benefits/suffers from residential energy conditions at parents' home. In the second period of her life, when she is adult, she supplies inelastically one unit of labor. She retires when old. Population evolves at a constant rate of growth $n \in]-1, +\infty[$ such that $N_{t+1} = (1 + n)N_t$.

3.1 Health-status and energy efficiency

In a first step of the analysis, we consider the simple case where poor energy efficiency makes people sick when adult and/or old, but illness is cured during the period, thanks to healthcare expenditures, and the probability to be sick when old is not related to previous health condition. These simplifying assumptions enable us to present the main mechanisms of the model. Then we will present, in section 6.2, a more realistic case where poor energy efficiency could lead to chronic diseases.

In our basic simplifying framework, each generation can be in “bad” health when adult with a probability π_t^a and when old with a probability π_{t+1}^o . Each probability is unrelated and it depends on the efficiency of energy services each generation lived with in the previous period. Then the probability of being sick for the adult generation at time t (respectively for the old generation at time $t + 1$) is defined as:

$$\pi_t^a = \pi^a(\varepsilon_{r,t-1}) \quad \text{and} \quad \pi_{t+1}^o = \pi^o(\varepsilon_{r,t}) \quad (1)$$

with $\pi_t^a(\varepsilon_{r,t-1}) < 0$ and $\pi_{t+1}^o(\varepsilon_{r,t}) < 0$. $\varepsilon_{r,t}$ (resp. $\varepsilon_{r,t-1}$) denotes residential energy efficiency in period t (resp. $t - 1$).

For convenience we normalize “good” health to unity and we assume that “bad” health of an adult generation (resp. old generation) is denoted by $h_t^a < 1$ (resp. $h_{t+1}^o < 1$). To restore her health, the adult generation (resp. old generation) with poor health incurs healthcare expenditures denoted by m_t^a (resp. m_{t+1}^o). We assume that the higher the detrimental effects of bad energy efficiency the higher

healthcare expenditures requires to recover health. Therefore we define:

$$m_t^a = m^a(\varepsilon_{r,t-1}) \quad \text{and} \quad m_{t+1}^o = m^o(\varepsilon_{r,t}) \quad (2)$$

with $m^a(\varepsilon_{r,t}) < 0$ and $m^o(\varepsilon_{r,t+1}) < 0$. This is in accordance with Gutiérrez (2008) and Wang, Zhao, and Bhattacharya (2015) who study the negative impact of ambient air pollution. We follow their assumption that healthcare expenditures enable agents to recover full health. Nevertheless, conversely to these authors, we assume that health recovery takes time. When sick, agents need a portion $z^a \in [0, \bar{z}^a]$ of their adult lifetime (resp. $z^o \in [0, \bar{z}^o]$ when old) to get full health back ($\bar{z}^i < 1$ for $i = a, o$).⁴ As a consequence, remembering that full health is normalized to unity, “bad” health-status during each period of life is defined as:

$$h_t^a = h^a = 1 - z^a > 0 \quad \text{and} \quad h_{t+1}^o = h^o = 1 - z^o > 0 \quad (3)$$

3.2 Households

At each period, adult and old generations consume non-energy goods and energy services whose efficiency may affect positively their health. The expected intertemporal utility of an adult is:⁵

$$EU_t = \pi_t^a \log \bar{c}_t^b + (1 - \pi_t^a) \log \bar{c}_t^g + \beta \left[\pi_{t+1}^o \log \bar{d}_{t+1}^b + (1 - \pi_{t+1}^o) \log \bar{d}_{t+1}^g \right] \quad (4)$$

where $\beta > 0$ is the subjective discounting parameter, and the composite consumption good of the adult generation and the old generation, respectively denoted by \bar{c}^i and \bar{d}^i , are given by:

$$\begin{aligned} \bar{c}_t^i &= \left[(1 - \nu) c_t^i \frac{\sigma_{C,E} - 1}{\sigma_{C,E}} + \nu \left(\varepsilon_{r,t} E_{r,t}^{a,i} \right) \frac{\sigma_{C,E} - 1}{\sigma_{C,E}} \right]^{\frac{\sigma_{C,E}}{\sigma_{C,E} - 1}} \quad \text{and} \\ \bar{d}_{t+1}^i &= \left[(1 - \nu) d_{t+1}^i \frac{\sigma_{C,E} - 1}{\sigma_{C,E}} + \nu \left(\varepsilon_{r,t+1} E_{r,t+1}^{o,i} \right) \frac{\sigma_{C,E} - 1}{\sigma_{C,E}} \right]^{\frac{\sigma_{C,E}}{\sigma_{C,E} - 1}} \quad \text{with} \quad \nu \in]0, 1[, \quad i = (b, g) \end{aligned}$$

c_t^i (respectively d_{t+1}^i) is the amount of non-energy goods the adult (resp. old) generation consumes with a health condition i (with $i = b$ when agents are in bad health and $i = g$ when agents are in good health). $E_{r,t}^{a,i}$ (respectively $E_{r,t+1}^{o,i}$) is the amount of energy services (whose efficiency is captured by ε_r) the adult (resp. old) generation consumes with a health condition i . $\sigma_{C,E} \geq 0$ is the elasticity of substitution between non-energy goods and energy consumptions. Parameter ν is the share of energy consumption in the composite consumption good.

The program of each adult generation is to maximize intertemporal utility (4) under the following per-period budget constraints:⁶

$$s_t + c_t^b + p_r E_{r,t}^{a,b} + m^a(\varepsilon_{r,t-1}) = w_t \quad (5a)$$

$$s_t + c_t^g + p_r E_{r,t}^{a,g} = w_t \quad (5b)$$

$$d_{t+1}^b + p_r E_{r,t}^{o,b} + m^o(\varepsilon_{r,t}) = R_{t+1} s_t \quad (5c)$$

$$d_{t+1}^g + p_r E_{r,t}^{o,g} = R_{t+1} s_t \quad (5d)$$

$$c_t^i \geq 0, \quad d_{t+1}^i \geq 0, \quad E_{r,t}^{a,i} \geq 0, \quad E_{r,t+1}^{o,i} \geq 0 \quad (5e)$$

⁴We assume exogenous recovery time for simplification. Assuming that z^a and z^o are functions of residential energy efficiency would not modify the qualitative insights of the model.

⁵Levy and Nir (2012) and Finkelstein, Luttmer, and Notowidigdo (2013) amongst others documented that health status enters utility function by affecting the marginal utility of consumption. We abstract from such an influence in this basic framework. We will relax this assumption later in section 6.1.

⁶Note that agents attend work while ill. This phenomenon called *illness presenteeism* is well documented (Arnold, 2016; Aronsson and Gustafsson, 2005; Gosselin, Lemyre, and Corneil, 2013; Eurofound, 2012). To avoid complexity, we assume that the sick worker earns a wage equal to the wage of the healthy worker because firms are not able to discriminate between sick and healthy at work and therefore pay workers at the average productivity of the global workforce. This is consistent with empirical evidence (see Zhang, Sun, Woodcock, and Anis, 2015, for example). Alleviate this assumption, especially assuming that sick people earns a wage related to their productivity (which relies on their health status h^a), that is $w_t h^a$ would not modify the qualitative results but would complexify the exposition of the model.

where $R_{t+1} = 1 + r_{t+1}$ (with r_{t+1} the real interest rate), w_t is real wage, p_r is energy price. Budget constraints (5a) and (5c) (respectively (5b) and (5d)) represent budget constraints of sick (resp. healthy) people respectively when adult and when old.

The resolution of the decision problem leads to (see Appendix A):

$$\begin{aligned} E_{r,t}^{a,i} &= \mathfrak{E}(\varepsilon_{r,t})c_t^i & \text{with} & \quad \mathfrak{E}(\varepsilon_r) \equiv \left(\frac{\nu}{1-\nu}\right)^{\sigma_{c,E}} p_r^{-\sigma_{c,E}} \varepsilon_r^{\sigma_{c,E}-1} \text{ and } i = (b, g) \\ E_{r,t+1}^{o,i} &= \mathfrak{E}(\varepsilon_{r,t+1})d_{t+1}^i \end{aligned} \quad (6)$$

and

$$\begin{aligned} \bar{c}_t^i &= \mathfrak{U}(\varepsilon_{r,t})c_t^i & \text{with} & \quad \mathfrak{U}(\varepsilon_r) \equiv \left[(1-\nu) + \nu (\varepsilon_r \mathfrak{E}(\varepsilon_r))^{\frac{\sigma_{c,E}-1}{\sigma_{c,E}}} \right]^{\frac{\sigma_{c,E}}{\sigma_{c,E}-1}} \\ \bar{d}_{t+1}^i &= \mathfrak{U}(\varepsilon_{r,t+1})d_{t+1}^i \end{aligned} \quad (7)$$

LEMMA 1. $\partial \mathfrak{E}(\varepsilon_r)/\partial \varepsilon_r \leq 0$ for $\sigma_{c,E} \leq 1$ and $\partial \mathfrak{U}(\varepsilon_r)/\partial \varepsilon_r > 0 \forall \sigma_{c,E}$.

Proof. From equations (6) and (7). □

Using budget constraints and (6), the expression of saving chosen by the adult generation is:

$$\begin{aligned} \left(\frac{\mathfrak{U}(\varepsilon_{r,t})}{\mathfrak{U}(\varepsilon_{r,t+1})}\right)^{-\frac{\sigma_{c,E}-1}{\sigma_{c,E}}} (1 + p_r \mathfrak{E}(\varepsilon_{r,t})) & \left\{ \frac{\pi^a(\varepsilon_{r,t-1})}{w_t - s_t - m^a(\varepsilon_{r,t-1})} + \frac{1 - \pi^a(\varepsilon_{r,t-1})}{w_t - s_t} \right\} \\ - R_{t+1} \beta (1 + p_r \mathfrak{E}(\varepsilon_{r,t+1})) & \left\{ \frac{\pi^o(\varepsilon_{r,t})}{R_{t+1} s_t - m^o(\varepsilon_{r,t})} + \frac{1 - \pi^o(\varepsilon_{r,t})}{R_{t+1} s_t} \right\} = 0 \end{aligned} \quad (8)$$

3.3 Firms

Firms produce an homogenous good, denoted Y , used to final and energy consumption in residential and producing sectors as well as to physical capital accumulation. They operate under perfect competition with a technology defined by the following nested Constant Elasticity Substitution (CES) production function:

$$Y_t = A \left[(1 - \eta) \left(A_Q K_t^\alpha (B_t L)^{1-\alpha} \right)^{\frac{\sigma_{KL,E}-1}{\sigma_{KL,E}}} + \eta (\varepsilon_f E_{f,t})^{\frac{\sigma_{KL,E}-1}{\sigma_{KL,E}}} \right]^{\frac{\sigma_{KL,E}}{\sigma_{KL,E}-1}} \quad \text{with} \quad (\eta, \alpha) \in]0, 1[, \quad (9)$$

where K_t denotes the aggregate stock of physical capital, L is labor force (exogenous because adult generation supplies inelastically on unit of labor), E_f the energy consumption in production and ε_f its efficiency. Physical capital fully depreciates during the period. We note $\sigma_{KL,E} \geq 0$ the elasticity of substitution between capital/labor and energy.⁷

Relying on the empirical evidence reported in introduction, we assume that the aggregate productivity of labor, B_t , is related to workers' health-status. Because of uncertainty on health-status, B_t is actually the expected labor productivity and is defined as:⁸

$$B_t = \mathcal{B}(\varepsilon_{r,t-1}) \equiv B [1 - z^a \pi^a(\varepsilon_{r,t-1})]^\psi \quad \text{with} \quad \mathcal{B}'(\varepsilon_{r,t-1}) \geq 0, \quad \psi \in [0, 1] \quad (10)$$

Parameter ψ measure the intensity of health externality on labor productivity and parameter B measures labor productivity when health does not influence labor productivity. In the presence of instantaneous full recovery, we have $z^a = 0$ and, therefore, labor productivity is independent from the probability to be sick. Conversely, when full recovery takes time, $z^a \in]0, 1[$, labor productivity depends on the probability to be sick and increases in residential energy efficiency.

⁷We assumed that the technology linking physical capital and labor is Cobb-Douglas for convenience. Assuming that the elasticity of substitution between capital and labor is different from unity would not modify the qualitative results of the model. Proof upon request.

⁸We have $B_t = B [(1 - \pi^a(\varepsilon_{r,t-1})) \times 1 + \pi^a(\varepsilon_{r,t-1})h^a]$ with h^a given by equation (3).

Firms maximize their profits $Y_t - R_t K_t - w_t L_t - p_f E_{f,t}$, where p_f is the price of energy in production, leading to the following (inverse) demands:

$$R_t = \mathcal{R}(k_t, \varepsilon_{r,t-1}, \varepsilon_{f,t}) \equiv \mathcal{A}(\varepsilon_{f,t}) \alpha k_t^{\alpha-1} \mathcal{B}(\varepsilon_{r,t-1})^{1-\alpha} \quad (11a)$$

$$w_t = \mathcal{W}(k_t, \varepsilon_{r,t-1}, \varepsilon_{f,t}) \equiv \mathcal{A}(\varepsilon_{f,t}) (1 - \alpha) k_t^\alpha \mathcal{B}(\varepsilon_{r,t-1})^{1-\alpha} \quad (11b)$$

$$E_{f,t} = \frac{A_Q k_t^\alpha \mathcal{B}(\varepsilon_{r,t-1})^{1-\alpha}}{\varepsilon_f \Omega(\varepsilon_f)} \quad (11c)$$

with $k \equiv K/L$ the ratio capital labor and

$$\Omega(\varepsilon_f) \equiv \left(\frac{\left(\frac{p_f}{A \eta \varepsilon_f} \right)^{\sigma_{\text{KL},\text{E}} - 1} - \eta}{1 - \eta} \right)^{\frac{\sigma_{\text{KL},\text{E}}}{\sigma_{\text{KL},\text{E}} - 1}} > 0 \quad (12a)$$

$$\mathcal{A}(\varepsilon_{f,t}) \equiv (1 - \eta) A_Q A \left(\frac{1 - \eta \left(\frac{p_f}{A \eta \varepsilon_f} \right)^{1 - \sigma_{\text{KL},\text{E}}}}{1 - \eta} \right)^{\frac{1}{1 - \sigma_{\text{KL},\text{E}}}} \quad (12b)$$

LEMMA 2.

1. $\partial \Omega(\varepsilon_f) / \partial \varepsilon_f < 0$ and $\partial \Omega(\varepsilon_f) / \partial p_f > 0$, $\forall \sigma_{\text{KL},\text{E}}$.
2. $\partial \mathcal{A}(\varepsilon_{f,t}) / \partial \varepsilon_f > 0$ and $\partial \mathcal{R}(k_t, \varepsilon_{r,t-1}, \varepsilon_{f,t}) / \partial k_t < 0$ and $\partial \mathcal{R}(k_t, \varepsilon_{r,t-1}, \varepsilon_{f,t}) / \partial \varepsilon_r > 0$ and $\partial \mathcal{R}(k_t, \varepsilon_{r,t-1}, \varepsilon_{f,t}) / \partial \varepsilon_f > 0$.
3. $\partial \mathcal{W}(k_t, \varepsilon_{r,t-1}, \varepsilon_{f,t}) / \partial \varepsilon_f > 0$ and $\partial \mathcal{W}(k_t, \varepsilon_{r,t-1}, \varepsilon_{f,t}) / \partial k_t > 0 \forall \sigma_{\text{KL},\text{E}}$ and $\partial \mathcal{W}(k_t, \varepsilon_{r,t-1}, \varepsilon_{f,t}) / \partial \varepsilon_r > 0$.

Proof. Straightforward from expression of $\Omega(\varepsilon_f)$ in (12a) and Lemma 2.1. \square

3.4 The general equilibrium

Because each adult supplies one unit of labor and the size of adult population at time t is N_t , labor market equilibrium implies that:

$$L_t = N_t \quad (13)$$

Finally, general equilibrium condition is:

$$N_t s_t = K_{t+1} \quad (14)$$

Defining $\mu^i(\varepsilon_r)$ as the ratio healthcare expenditures to non energy consumption for generation $i = a, o$, such that $\mu^a(\varepsilon_{r,t-1}) \equiv m^a(\varepsilon_{r,t-1})/c^b$ and $\mu^o(\varepsilon_{r,t}) \equiv m^o(\varepsilon_{r,t})/d^b$ with $\mu^a(\varepsilon_{r,t-1}), \mu^o(\varepsilon_{r,t}) \in]0, 1[$ and $\mu^{i'}(\varepsilon_r) < 0$ ($i = a, o$), it comes from equation (8) and the budget constraints (5a) to (5e):⁹

PROPOSITION 1.

$$s_t = \frac{\beta}{\Psi(\varepsilon_{r,t-1}, \varepsilon_{r,t}, \varepsilon_{r,t+1}) + \beta} \mathcal{W}(k_t, \varepsilon_{r,t-1}, \varepsilon_{f,t}) \quad (15)$$

where

$$\Psi(\varepsilon_{r,t-1}, \varepsilon_{r,t}, \varepsilon_{r,t+1}) \equiv \left[\frac{\mathfrak{U}(\varepsilon_{r,t})}{\mathfrak{U}(\varepsilon_{r,t+1})} \right]^{\frac{1 - \sigma_{\text{C},\text{E}}}{\sigma_{\text{C},\text{E}}}} \times \left\{ \frac{\pi^a(\varepsilon_{r,t-1}) \mu^a(\varepsilon_{r,t-1}) + 1 + p_r \mathfrak{E}(\varepsilon_{r,t})}{\pi^o(\varepsilon_{r,t}) \mu^o(\varepsilon_{r,t}) + 1 + p_r \mathfrak{E}(\varepsilon_{r,t+1})} \right\},$$

⁹We introduced the ratio healthcare expenditures to non energy consumption for convenience, in order to obtain an explicit expression of saving and clarify the exposition of the main mechanisms. Rather keeping $m^{i'}(\varepsilon_r)$ ($i = a, o$) would not modify the qualitative results we obtain but would not enable us to have an explicit expression of saving with no substantial gains. Proof upon request.

Proof. From equation (8). □

Proposition 1 highlights that residential energy efficiency affects savings through two channels, both linked to health. The first influence goes through the propensity to save (the first ratio) like in Kotlikoff (1989). We call it the *saving propensity effect* of the detrimental impact of low residential energy efficiency on health. On the one hand, the influence of $\varepsilon_{r,t-1}$ and $\varepsilon_{r,t}$ is directly derived from the probability to be sick (respectively when adult and old) as shown by equation (8). On the other hand, the influence of $\varepsilon_{r,t+1}$ and a second influence of $\varepsilon_{r,t}$ comes from the first ratio of the left-hand side of equation (8) which represents the relative importance of the ratio of the composite consumption good to non-energy consumption $\mathfrak{U}(\cdot)$ at date t and $t + 1$ (see equation (7)).¹⁰

A second influence goes through wage $\mathcal{W}(k_t, \varepsilon_{r,t-1}, \varepsilon_{f,t})$ and is linked to the health externality on labor productivity; a greater residential energy efficiency at $t - 1$ will reduce the probability of the adults to be sick at t and therefore will rise their expected labor productivity and their wage. It will lead to higher saving. We call this second influence the *disposal income effect*.

Equations (14) and (15) define the law of motion of per capita physical capital:

$$k_{t+1} = \frac{\beta \mathcal{A}(\varepsilon_{f,t})(1 - \alpha) \mathcal{B}(\varepsilon_{r,t-1})^{1-\alpha}}{(1 + n)(\Psi(\varepsilon_{r,t-1}, \varepsilon_{r,t}, \varepsilon_{r,t+1}) + \beta)} k_t^\alpha \quad (16)$$

where the right-hand side of the equality is increasing and concave with respect to k_t for realistic values of $\sigma_{\kappa,L}$.

4 The long-term equilibrium

Long-term equilibrium is such as $\varepsilon_{f,t} = \varepsilon_f^*$ and $\varepsilon_{r,t} = \varepsilon_{r,t+1} = \varepsilon_r^*$ and $k_t = k_{t+1} = k^*$ defined by:

$$k^* = \mathcal{B}(\varepsilon_r^*) \left[\frac{\beta \mathcal{A}(\varepsilon_f^*)(1 - \alpha)}{(1 + n)(\Psi(\varepsilon_r^*) + \beta)} \right]^{\frac{1}{1-\alpha}} \quad \text{with} \quad \Psi(\varepsilon_r^*) = \frac{\pi^a(\varepsilon_r^*)\mu^a(\varepsilon_r^*) + 1 + p_r \mathfrak{E}(\varepsilon_r^*)}{\pi^o(\varepsilon_r^*)\mu^o(\varepsilon_r^*) + 1 + p_r \mathfrak{E}(\varepsilon_r^*)} \quad (17)$$

LEMMA 3. We have $\partial \Psi(\varepsilon_r^*) \leq 1$ and $\partial \Psi(\varepsilon_r^*) / \partial \varepsilon_r^* \geq 0$ when $\pi^o(\varepsilon_r^*)\mu^o(\varepsilon_r^*) \geq \pi^a(\varepsilon_r^*)\mu^a(\varepsilon_r^*)$.

Proof. From Lemma 2.1. □

Lemma 3 means that if the expected health expenditures expressed in terms of consumption when old ($\pi^o(\varepsilon_r^*)\mu^o(\varepsilon_r^*)$) is higher than when adult ($\pi^a(\varepsilon_r^*)\mu^a(\varepsilon_r^*)$), the propensity to save is higher. Thus, agents make precautionary saving when adult in order to fund expected health care expenditures when old. Conversely, if the expected health expenditures expressed in terms of consumption when old ($\pi^o(\varepsilon_r^*)\mu^o(\varepsilon_r^*)$) is lower than when adult ($\pi^a(\varepsilon_r^*)\mu^a(\varepsilon_r^*)$), the propensity to save is lower because adult have to face higher current health expenditures. This is consistent with results found by Kotlikoff (1989) when health risk only exists for old. It is interesting to note that an increase in the probability to be sick in adulthood will decrease savings *ceteris paribus*.

We obtain:

PROPOSITION 2. When energy efficiency positively affects health ($\pi^a > 0$ and/or $\pi^o > 0$):

$$k^* = \kappa(\varepsilon_r^*, \varepsilon_f^*) \quad \text{with} \quad \kappa_{\varepsilon_f^*}(\cdot) > 0, \kappa_{\varepsilon_r^*}(\cdot) \geq 0 \quad (18)$$

Otherwise ($\pi^a = 0$ and $\pi^o = 0$):

$$k^* = \kappa^{nh}(\varepsilon_f^*) \quad \text{with} \quad \kappa_{\varepsilon_f^*}^{nh}(\cdot) > 0 \quad (19)$$

Proof. From equation (17). With no health effect of residential energy efficiency, equation (17) simplifies to $k^* = B \left[\frac{\beta \mathcal{A}(\varepsilon_f^*)(1-\alpha)}{(1+n)(1+\beta)} \right]^{\frac{1}{1-\alpha}}$. □

¹⁰This term vanishes at the steady-state equilibrium because $\varepsilon_{r,t} = \varepsilon_{r,t+1} = \varepsilon_r^*$.

COROLLARY 1. When energy efficiency positively affects health ($\pi^a > 0$ and/or $\pi^o > 0$):

$$w^* = \mathcal{W}(\varepsilon_r^*, \varepsilon_f^*) \quad \text{with} \quad \mathcal{W}_{\varepsilon_f^*}(\cdot) > 0, \mathcal{W}_{\varepsilon_r^*}(\cdot) \geq 0 \quad (20a)$$

$$R^* = \mathcal{R}(\varepsilon_r^*, \varepsilon_f^*) \quad \text{with} \quad \mathcal{R}_{\varepsilon_f^*}(\cdot) < 0, \mathcal{R}_{\varepsilon_r^*}(\cdot) > 0 \quad (\text{if } \pi^o(\varepsilon_r^*)\mu^o(\varepsilon_r^*) > \pi^a(\varepsilon_r^*)\mu^a(\varepsilon_r^*)) \quad (20b)$$

$$s^* = \mathcal{S}(\varepsilon_r^*, \varepsilon_f^*) \quad \text{with} \quad \mathcal{S}_{\varepsilon_f^*}(\cdot) > 0, \mathcal{S}_{\varepsilon_r^*}(\cdot) \geq 0 \quad (20c)$$

Otherwise ($\pi^a = 0$ and $\pi^o = 0$):

$$w^* = \mathcal{W}^{nh}(\varepsilon_f^*) \quad \text{with} \quad \mathcal{W}_{\varepsilon_f^*}^{nh}(\cdot) > 0 \quad (20d)$$

$$R^* = \mathcal{R}^{nh}(\varepsilon_f^*) \quad \text{with} \quad \mathcal{R}_{\varepsilon_f^*}^{nh}(\cdot) < 0 \quad (20e)$$

$$s^* = \mathcal{S}^{nh}(\varepsilon_f^*) \quad \text{with} \quad \mathcal{S}_{\varepsilon_f^*}^{nh}(\cdot) > 0 \quad (20f)$$

Proof. From equation (17). □

The indeterminate impact of $\Delta^+ \varepsilon_r^*$ on long-term physical capital, savings and the other key variables is linked to the two opposite effects we discussed in the previous section (captured by $\mathcal{B}(\varepsilon_r^*)$ and $\Psi(\varepsilon_r^*)$ in the expression of k^* at equation 17). The “saving propensity effect” (captured by $\Psi(\varepsilon_r^*)$) impacts negatively saving when the expected health expenditures expressed in terms of consumption at old age is greater than the expected health expenditures expressed in terms of consumption at adult age. An increase in residential energy efficiency will reduce the probability to be sick for both adults and elderly, leading to a decrease in precautionary saving. The “disposal income effect” (captured by $\mathcal{B}(\varepsilon_r^*)$) impacts positively savings. A increase in residential energy efficiency will reduce the probability to be sick for both adults and elderly, labor productivity of adults rises and therefore wage. The global effect is hard to disentangle analytically. Without health effect, steady-state capital labor ratio is not affected by residential energy efficiency.

The expected per capita residential energy consumption at the steady-state (both young and old of the previous generation) is:

$$E_r^* = \underbrace{\frac{\mathfrak{E}(\varepsilon_r^*)}{1 + p_r \mathfrak{E}(\varepsilon_r^*)}}_I \left\{ \underbrace{\frac{\Psi(\varepsilon_r^*) \mathcal{W}(\varepsilon_r^*, \varepsilon_f^*)}{\Psi(\varepsilon_r^*) + \beta}}_{II^a} \left[1 - \pi^a(\varepsilon_r^*) \frac{\overbrace{\mu^a(\varepsilon_r^*)}}{III^a}}{1 + p_r \mathfrak{E}(\varepsilon_r^*) + \mu^a(\varepsilon_r^*)} \right] + \underbrace{\frac{\mathcal{R}(\varepsilon_r^*, \varepsilon_f^*) \mathcal{S}(\varepsilon_r^*, \varepsilon_f^*)}{1 + n}}_{II^o} \left[1 - \pi^o(\varepsilon_r^*) \frac{\mu^o(\varepsilon_r^*)}{\underbrace{1 + p_r \mathfrak{E}(\varepsilon_r^*) + \mu^o(\varepsilon_r^*)}_{III^o}} \right] \right\} \quad (21)$$

Equation (21) highlights the different channels through which residential energy efficiency in the long-run ε_r^* affect the total energy consumption in the residential sector:¹¹

1. Independently from health effect, E_r^* is directly influenced by the demand effect (I in equation (21)) according to which an increase in residential energy efficiency will lead to a decrease in residential energy consumption if residential consumption is weakly substitutable with non-energy consumption $\sigma_{c,E} < 1$.
2. A second channel goes through general equilibrium effects (II^a and II^o in equation (21)) via the incomes of adult generation ($w^* - s^*$) and old generation ($R^* s^*$) which are indirectly affected by ε_r^* through health (see corollary 1).
3. Finally a third channel arises from the detrimental effect of bad residential energy efficiency on health for adult generation (III^a) and old generation (III^o). We expect that III^a and III^o will

¹¹It is clear that what we call here channels II and III are directly affected by the “saving propensity effect” and the “disposal income effect” we previously highlighted.

lead to positive impacts of ε_r^* on residential energy consumption in the long-term because a higher ε_r^* will reduce the expected healthcare expenditures for both generations and then, everything being equal elsewhere, will increase the resources to spend in both types of consumption.

Finally from (11c) and (13), steady-state per capita energy consumption in production:

$$E_f^* = \frac{A_q \mathcal{B}(\varepsilon_r^*)}{\varepsilon_f^* \Omega(\varepsilon_f^*)} \left[\frac{\beta \mathcal{A}(\varepsilon_f^*) (1 - \alpha)}{(1 + n) (\Psi(\varepsilon_r^*) + \beta)} \right]^{\frac{\alpha}{1 - \alpha}} \quad (22)$$

Long-term energy consumption in the production sector is influenced by the energy efficiency in the residential sector through the positive probability to be sick in the presence of a low energy efficiency in the residential sector. This influence goes through two channels. A direct channel is associated with the health externality on labor productivity (captured by the term $\mathcal{B}(\varepsilon_r^*)$). An indirect channel is linked to the “saving propensity effect” which affects savings and then long-term physical capital (captured by the term $\Psi(\varepsilon_r^*)$).

5 Numerical exercises

In this section, we present numerical simulations of our theoretical model that we calibrate on the U.S. economy for the period going from 1985 to 2018. In sub-section 5.1, we explain calibration of preference, technology and energy parameters. In sub-section 5.2, we discuss results.

5.1 Calibration

Preference parameters: The value of the elasticity of substitution between energy and non-energy consumption in utility comes from Lemoine (2020) and is very close to the value reported by de Miguel and Manzano (2011). Following De La Croix and Michel (2002) we assume that the quarterly psychological discount factor is equal to 0.99. The parameter β is thus equal to $0.99^{120} = 0.3$. Finally, we calibrate energy consumption share in utility, ν , in order to match the average value of the residential energy consumption to GDP ratio in the U.S. economy during the period 1985-2018. The residential energy consumption is extracted from the U.S. Energy Information Administration (EIA) State Energy data set. The real GDP comes from the Federal Reserve Economic Data (FRED).

Technology parameters: We follow the Real Business Cycle literature to set the share of physical capital in production a at $1/3$. As for elasticities of substitution in the production function, we consider the elasticity of substitution between energy E_f and non-energy factors Z as well as between physical capital and labor. The question of what values to attribute to these elasticities is the subject of recurrent debate in the current state of empirical literature. As a consequence, in our model, we use as benchmark values the estimations made by van der Werf (2008) for the U.S. and we will also investigate alternative values estimated by other authors. Otherwise, we approximate the share of energy in industrial production a by the average value of the ratio energy expenditures in the industrial sector to GDP during the period 1985-2018, using the data from the U.S. EIA State Energy Data set. Following De La Croix and Michel (2002); Wang, Zhao, and Bhattacharya (2015), we set a steady-state target value of k around 1.2 and with a steady-state quarterly interest rate equal to 1% (that is $R^* = (1.01)^{120}$) from equation (11a), we get the labor productivity parameter $B = 3.84$ for scale parameters $A = 9.155$ and $A_q = 0.44$.

Energy parameters: Values of energy efficiencies are extracted from the 2016 American Council for an Energy-Efficient Economy (ACEEE) report. Energy prices are computed as the average value of prices (respectively for residential and for industrial sector) during the period 1985-2018. They come from the 2018 U.S. EIA State Energy Data set.

Health parameters: The impact of energy efficiency on health is less documented than the detrimental influence of air pollution especially with respect to the specification of the health risk function. That is why we make the simple assumption of a sigmoidal negative influence of energy efficiency on the probability to becoming sick which captures non linear impact. We will investigate the influence of different sets of parameter value to check the robustness of numerical simulations. We discuss the shape of the health risk function in Appendix B. Especially, we assumed that when $\varepsilon_r < 0.2$ the probability to becoming sick is at its maximum ($\pi^a = 1$) and when $\varepsilon_r > 0.8$, the probability to becoming sick is null ($\pi^a = 0$).¹² We always assume that for a given ε_r , the probability to becoming sick is higher for the elderly. Following Gutiérrez (2008), we consider that health expenditures are linear with respect to energy efficiency, such that we define $m^a(\varepsilon_{r,t-1}) \equiv \tilde{m}^a \times (1 - \varepsilon_{r,t-1})$ with $\tilde{m}^a > 0$ and $m^o(\varepsilon_{r,t}) \equiv \tilde{m}^o \times (1 - \varepsilon_{r,t})$ with $\tilde{m}^o > 0$. Similarly, we assume that recovery time is linearly linked to energy efficiency: $z^a \equiv \tilde{z}^a \times (1 - \varepsilon_{r,t-1})$ with $\tilde{z}^a > 0$ and $z^o \equiv \tilde{z}^o \times (1 - \varepsilon_{r,t})$ with $\tilde{z}^o > 0$. Parameters \tilde{z}^j ($j = \{a, o\}$) are arbitrarily fixed such that $\tilde{z}^a = 0.3$ and $\tilde{z}^o = 0.5$ (elderly remain sick a longer time than adults) and parameters \tilde{m}^j match steady-state targets. Finally, we set arbitrarily the intensity of health externality on labor productivity to the medium value $\psi = 0.5$ even of empirical evidence would suggest that it is stronger (see section 2.2). We will investigate how different values of ψ could modify our numerical simulations.

In Table 1, we summarize benchmark parameter values and their sources.

Table 1: Benchmark parameter values

Parameter		Value	Source
Preference			
- Elasticity of substitution between energy and non-energy consumption	$\sigma_{C,E}$	0.9	Lemoine (2020)
- Subjective rate of time preference	β	0.30	De La Croix and Michel (2002)
- Energy consumption share	ν	0.144	<i>Matches steady-state targets</i>
Technology			
- Share of physical capital in production	α	0.30	De La Croix and Michel (2002)
- Scale parameter	A	9.155	<i>matches steady-state targets</i>
- Scale parameter	A_Q	0.44	<i>matches steady-state targets</i>
- Labor productivity parameter	B	3.84	<i>matches steady-state targets</i>
- Share of energy in industrial production	η	0.4787	<i>matches steady-state targets</i>
- Elasticity of substitution between energy and non-energy in production	$\sigma_{KL,E}$	0.5470	van der Werf (2008)
Energy			
- Energy efficiency in the residential sector	ε_r	0.65	ACEEE (2016)
- Energy efficiency in the industrial sector	ε_f	0.7	ACEEE (2016)
- Unitary price of residential energy services	p_r	16.83	U.S. EIA State Energy Data
- Unitary price of firm energy services	p_f	8.11	U.S. EIA State Energy Data
Health			
- Adult health expenditures (% of consumption)	μ^a	0.0673	U.S. BLS & author calculus
- Old health expenditures (% of consumption)	μ^o	0.1270	U.S. BLS & author calculus
- Adult recovery time constant	\tilde{z}^a	0.3	<i>arbitrary fixed</i>
- Old recovery time constant	\tilde{z}^o	0.5	<i>arbitrary fixed</i>
- Externality of health on labor productivity	ψ	0.5	<i>arbitrary fixed</i>

5.2 Main results

Figure 1 below summarizes health channels through which energy efficiency effect on energy consumption occurs.

Figure 2 reports the results of our numerical exercise where the benchmark case is represented by blue plain curves and the “no health effect” case is represented by the red dashed curve. Figure 2 reports the steady-state values of the physical capital, energy and non-energy consumptions as well as welfare, with respect to the value of residential energy efficiency.

We firstly investigate the case where the residential energy efficiency does not affect health (red dashed curves). In such case, our results show that the steady-state per capita physical capital and energy consumption in production are not affected by residential energy efficiency (see graphs 1 and 3 in Figure 2). Welfare and non-energy consumption during adulthood and in the old-age are slightly positively influenced by the residential energy efficiency, for low values of ε_r^* (see graphs 4 to 6 in Figure 2). Finally, residential energy consumption diminishes with residential energy efficiency (see graph 2 in Figure 2). Interestingly, variables which were not affected by, or just slightly affected by residential

¹²That is the reason why, in subsequent figures, only the range of ε_r between 0.2 and 0.8 is of interest.

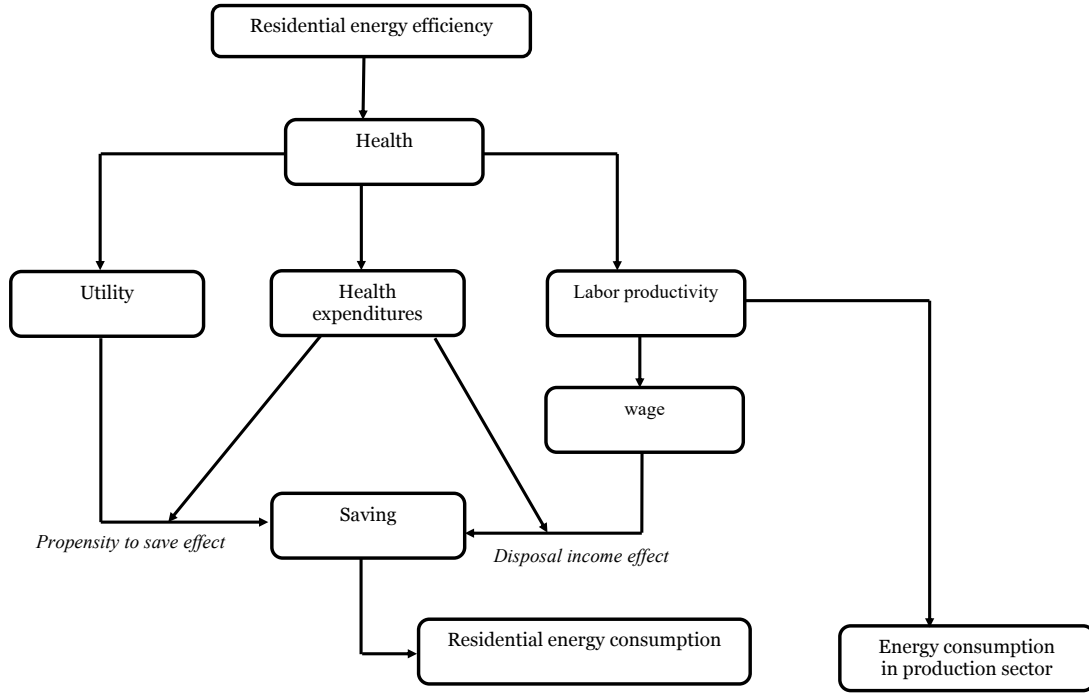


Figure 1. Basic mechanisms

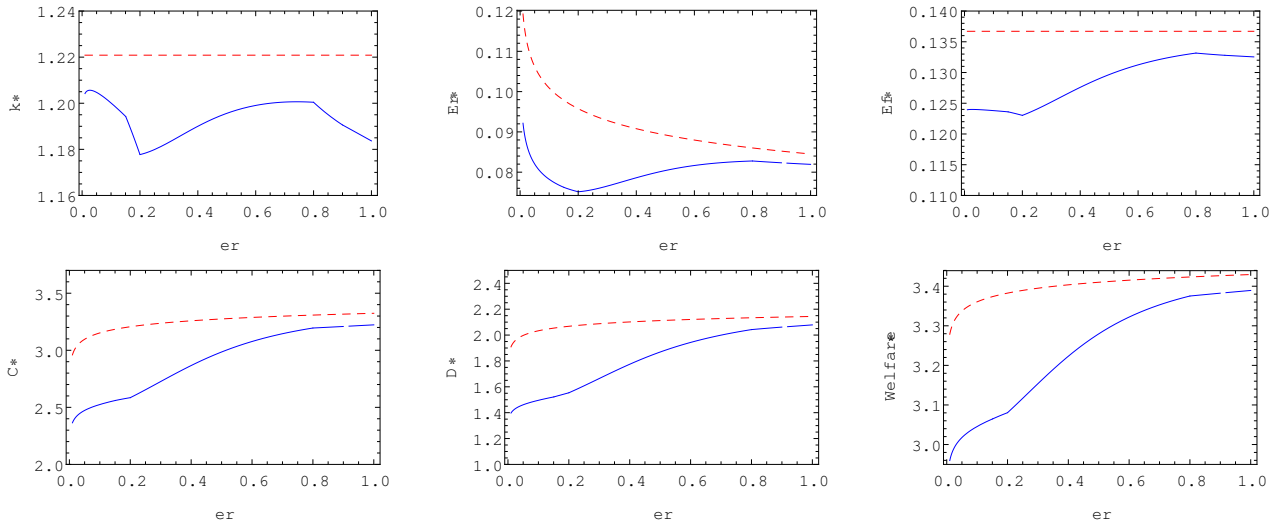


Figure 2: Influence of ε_r (Benchmark case (blue) vs no health effect (red large dashes))

energy efficiency in the “no health” case are now significantly affected positively when health is taken into account. In particular, a rise in residential energy efficiency in the presence of detrimental health effect would increase energy consumption in both sectors (see graphs 2 and 3 in Figure 2).

When health is negatively impacted by residential energy efficiency, all steady-state variables (blue plain curves in Figure 2) are lower with respect to the “no health effect” case. In fact, the probability of being sick induced by residential energy inefficiency reduces non-energy consumption and savings, i.e. health expenditures and lower productivity of labor which reduces wage and interest rate, everything being equal. As a result, per capita physical capital, energy consumptions and welfare are reduced as well.

In the next subsections, we investigate how channels highlighted above operate to give these patterns.

5.2.1 The “saving propensity effect”: Adulthood sickness vs old-age sickness

In his seminal article, Kotlikoff (1989) assumed that only elderly are sick. As a consequence, he demonstrated that elderly decide to save more when young, i.e. precautionary saving. Assuming as we did, that young individuals could be sick as well, has significant implications for saving decisions and for the influence of residential energy efficiency on energy and non-energy consumptions.

In Figure (3), we plot the results of numerical simulations when we assume either only elderly are sick due to low residential energy efficiency when young (tiny dashed line) or only young are sick (straight line). We compare both situations to the conventional case with no detrimental health effect of low residential energy efficiency (dashed line).

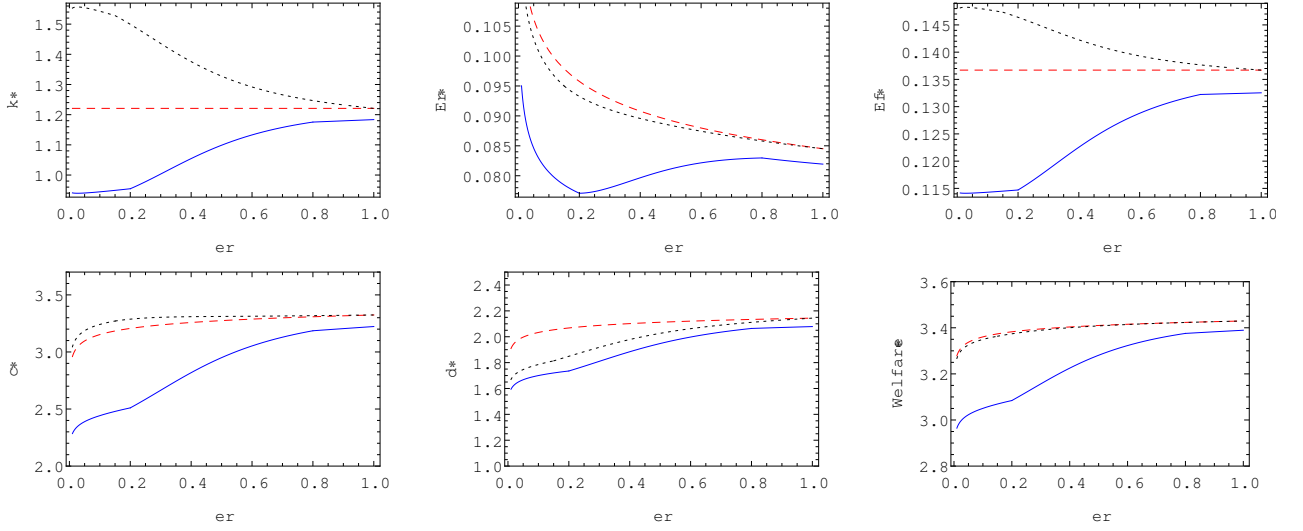


Figure 3: “Sickness by age” and influence of ϵ_r (health effect during adulthood only (blue), no health effect during old age only (black tiny dashed) and no health effect (red large dashes))

The first graph of Figure (3) shows the influence of illness on saving. If agents are sick only when old, they increase their saving for precautionary purpose in their adulthood. As a consequence, the steady-state of physical capital is higher than its level with no health effect. Conversely, if agents are sick only when adult, they reduce saving in order to fund their health expenditures. Therefore, the steady-state level of physical capital is lower than its level with no health effect. In the benchmark case, agents are sick during adulthood and old age and, globally, the detrimental impact of low residential energy efficiency reduces saving, the lower the residential energy efficiency is.

As for impacts on energy consumption, we firstly note that energy consumption in production evolves like physical capital. Then, if considering the residential energy consumption, we also note that when agents are sick only in old age, energy consumption is not quite different than its level with no health effects. It is very different when agents are sick in adulthood only. In this case, their consumption of energy is very low when residential energy efficiency is low and it rises with it, at the opposite of the case with no health effects. This can be explained by the shape of non-energy consumptions in both ages, which are very reduced with respect to “no-health effects” case (see graphs 4 and 5). Such a pattern comes from health expenditures in adulthood which reduce both adulthood and old-age consumption. As noted by Kotlikoff (1989), when only elderly have a probability to be sick, they have precautionary savings. As a consequence, physical capital is higher than its value with “no health effect” (see the black dotted curve with respect to the dashed red curve in graph 1 of Figure 3) and because it rises wages. It also rises non-energy consumption c^* in the adult period (see graph 4 in Figure 3) while non-consumption in old-age period d^* is reduced (see graph 5 in Figure 3). Globally, when agents has a probability to be sick only when old, welfare is not slightly different from the “no health effect” case (see the black dotted curve with respect to the dashed red curve in graph 6 of Figure 3). The evolution of adulthood consumption with respect to ϵ_r^* explains why residential energy consumption decreases in ϵ_r^* when only old-age are sick.

Conversely, when agents has a probability to be sick only when adult, they do not make precau-

tionary saving but rather reduce their savings to fund current health expenditures. As a consequence steady-state per capita physical capital and non-energy consumptions are lower than their respective “no health effect” values (see the black dotted curve with respect to the dashed red curve respectively in graph 1 and in graph 4 of Figure 3). That explains the lower values of energy consumptions and welfare (see the black dotted curve with respect to the dashed red curve respectively in graph 2, 3 and in graph 6 of Figure 3). Because a higher residential energy efficiency (ε_r^*) reduces the probability to be sick, thus, decreases health expenditures and rising savings, all variables are positively affected by ε_r^* .

Because our benchmark case is a mix between the two polar cases we just investigated, with a higher probability of old to be sick with respect to adulthood, we would expect that benchmark (blue) curves in Figure 2 would mimic rather the black dotted curves in Figure 3 than the blue curves in Figure 3. Unfortunately, this is not the case, because of the “disposal income effect”. To understand why, we investigate below the influence of the health externality on labor productivity.

5.2.2 The “disposal income effect”

We have taken into account the empirical evidence that labor productivity is affected by health-status and, therefore in our basic framework, workforce productivity is positively affected by residential energy efficiency. This impacts the income of agents and therefore their saving. Here, we investigate how important is this transmission channel when analyzing the relationship between energy efficiency improvement and energy and non-energy consumption. For simplicity, we investigate the cases where there is no health externality on labor productivity ($\psi = 0$ in equation 10) and where the intensity of health externality on labor productivity is maximal ($\psi = 1$). We compare them to the benchmark case and to the “no health effect” case.

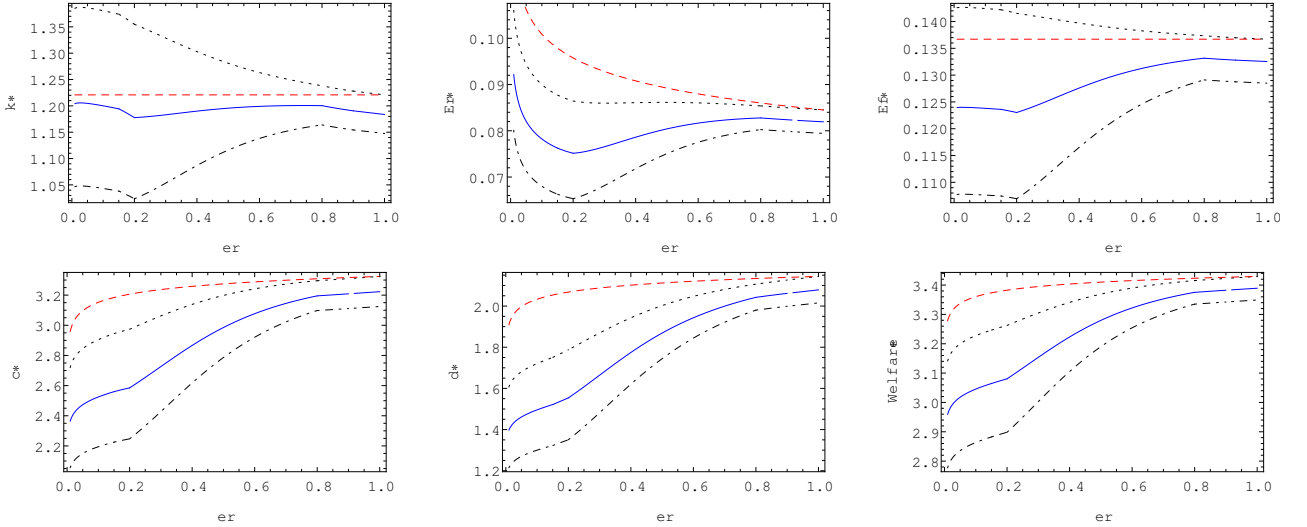


Figure 4: Health externality on labor productivity & influence of ε_r (Benchmark case (blue) vs no health externality (black dotted) vs full health externality $\psi = 1$ (black dotted dashed)). Red large dashed depicts the case with no health effect.

Figure (4) shows the results of our numerical simulations. It highlights that health externality on productivity has a significant role when considering the impact of the residential energy efficiency on steady-state per worker physical capital. Indeed, when there is no health externality on labor productivity ($\psi = 0$ in equation (10)), everything being equal, B_t and, therefore, wage given by equation (11b) are independent from ε_r^* . As a consequence, per worker physical capital is at its maximum level (with respect to the case where there is health externality on labor productivity). It comes from Proposition 1 and Lemma 3 that savings and k^* are at their highest level (see equation 17 where $\mathcal{B}(\varepsilon_r^*)$ is independent from ε_r^*). That is the reason why, in Graph 1 of Figure 4, the steady-state value of physical capital is always higher than the “no health effect” case and it is the lower, the greater

the health externality on labor productivity ($\psi = 1$).¹³

Note that, with our choice of parameter values, with no health externality on labor productivity, a higher residential energy efficiency ε_r^* leads to lower steady-state energy consumption in, both, residential and production sectors (see Graphs 2 and 3 in Figure 4). This is due to the fact that ε_r^* has general equilibrium effects (through health) which affect energy consumptions compared with the “no health” case. Generally, the lower the health externality on labor productivity, the lower the welfare gain associated with residential energy efficiency improvements (see Graph 6 in Figure 4).

6 Extensions

In this section, we extend the basic framework in order to investigating how taking into account health in utility function and/or linking probability to be sick in adulthood with probability to be sick in great age, would modify the results of the basic framework.

6.1 Health in utility

Levy and Nir (2012) and Finkelstein, Luttmer, and Notowidigdo (2013), amongst others, documented that health status enters utility function by affecting the marginal utility of consumption. In the basic framework, we abstracted from this to highlight the main channels. In this section, we take into account this extension. As a result, the expected inter-temporal utility of an adult becomes:

$$EU_t = \pi_t^a \Phi(h_t^a) \log \bar{c}_t^b + (1 - \pi_t^a) \log \bar{c}_t^g + \beta \left[\pi_{t+1}^o \Phi(h_{t+1}^o) \log \bar{d}_{t+1}^b + (1 - \pi_{t+1}^o) \log \bar{d}_{t+1}^g \right] \quad (23)$$

where $\Phi(h^i) \in]0, 1[$, with $i = (a, o)$ and $\Phi(1) = 1$, captures the influence of health in utility. Because we assume that full health is normalized to unity, it does not appear in front of utility of not sick agents.

PROPOSITION 3.

$$\hat{s}_t = \hat{\mathcal{S}}(\varepsilon_{f,t}, \varepsilon_{r,t-1}, \varepsilon_{r,t}, \varepsilon_{r,t+1}) \equiv \frac{\beta}{\Theta(\varepsilon_{r,t-1}, \varepsilon_{r,t}, \varepsilon_{r,t+1}) + \beta} \mathcal{W}(k_t, \varepsilon_{r,t-1}, \varepsilon_{f,t}) \quad (24)$$

where

$$\Theta(\varepsilon_{r,t-1}, \varepsilon_{r,t}, \varepsilon_{r,t+1}) \equiv \left[\frac{\mathfrak{U}(\varepsilon_{r,t})}{\mathfrak{U}(\varepsilon_{r,t+1})} \right]^{\frac{1-\sigma_{C,E}}{\sigma_{C,E}}} \times \left\{ \frac{\pi^a(\varepsilon_{r,t-1}) \Phi(h^a) \mu^a(\varepsilon_{r,t-1}) + [1 - (1 - \Phi(h^a)) \pi^a(\varepsilon_{r,t-1})] (1 + p_r \mathfrak{E}(\varepsilon_{r,t}))}{\pi^o(\varepsilon_{r,t}) \Phi(h^o) \mu^o(\varepsilon_{r,t}) + [1 - (1 - \Phi(h^o)) \pi^o(\varepsilon_{r,t})] (1 + p_r \mathfrak{E}(\varepsilon_{r,t+1}))} \right\},$$

Proof. From equation (24). □

Results of numerical simulations are summarized in Figure 5.¹⁴ It shows that the influence of residential energy efficiency on energy consumption is not qualitatively modified in the presence of health in utility with respect to the basic framework, especially for residential energy consumption, non-energy consumptions and, therefore, welfare (respectively graphs 2, 4, 5 & 6). For physical capital and energy consumption in production (graphs 1 & 3), the shape is quite the same but the magnitude of variations is higher with a significant lower value at small residential energy efficiency.

6.2 Chronic diseases

In this section, we assume that bad energy efficiency in the residential sector has a permanent effect on health. It creates chronic disease.¹⁵ As a result, adult agents when sick never fully recover, i.e. $z^a = \bar{z}^a$,

¹³Note that this is not true in the case where only adults are suffering from low residential energy efficiency. In this case, steady-state physical capital always remains under its “no health effect” case value (proof upon request).

¹⁴For convenience we set $\Phi(h^i) = \sqrt{h^i}$ with $i = (a, o)$.

¹⁵We use the modeling of chronic disease from Pautrel (2022).

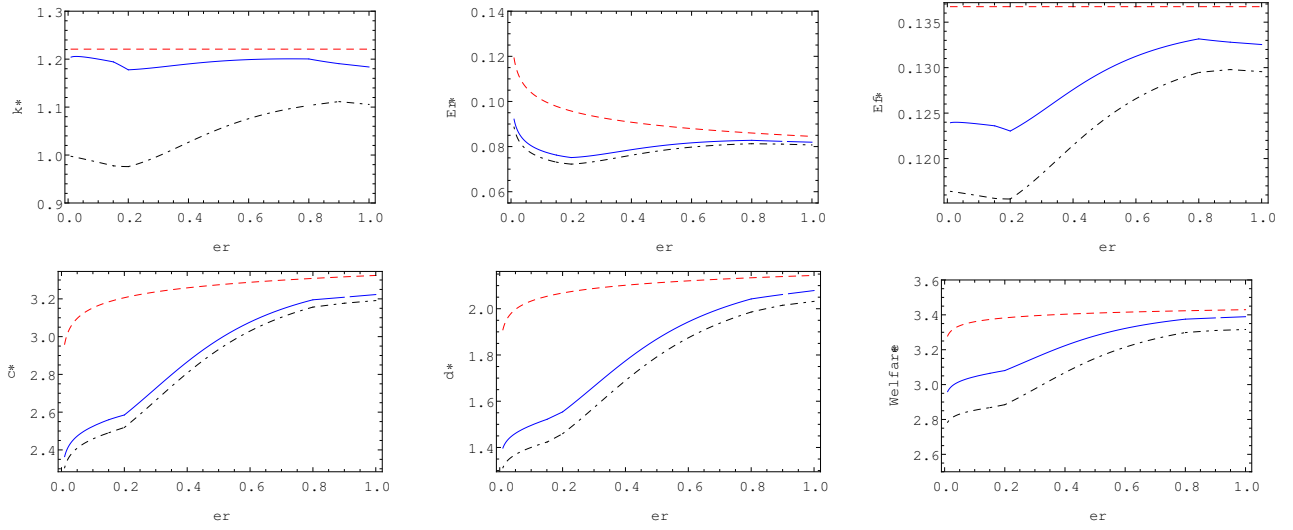
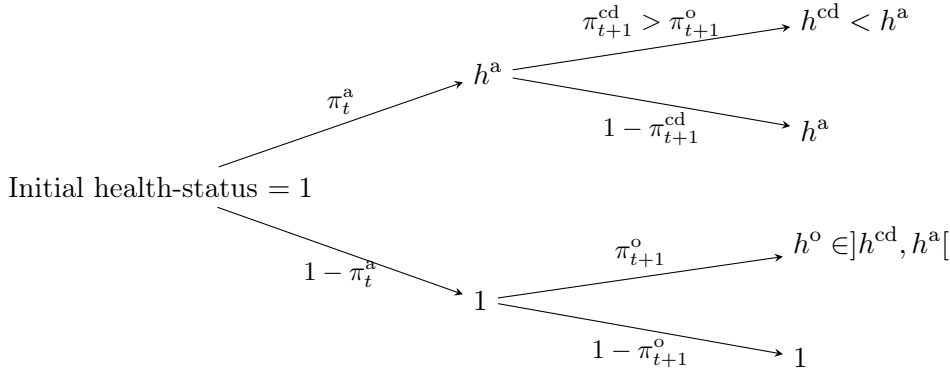


Figure 5: Health in utility and influence of ε_r (Benchmark case (blue) vs no health effect (red large dashes) vs health in utility case (black dot-dashed))

and health condition when old is related to health condition when adult. First, an individual who was sick during adulthood will have a higher probability to be sick when elderly than an individual who wasn't sick during adulthood. Second, an individual who was sick during adulthood will suffer from a greater loss of health than an individual who wasn't sick during adulthood. We represent this logical by the following tree:



where π_{t+1}^{cd} ("cd" for chronic disease) is the probability that old generation becomes sick when she has been sick during adulthood. The tree shows that the old generation can experience four different health conditions:

- (i) 1 (perfect health) with a probability $(1 - \pi_t^a)(1 - \pi_{t+1}^o)$;
- (ii) h^a (similar deteriorated health than during adulthood) with a probability $\pi_t^a(1 - \pi_{t+1}^{cd})$;
- (iii) $h^o < h^a$ (more deteriorated health than during adulthood) with a probability $(1 - \pi_t^a)\pi_{t+1}^o$;
- (iv) $h^{cd} < h^o < h^a$ (very deteriorated health with respect to adulthood) with a probability $\pi_t^a\pi_{t+1}^{cd}$.

Furthermore, we make the following realistic assumptions:

1. Elderly early sick when adult have a higher probability to be sick than elderly not sick when adult: $\pi_{t+1}^{cd} > \pi_{t+1}^o$.
2. Elderly not sick when adult have a higher probability to be sick than adult: $\pi_{t+1}^o > \pi_t^a$.

The expected intertemporal utility of an adult becomes:¹⁶

$$EU_t = \pi^a(\varepsilon_{r,t-1}) \left\{ \log \bar{c}_t^b + \beta \left[\pi^{cd}(\varepsilon_{r,t}) \log \bar{d}_{t+1}^{cdb} + (1 - \pi^{cd}(\varepsilon_{r,t})) \log \bar{d}_{t+1}^{cdg} \right] \right\} \\ + (1 - \pi^a(\varepsilon_{r,t-1})) \left\{ \log \bar{c}_t^g + \beta \left[\pi^o(\varepsilon_{r,t}) \log \bar{d}_{t+1}^b + (1 - \pi^o(\varepsilon_{r,t})) \log \bar{d}_{t+1}^g \right] \right\} \quad (25)$$

with the following budget constraints:

$$s_t + c_t^b + p_r E_{r,t}^{a,b} + m^a(\varepsilon_{r,t-1}) = w_t \quad (26a)$$

$$s_t + c_t^g + p_r E_{r,t}^{a,g} = w_t \quad (26b)$$

$$d_{t+1}^b + p_r E_{r,t}^{o,b} + m^o(\varepsilon_{r,t}) = R_{t+1} s_t \quad (26c)$$

$$d_{t+1}^g + p_r E_{r,t}^{o,g} = R_{t+1} s_t \quad (26d)$$

$$d_{t+1}^{cdb} + p_r E_{r,t}^{o,cdb} + m^{o,cdb}(\varepsilon_{r,t}) = R_{t+1} s_t \quad (26e)$$

$$d_{t+1}^{cdg} + p_r E_{r,t}^{o,cdg} + m^{o,cdg}(\varepsilon_{r,t}) = R_{t+1} s_t \quad (26f)$$

$$c_t^i \geq 0, d_{t+1}^j \geq 0, E_{r,t}^{a,i} \geq 0, E_{r,t+1}^{o,j} \geq 0 \quad (26g)$$

where $m^{o,cdj}$ with $j = b, g$ is health expenditures for old suffering from chronic disease.

Because, in the presence of chronic disease, sick agents during adulthood never fully recover, that is $z^a = \bar{z}^a$, from equation (10), we have:

$$B_t = \tilde{\mathcal{B}}(\varepsilon_{r,t-1}) \equiv B[1 - \bar{z}^a \pi^a(\varepsilon_{r,t-1})]^\psi \quad (27)$$

and,

$$w_t = \tilde{\mathcal{W}}(k_t, \varepsilon_{r,t}, \varepsilon_{f,t}) \equiv \mathcal{A}(\varepsilon_{f,t})(1 - \alpha)k_t^\alpha \tilde{\mathcal{B}}(\varepsilon_{r,t-1})^{1-\alpha} \quad (28)$$

$$R_t = \tilde{\mathcal{R}}(k_t, \varepsilon_{r,t-1}, \varepsilon_{f,t}) \equiv \mathcal{A}(\varepsilon_{f,t})\alpha k_t^{\alpha-1} \tilde{\mathcal{B}}(\varepsilon_{r,t-1})^{1-\alpha} \quad (29)$$

Let remark that, because $\bar{z}^a \geq z^a$, with chronic disease the impact of $\pi^a(\varepsilon_{r,t-1})$ on labor productivity is always higher than is impact in the benchmark case: $\tilde{\mathcal{B}}(\varepsilon_{r,t-1}) \geq \mathcal{B}(\varepsilon_{r,t-1})$.

PROPOSITION 4. *In the presence of chronic disease,*

1. *Savings is defined as:*

$$\tilde{s}_t = \tilde{\mathcal{S}}(\varepsilon_{r,t-1}, \varepsilon_{r,t}, \varepsilon_{r,t+1}, \varepsilon_f) \equiv \frac{\beta}{\Upsilon(\varepsilon_{r,t-1}, \varepsilon_{r,t}, \varepsilon_{r,t+1}) + \beta} \tilde{\mathcal{W}}(k_t, \varepsilon_{r,t}, \varepsilon_{f,t}) \quad (30)$$

where

$$\Upsilon(\varepsilon_{r,t-1}, \varepsilon_{r,t}, \varepsilon_{r,t+1}) \equiv \left[\frac{\mathfrak{U}(\varepsilon_{r,t})}{\mathfrak{U}(\varepsilon_{r,t+1})} \right]^{\frac{\sigma_{c,E}-1}{\sigma_{c,E}}} \times \left\{ \frac{\pi^a(\varepsilon_{r,t-1})\mu^a(\varepsilon_{r,t-1})+1+p_r\mathfrak{E}(\varepsilon_{r,t})}{(1-\pi^a(\varepsilon_{r,t-1}))\pi^o(\varepsilon_{r,t})\mu^o(\varepsilon_{r,t})+\pi^a(\varepsilon_{r,t-1})[\pi^{cd}(\varepsilon_{r,t})\mu^{ocdb}(\varepsilon_{r,t})+(1-\pi^{cd}(\varepsilon_{r,t}))\mu^{ocdg}(\varepsilon_{r,t})+1+p_r\mathfrak{E}(\varepsilon_{r,t})]} \right\} \quad (31)$$

2. *Propensity to save the income is higher (with respect to the benchmark case without chronic disease), $\Upsilon(\varepsilon_{r,t-1}, \varepsilon_{r,t}, \varepsilon_{r,t+1}) \leq \Psi(\varepsilon_{r,t-1}, \varepsilon_{r,t}, \varepsilon_{r,t+1})$, when $\pi^{cd}(\varepsilon_{r,t})\mu^{ocdb}(\varepsilon_{r,t}) \geq \pi^o(\varepsilon_{r,t})\mu^o(\varepsilon_{r,t})$.*

Proof. Point (1) from Appendix. Point (2) from equation (31), $\Upsilon(\varepsilon_{r,t-1}, \varepsilon_{r,t}, \varepsilon_{r,t+1}) \leq \Psi(\varepsilon_{r,t-1}, \varepsilon_{r,t}, \varepsilon_{r,t+1})$ iff $\pi^{cd}(\varepsilon_{r,t})\mu^{ocdb}(\varepsilon_{r,t}) \geq \pi^o(\varepsilon_{r,t})\mu^o(\varepsilon_{r,t})$. \square

¹⁶We continue to abstract from the influence of health on marginal utility of consumption to simplify. See Appendix for the general case.

Proposition 4.2 means that if the share of expected expenditures in consumption when old with chronic disease is higher than the share of expected healthcare expenditures in consumption when old with no chronic disease (which is a realistic assumption), the propensity to save is higher with chronic disease than without. It means that chronic disease rises precautionary saving motive.

Long-term per worker physical capital is given by:

$$\tilde{k}^* = \tilde{\mathcal{B}}(\varepsilon_r^*) \left[\frac{\beta \mathcal{A}(\varepsilon_f^*)(1 - \alpha)}{(1 + n)(\Upsilon(\varepsilon_r^*) + \beta)} \right]^{\frac{1}{1-\alpha}}$$

with $\Upsilon(\varepsilon_r^*) = \frac{\pi^a(\varepsilon_r^*)\mu^a(\varepsilon_r^*) + 1 + p_r \mathfrak{E}(\varepsilon_r^*)}{(1 - \pi^a(\varepsilon_r^*))\pi^o(\varepsilon_r^*)\mu^o(\varepsilon_r^*) + \pi^a(\varepsilon_r^*)[\pi^{cd}(\varepsilon_r^*)\mu^{ocdb}(\varepsilon_r^*) + (1 - \pi^{cd}(\varepsilon_r^*))\mu^{ocdg}(\varepsilon_r^*) + 1 + p_r \mathfrak{E}(\varepsilon_r^*)]}$

(32)

The expected per capita residential energy consumption at the steady-state (both young and old of the previous generation) is now:

$$\begin{aligned} \tilde{E}_r^* = & \left[\frac{\mathfrak{E}(\varepsilon_r^*)}{1 + p_r \mathfrak{E}(\varepsilon_r^*)} \right] \left\{ \left[\frac{\Upsilon(\varepsilon_r^*)}{\Upsilon(\varepsilon_r^*) + \beta} \tilde{\mathcal{W}}(\varepsilon_r^*, \varepsilon_f^*) \right] \times \left[1 - \pi^a(\varepsilon_r^*) \frac{\mu^a(\varepsilon_r^*)}{1 + p_r \mathfrak{E}(\varepsilon_r^*) + \mu^a(\varepsilon_r^*)} \right] \right. \\ & + \frac{\tilde{\mathcal{R}}(\varepsilon_r^*, \varepsilon_f^*) \tilde{\mathcal{S}}(\varepsilon_r^*, \varepsilon_f^*)}{1 + n} \left[(1 - \pi^a(\varepsilon_r^*)) \left(1 - \pi^o(\varepsilon_r^*) \frac{\mu^o(\varepsilon_r^*)}{1 + p_r \mathfrak{E}(\varepsilon_r^*) + \mu^o(\varepsilon_r^*)} \right) \right. \\ & \left. \left. + \pi^a(\varepsilon_r^*) (1 + p_r \mathfrak{E}(\varepsilon_r^*)) \left(\frac{\pi^{cd}(\varepsilon_r^*)\mu^{ocdg}(\varepsilon_r^*) + (1 - \pi^{cd}(\varepsilon_r^*))\mu^{ocdb}(\varepsilon_r^*) + 1 + p_r \mathfrak{E}(\varepsilon_r^*)}{(1 + p_r \mathfrak{E}(\varepsilon_r^*) + \mu^{ocdb}(\varepsilon_r^*)) (1 + p_r \mathfrak{E}(\varepsilon_r^*) + \mu^{ocdg}(\varepsilon_r^*))} \right) \right] \right\} \end{aligned} \quad (33)$$

We find an expression of the long-term residential energy consumption quite similar to the one obtained in equation (21), except the last term into square brackets at the second line of equation (21) became the last term into square brackets of the second and third lines of equation (33). This is explained by the fact that in the presence of chronic disease, the expected health expenditures when old is connected with the probability to be sick when adult ($\pi^a(\varepsilon_r^*)$).

Note that when $\pi^a(\varepsilon_r^*)$ is getting closer to 0 (that is when ε_r^* is high), the last term into square brackets in (33) is getting closer to the last term into square brackets in (21). That is why, when ε_r^* is high, the benchmark curve (the blue one) is getting closer to the “chronic disease” curve in Figure 6.

Steady-state per capita energy consumption in production is now:

$$\tilde{E}_f^* = \frac{A_Q \mathcal{B}(\varepsilon_r^*)}{\varepsilon_f^* \Omega(\varepsilon_f^*)} \left[\frac{\beta \mathcal{A}(\varepsilon_f^*)(1 - \alpha)}{(1 + n)(\Upsilon(\varepsilon_r^*) + \beta)} \right]^{\frac{\alpha}{1-\alpha}} \quad (34)$$

Figure 6 shows that, when chronic diseases are taken into account, all key variables are lower with respect to the benchmark case. According to Proposition 4, it seems surprising that the steady-state values of the per worker physical capital is lower than the benchmark case because we noted that a higher precautionary saving is expected in the presence of chronic diseases. Nevertheless, we also noted that the “disposal income effect” is expected to be lower in the presence of chronic diseases with respect to the benchmark. In our assumption of a medium intensity of health externality on labor productivity ($\psi = 0.5$), the decrease in “disposal income effect” offsets the rise in “saving propensity effect” and steady-state saving reduces, as well as the other key variables because health expenditures in the presence of chronic disease are higher.

Figure 7 investigates how the effect of the health externality on labor productivity affects the influence of ε_r in the presence of chronic disease. As already highlighted in section 5.2.2, the magnitude of health externality on labor productivity plays an important role. Especially, with no health externality on labor productivity, the steady-state per worker physical capital is higher than the “no health” case because the “saving propensity effects” offsets the “disposal income effect” (which is null) and adults rise precautionary saving. As a result, in the presence of chronic disease, the influence of ε_r becomes negative for physical capital and energy consumptions (see black dotted curves in graphs 1, 2 & 3 of

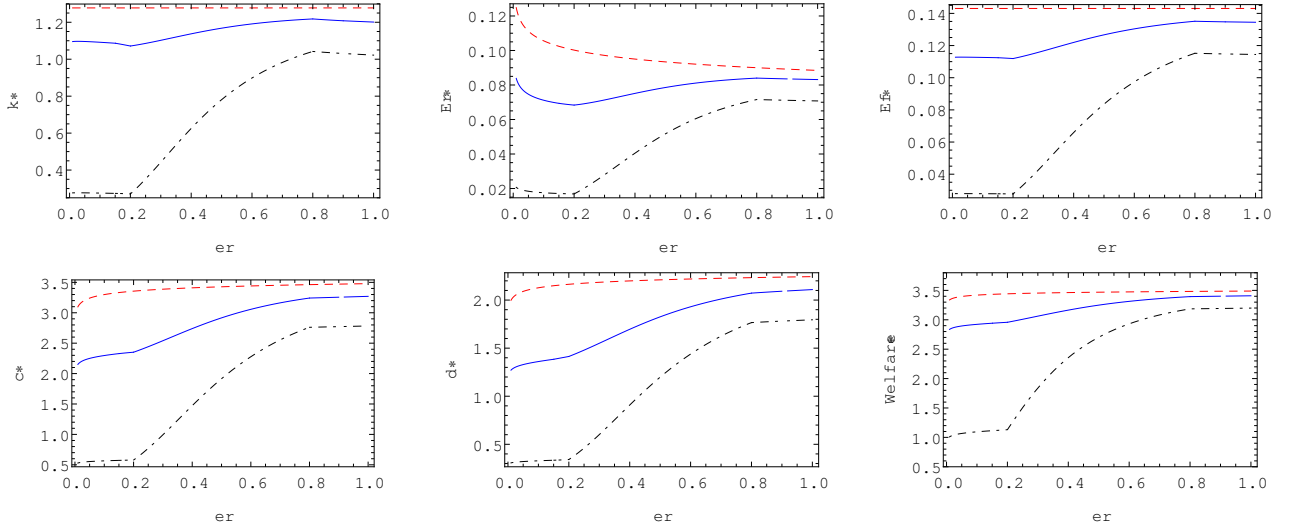


Figure 6: Chronic Disease and influence of ϵ_r (Benchmark case (blue) vs no health effect (red large dashes) vs Chronic disease case (black dot-dashed))

Figure 7). Conversely, when the health externality on labor productivity is at its highest level ($\psi = 1$), the positive influence of ϵ_r on all key variables is more important (see black dotted dashed curves in Figure 7) because the “disposal income effect” is also more important.

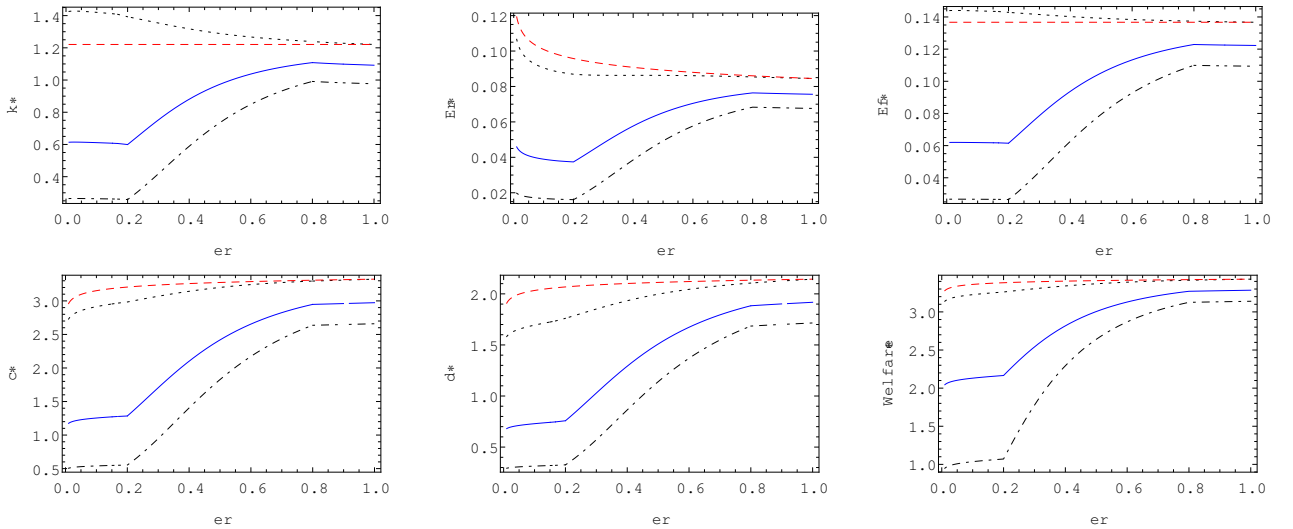


Figure 7: Chronic Disease and influence of ϵ_r with respect to the amplitude of the health externality on labor productivity (limited health externality $\psi = 0.5$ (blue) vs no health externality (black dotted) vs full health externality $\psi = 1$ (black dotted dashed)). Red large dashed depicts the case with no health effect.

7 Conclusion and policy implications

This paper proposes a 3-periods overlapping generations model, with probability to be sick during adulthood and old-age, in order to investigate how health (via morbidity effects) influences the impacts of residential energy efficiency on energy consumption. Based on empirical evidences from two bodies of economic and interdisciplinary literatures, we assumed that these morbidity effects are induced by residential energy inefficiency and influence the workforce productivity.

Our results show that health channels we introduced could significantly shape the change in energy consumption induced by an energy efficiency variation, through their impacts on the propensity to save and the disposal income. In particular, our results suggest that if mostly old (resp. young)

people health is affected, the health impact of residential energy efficiency should have a backfire (resp. rebound) influence on residential energy consumption, by promoting precautionary saving (resp. by rising labor productivity). However, our results show that if health externality on labor productivity is strong enough, as suggested by recent empirical evidence, an energy efficiency improvement should rise the energy consumption not only in the residential sector but also in production sector.

When our theoretical framework is extended to take into account the influence of health status on utility and to integrate chronic disease associated with residential energy inefficiency, our results interestingly show that chronic disease introduces two opposite effects. On the one hand, it reinforces the “propensity to save” effect by increasing precautionary saving. In fact, the probability to be sick when old increases due to chronic disease. On the other, the chronic disease reinforces the “disposal income” effect, because an improvement of residential energy efficiency (as greater impact of labor productivity) induces a better health-status during adulthood, thus, labor productivity. The global effect mainly depends on the magnitude of the health externality on labor productivity and on the probabilities of elderly of becoming sick.

In policy terms, this study adds to the debate about two main issues. The first deals with the rebound effect. From an empirical point of view, it is largely recognized that rebound effect is difficult to estimate. Usually, only a part of the rebound effect is estimated, i.e. direct effect. Although, theoretical studies give additional insights on this complex phenomena, several related questions are still open mainly regarding the interaction with the macroeconomic side. In our analysis, by introducing health and focusing on identifying health channels through which a variation in energy efficiency may affect energy consumption, we take an original tack to consider the rebound effect, thus, we introduce a new conceptual framework that may help defining new strategies to mitigate this phenomenon as well as new policy pathways. In fact, Vivanco et al. (2016) argue that policy inaction on rebound effect is partly explained by the unsuccessful push from academics. Our contribution particularly shows that considering health channels when analyzing the relationship between energy efficiency and energy consumption may help understanding the rebound effect by stressing some unexpected interactions with labor productivity and (precautionary) saving.

From a more general point of view, today, a growing number of academic researches show that energy consumption (or more generally, environment), health and growth policies are intertwined. From a microeconomic point of view, our research also adds to the policy debate when considering the relationship between individual energy conservation policies and health (self-insurance) policies. We show in our model that -in some cases- energy inefficiency in the residential sector may increase the likelihood of precautionary saving in order to self-insure against future expected medical expenses. As a consequence, it is crucial to reconsider the question of households energy and health expenditures trade-offs and the subsequent question of impacts individual decisions may have on macroeconomic health, energy and growth programs. The international community recently pointed out the significant increase in energy bills during the global pandemic, i.e. lock-down periods and online working, and its dramatic consequences on households expenditures trade-offs. New rules of deprivation have been self-implemented and new forms of micro and macro-vulnerabilities have been revealed. These new observations reflect how crucial and complex are for governments the discussion and policy action dealing with the connection between energy conservation policies and health insurance system and growth.

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A The resolution of the general framework

The expected intertemporal utility of an adult is:

$$\begin{aligned}
EU_t = & \pi^a(\varepsilon_{r,t-1})\Phi(h_t^a) \log \bar{c}_t^b + (1 - \pi^a(\varepsilon_{r,t-1})) \log \bar{c}_t^g \\
& + \beta\pi^a(\varepsilon_{r,t-1}) \left[\pi^{cd}(\varepsilon_{r,t})\Phi(h_{t+1}^{cd}) \log \bar{d}_{t+1}^{cdb} + (1 - \pi^{cd}(\varepsilon_{r,t}))\Phi(h_t^a) \log \bar{d}_{t+1}^{cdg} \right] \\
& + \beta(1 - \pi^a(\varepsilon_{r,t-1})) \left[\pi^o(\varepsilon_{r,t})\Phi(h_{t+1}^o) \log \bar{d}_{t+1}^b + (1 - \pi^o(\varepsilon_{r,t})) \log \bar{d}_{t+1}^g \right] \quad (A.1)
\end{aligned}$$

where the composite consumption good of the adult generation and the old generation, respectively denoted by \bar{c}^i and \bar{d}^i , are given by

$$\bar{c}_t^i = \left[(1 - \nu)c_t^i \frac{\sigma_{C,E}-1}{\sigma_{C,E}} + \nu \left(\varepsilon_{r,t} E_{r,t}^{a,i} \right) \frac{\sigma_{C,E}-1}{\sigma_{C,E}} \right]^{\frac{\sigma_{C,E}}{\sigma_{C,E}-1}} \quad \text{with} \quad i = (b, g) \text{ and} \quad (A.2)$$

$$\bar{d}_{t+1}^j = \left[(1 - \nu)d_{t+1}^j \frac{\sigma_{C,E}-1}{\sigma_{C,E}} + \nu \left(\varepsilon_{r,t+1} E_{r,t+1}^{o,j} \right) \frac{\sigma_{C,E}-1}{\sigma_{C,E}} \right]^{\frac{\sigma_{C,E}}{\sigma_{C,E}-1}} \quad \text{with} \quad \nu \in]0, 1[, \quad j = (cdb, cdg, b, g) \quad (A.3)$$

c_t^i (respectively d_{t+1}^j) is the amount of non-energy goods the adult (resp. old) generation consumes with a health condition i . $E_{r,t}^{a,i}$ (respectively $E_{r,t+1}^{o,j}$) is the amount of energy services (whose efficiency is captured by ε_r) the adult (resp. old) generation consumes with a health condition i . $\sigma_{C,E} \geq 0$ is the elasticity of substitution between non-energy goods and energy consumptions.

Per-period budget constraints are:

$$s_t + c_t^b + p_r E_{r,t}^{a,b} + m^a(\varepsilon_{r,t-1}) = w_t \quad (A.4)$$

$$s_t + c_t^g + p_r E_{r,t}^{a,g} = w_t \quad (A.5)$$

$$d_{t+1}^b + p_r E_{r,t}^{o,b} + m^o(\varepsilon_{r,t}) = R_{t+1} s_t \quad (A.6)$$

$$d_{t+1}^g + p_r E_{r,t}^{o,g} = R_{t+1} s_t \quad (A.7)$$

$$d_{t+1}^{cdb} + p_r E_{r,t}^{o,cdb} + m^{o,cdb}(\varepsilon_{r,t}) = R_{t+1} s_t \quad (A.8)$$

$$d_{t+1}^{cdg} + p_r E_{r,t}^{o,cdg} + m^{o,cdg}(\varepsilon_{r,t}) = R_{t+1} s_t \quad (A.9)$$

$$c_t^i \geq 0, \quad d_{t+1}^j \geq 0, \quad E_{r,t}^{a,i} \geq 0, \quad E_{r,t+1}^{o,j} \geq 0 \quad (A.10)$$

where $R_{t+1} = (1 + r_{t+1})$ with r the real interest rate, w_t is real wage, p_r is energy price. The maximization program is:

$$\begin{aligned}
\mathcal{L} = & \pi^a(\varepsilon_{r,t-1})\Phi(h_t^a) \log \bar{c}_t^b + (1 - \pi^a(\varepsilon_{r,t-1})) \log \bar{c}_t^g \\
& + \beta\pi^a(\varepsilon_{r,t-1}) \left[\pi^{cd}(\varepsilon_{r,t})\Phi(h_{t+1}^{cd}) \log \bar{d}_{t+1}^{cdb} + (1 - \pi^{cd}(\varepsilon_{r,t}))\Phi(h_t^a) \log \bar{d}_{t+1}^{cdg} \right] \\
& + \beta(1 - \pi^a(\varepsilon_{r,t-1})) \left[\pi^o(\varepsilon_{r,t})\Phi(h_{t+1}^o) \log \bar{d}_{t+1}^b + (1 - \pi^o(\varepsilon_{r,t})) \log \bar{d}_{t+1}^g \right] \\
& + \lambda_1 \left[w_t - s_t - c_t^b - p_r E_{r,t}^{a,b} - m^a(\varepsilon_{r,t-1}) \right] + \lambda_2 \left[w_t - s_t - c_t^g - p_r E_{r,t}^{a,g} \right] \\
& + \lambda_3 \left[R_{t+1} s_t - d_{t+1}^b - p_r E_{r,t+1}^{o,b} - m^o(\varepsilon_{r,t}) \right] + \lambda_4 \left[R_{t+1} s_t - d_{t+1}^g - p_r E_{r,t+1}^{o,g} \right] \\
& + \lambda_5 \left[R_{t+1} s_t - d_{t+1}^{cdb} - p_r E_{r,t+1}^{o,cdb} - m^{o,cdb}(\varepsilon_{r,t}) \right] + \lambda_6 \left[R_{t+1} s_t - d_{t+1}^{cdg} - p_r E_{r,t+1}^{o,cdg} - m^{o,cdg}(\varepsilon_{r,t}) \right] \quad (A.11)
\end{aligned}$$

First-order conditions give:

$$c^b \Rightarrow \pi^a(\varepsilon_{r,t-1})\Phi(h_t^a)(1-\nu)\left(\frac{c^b}{\bar{c}^b}\right)^{\frac{\sigma_{C,E}-1}{\sigma_{C,E}}}c^{b-1} = \lambda_1 \quad (\text{A.12})$$

$$E_r^{a,b} \Rightarrow \pi^a(\varepsilon_{r,t-1})\Phi(h_t^a)\nu\left(\frac{\varepsilon_{r,t}E_r^{a,b}}{\bar{c}^b}\right)^{\frac{\sigma_{C,E}-1}{\sigma_{C,E}}}E_r^{a,b-1} = \lambda_1 p_r \quad (\text{A.13})$$

$$c^g \Rightarrow (1-\pi^a(\varepsilon_{r,t-1}))(1-\nu)\left(\frac{c^g}{\bar{c}^g}\right)^{\frac{\sigma_{C,E}-1}{\sigma_{C,E}}}c^{g-1} = \lambda_2 \quad (\text{A.14})$$

$$E_r^{a,g} \Rightarrow (1-\pi^a(\varepsilon_{r,t-1}))\nu\left(\frac{\varepsilon_{r,t}E_r^{a,g}}{\bar{c}^g}\right)^{\frac{\sigma_{C,E}-1}{\sigma_{C,E}}}E_r^{a,g-1} = \lambda_2 p_r \quad (\text{A.15})$$

$$d^b \Rightarrow \beta(1-\pi^a(\varepsilon_{r,t-1}))\pi_{t+1}^o(\varepsilon_{r,t+1})\Phi(h_{t+1}^o)(1-\nu)\left(\frac{d^b}{\bar{d}^b}\right)^{\frac{\sigma_{C,E}-1}{\sigma_{C,E}}}d^{b-1} = \lambda_3 \quad (\text{A.16})$$

$$E_r^{o,b} \Rightarrow \beta(1-\pi^a(\varepsilon_{r,t-1}))\pi_{t+1}^o(\varepsilon_{r,t+1})\Phi(h_{t+1}^o)\nu\left(\frac{\varepsilon_{r,t+1}E_r^{o,b}}{\bar{d}^b}\right)^{\frac{\sigma_{C,E}-1}{\sigma_{C,E}}}E_r^{o,b-1} = \lambda_3 p_r \quad (\text{A.17})$$

$$d^g \Rightarrow \beta(1-\pi^a(\varepsilon_{r,t-1}))(1-\pi_{t+1}^o(\varepsilon_{r,t+1}))(1-\nu)\left(\frac{d^g}{\bar{d}^g}\right)^{\frac{\sigma_{C,E}-1}{\sigma_{C,E}}}d^{g-1} = \lambda_4 \quad (\text{A.18})$$

$$E_r^{o,g} \Rightarrow \beta(1-\pi^a(\varepsilon_{r,t-1}))(1-\pi_{t+1}^o(\varepsilon_{r,t+1}))\nu\left(\frac{\varepsilon_{r,t+1}E_r^{o,g}}{\bar{d}^g}\right)^{\frac{\sigma_{C,E}-1}{\sigma_{C,E}}}E_r^{o,g-1} = \lambda_4 p_r \quad (\text{A.19})$$

$$d^{cdb} \Rightarrow \beta\pi^a(\varepsilon_{r,t-1})\pi^{cd}(\varepsilon_{r,t})\Phi(h_{t+1}^{cd})(1-\nu)\left(\frac{d^{cdb}}{\bar{d}^{cdb}}\right)^{\frac{\sigma_{C,E}-1}{\sigma_{C,E}}}d^{cdb-1} = \lambda_5 \quad (\text{A.20})$$

$$E_r^{o,cdb} \Rightarrow \beta\pi^a(\varepsilon_{r,t-1})\pi^{cd}(\varepsilon_{r,t})\Phi(h_{t+1}^{cd})\nu\left(\frac{\varepsilon_{r,t+1}E_r^{o,cdb}}{\bar{d}^{cdb}}\right)^{\frac{\sigma_{C,E}-1}{\sigma_{C,E}}}E_r^{o,cdb-1} = \lambda_5 p_r \quad (\text{A.21})$$

$$d^{cdg} \Rightarrow \beta\pi^a(\varepsilon_{r,t-1})(1-\pi^{cd}(\varepsilon_{r,t}))(1-\nu)\left(\frac{d^{cdg}}{\bar{d}^{cdg}}\right)^{\frac{\sigma_{C,E}-1}{\sigma_{C,E}}}d^{cdg-1} = \lambda_6 \quad (\text{A.22})$$

$$E_r^{o,cdg} \Rightarrow \beta\pi^a(\varepsilon_{r,t-1})(1-\pi^{cd}(\varepsilon_{r,t}))\nu\left(\frac{\varepsilon_{r,t+1}E_r^{o,cdg}}{\bar{d}^g}\right)^{\frac{\sigma_{C,E}-1}{\sigma_{C,E}}}E_r^{o,cdg-1} = \lambda_6 p_r \quad (\text{A.23})$$

$$s \Rightarrow \lambda_1 + \lambda_2 = R_{t+1}(\lambda_3 + \lambda_4 + \lambda_5 + \lambda_6) \quad (\text{A.24})$$

From (A.12) and (A.13) and (A.14) and (A.15), and from (A.16) and (A.17) and (A.18) and (A.19), we get

$$\begin{aligned} E_{r,t}^{a,i} &= \mathfrak{E}(\varepsilon_{r,t})c_t^i \\ E_{r,t+1}^{o,i} &= \mathfrak{E}(\varepsilon_{r,t+1})d_{t+1}^j \end{aligned} \quad \text{with} \quad \mathfrak{E}(\varepsilon_r) \equiv \left(\frac{\nu}{1-\nu}\right)^{\sigma_{C,E}} p_r^{-\sigma_{C,E}} \varepsilon_r^{\sigma_{C,E}-1} \text{ and } i = (b, g), j = (cdb, cdg, b, g) \quad (\text{A.25})$$

Furthermore

$$\begin{aligned} \bar{c}_t^i &= \mathfrak{U}(\varepsilon_{r,t})c_t^i \\ \bar{d}_{t+1}^j &= \mathfrak{U}(\varepsilon_{r,t+1})d_{t+1}^j \end{aligned} \quad \text{with} \quad \mathfrak{U}(\varepsilon_r) \equiv \left[(1-\nu) + \nu(\varepsilon_r \mathfrak{E}(\varepsilon_r))^{\frac{\sigma_{C,E}-1}{\sigma_{C,E}}} \right]^{\frac{\sigma_{C,E}}{\sigma_{C,E}-1}} \quad (\text{A.26})$$

and $\varepsilon_r = \varepsilon_{r,t}$ for a and $\varepsilon_r = \varepsilon_{r,t+1}$ for o. Then

$$\pi^a(\varepsilon_{r,t-1})\Phi(h_t^a)(1-\nu)\mathfrak{U}(\varepsilon_{r,t})\frac{\sigma_{C,E}-1}{\sigma_{C,E}}c^{b-1} = \lambda_1 \quad (\text{A.27})$$

$$(1-\pi^a(\varepsilon_{r,t-1}))(1-\nu)\mathfrak{U}(\varepsilon_{r,t})\frac{\sigma_{C,E}-1}{\sigma_{C,E}}c^{g-1} = \lambda_2 \quad (\text{A.28})$$

$$\beta(1-\pi^a(\varepsilon_{r,t-1}))\pi_{t+1}^o(\varepsilon_{r,t+1})\Phi(h_{t+1}^o)(1-\nu)\mathfrak{U}(\varepsilon_{r,t+1})\frac{\sigma_{C,E}-1}{\sigma_{C,E}}d^{b-1} = \lambda_3 \quad (\text{A.29})$$

$$\beta(1-\pi^a(\varepsilon_{r,t-1}))(1-\pi_{t+1}^o(\varepsilon_{r,t+1}))(1-\nu)\mathfrak{U}(\varepsilon_{r,t+1})\frac{\sigma_{C,E}-1}{\sigma_{C,E}}d^{g-1} = \lambda_4 \quad (\text{A.30})$$

$$\beta\pi^a(\varepsilon_{r,t-1})\pi^{\text{cd}}(\varepsilon_{r,t})\Phi(h_{t+1}^{\text{cd}})(1-\nu)\mathfrak{U}(\varepsilon_{r,t+1})\frac{\sigma_{C,E}-1}{\sigma_{C,E}}d^{\text{cdb}-1} = \lambda_5 \quad (\text{A.31})$$

$$\beta\pi^a(\varepsilon_{r,t-1})(1-\pi^{\text{cd}}(\varepsilon_{r,t}))(1-\nu)\mathfrak{U}(\varepsilon_{r,t+1})\frac{\sigma_{C,E}-1}{\sigma_{C,E}}d^{\text{cdg}-1} = \lambda_6 \quad (\text{A.32})$$

$$(\text{A.33})$$

that is

$$\begin{aligned} & \left(\frac{\pi^a(\varepsilon_{r,t-1})\Phi(h_t^a)}{c^b} + \frac{(1-\pi^a(\varepsilon_{r,t-1}))}{c^g} \right) = \beta R_{t+1} \left[\frac{(1-\pi^a(\varepsilon_{r,t-1}))\pi^o(\varepsilon_{r,t})\Phi(h_{t+1}^o)}{d^b} \right. \\ & \left. + \frac{(1-\pi^a(\varepsilon_{r,t-1}))(1-\pi^o(\varepsilon_{r,t}))}{d^g} + \frac{\pi^a(\varepsilon_{r,t-1})\pi^{\text{cd}}(\varepsilon_{r,t})\Phi(h_{t+1}^{\text{cd}})}{d^{\text{cdb}}} + \frac{\pi^a(\varepsilon_{r,t-1})(1-\pi^{\text{cd}}(\varepsilon_{r,t}))\Phi(h_{t+1}^a)}{d^{\text{cdg}}} \right] \end{aligned} \quad (\text{A.34})$$

that is

$$\begin{aligned} & \frac{\pi^a(\varepsilon_{r,t-1})\Phi(h_t^a)(1+p_r\mathfrak{E}(\varepsilon_{r,t})+\mu^a(\varepsilon_{r,t-1}))}{w_t-s_t} + \frac{(1-\pi^a(\varepsilon_{r,t-1}))(1+p_r\mathfrak{E}(\varepsilon_{r,t}))}{w_t-s_t} \\ & = \beta R_{t+1} \left[\frac{(1-\pi^a(\varepsilon_{r,t-1}))\pi^o(\varepsilon_{r,t})\Phi(h_{t+1}^o)(1+p_r\mathfrak{E}(\varepsilon_{r,t})+\mu^o(\varepsilon_{r,t}))}{R_{t+1}s_t} \right. \\ & \left. + \frac{(1-\pi^a(\varepsilon_{r,t-1}))(1-\pi^o(\varepsilon_{r,t}))(1+p_r\mathfrak{E}(\varepsilon_{r,t}))}{R_{t+1}s_t} \right. \\ & \left. + \frac{\pi^a(\varepsilon_{r,t-1})\pi^{\text{cd}}(\varepsilon_{r,t})\Phi(h_{t+1}^{\text{cd}})(1+p_r\mathfrak{E}(\varepsilon_{r,t})+\mu^{\text{ocdb}}(\varepsilon_{r,t}))}{R_{t+1}s_t} \right. \\ & \left. + \frac{\pi^a(\varepsilon_{r,t-1})(1-\pi^{\text{cd}}(\varepsilon_{r,t}))\Phi(h_{t+1}^a)(1+p_r\mathfrak{E}(\varepsilon_{r,t})+\mu^{\text{ocdg}}(\varepsilon_{r,t}))}{R_{t+1}s_t} \right] \end{aligned} \quad (\text{A.35})$$

that is

$$\begin{aligned} & s_t [\pi^a(\varepsilon_{r,t-1})\Phi(h_t^a)(1+p_r\mathfrak{E}(\varepsilon_{r,t})+\mu^a(\varepsilon_{r,t-1})) + (1-\pi^a(\varepsilon_{r,t-1}))(1+p_r\mathfrak{E}(\varepsilon_{r,t}))] \\ & = \beta (w_t-s_t) [(1-\pi^a(\varepsilon_{r,t-1}))\pi^o(\varepsilon_{r,t})\Phi(h_{t+1}^o)(1+p_r\mathfrak{E}(\varepsilon_{r,t})+\mu^o(\varepsilon_{r,t})) \\ & + (1-\pi^a(\varepsilon_{r,t-1}))(1-\pi^o(\varepsilon_{r,t}))(1+p_r\mathfrak{E}(\varepsilon_{r,t})) \\ & + \pi^a(\varepsilon_{r,t-1})\pi^{\text{cd}}(\varepsilon_{r,t})\Phi(h_{t+1}^{\text{cd}})(1+p_r\mathfrak{E}(\varepsilon_{r,t})+\mu^{\text{ocdb}}(\varepsilon_{r,t})) \\ & \quad + \pi^a(\varepsilon_{r,t-1})(1-\pi^{\text{cd}}(\varepsilon_{r,t}))\Phi(h_{t+1}^a)(1+p_r\mathfrak{E}(\varepsilon_{r,t})+\mu^{\text{ocdg}}(\varepsilon_{r,t}))] \end{aligned} \quad (\text{A.36})$$

that is

$$\begin{aligned} & s_t [\pi^a(\varepsilon_{r,t-1})\Phi(h_t^a)(1+p_r\mathfrak{E}(\varepsilon_{r,t})+\mu^a(\varepsilon_{r,t-1})) + (1-\pi^a(\varepsilon_{r,t-1}))(1+p_r\mathfrak{E}(\varepsilon_{r,t}))] \\ & = \beta (w_t-s_t) \{ (1-\pi^a(\varepsilon_{r,t-1}))\pi^o(\varepsilon_{r,t})\Phi(h_{t+1}^o)(1+p_r\mathfrak{E}(\varepsilon_{r,t})+\mu^o(\varepsilon_{r,t})) \\ & + (1-\pi^a(\varepsilon_{r,t-1}))(1-\pi^o(\varepsilon_{r,t}))(1+p_r\mathfrak{E}(\varepsilon_{r,t})) \\ & + \pi^a(\varepsilon_{r,t-1})\pi^{\text{cd}}(\varepsilon_{r,t})\Phi(h_{t+1}^{\text{cd}})(1+p_r\mathfrak{E}(\varepsilon_{r,t})+\mu^{\text{ocdb}}(\varepsilon_{r,t})) \\ & \quad + \pi^a(\varepsilon_{r,t-1})(1-\pi^{\text{cd}}(\varepsilon_{r,t}))\Phi(h_{t+1}^a)(1+p_r\mathfrak{E}(\varepsilon_{r,t})+\mu^{\text{ocdg}}(\varepsilon_{r,t})) \} \end{aligned} \quad (\text{A.37})$$

that is

$$s_t = \frac{\beta}{\Upsilon(\varepsilon_{r,t-1}, \varepsilon_{r,t}, \varepsilon_{r,t+1}) + \beta} w_t \quad (\text{A.38})$$

with

$$\begin{aligned} \Upsilon(\varepsilon_{r,t-1}, \varepsilon_{r,t}, \varepsilon_{r,t+1}) \equiv & \left[\frac{\mathfrak{U}(\varepsilon_{r,t})}{\mathfrak{U}(\varepsilon_{r,t+1})} \right]^{\frac{\sigma_{C,E}-1}{\sigma_{C,E}}} \times \\ & [\pi^a(\varepsilon_{r,t-1})\Phi(h_t^a) (1 + p_r \mathfrak{E}(\varepsilon_{r,t}) + \mu^a(\varepsilon_{r,t-1})) + (1 - \pi^a(\varepsilon_{r,t-1})) (1 + p_r \mathfrak{E}(\varepsilon_{r,t}))] \times \\ & \{ (1 - \pi^a(\varepsilon_{r,t-1})) [\pi^o(\varepsilon_{r,t})\Phi(h_{t+1}^o) (1 + p_r \mathfrak{E}(\varepsilon_{r,t}) + \mu^o(\varepsilon_{r,t})) + (1 - \pi^o(\varepsilon_{r,t})) (1 + p_r \mathfrak{E}(\varepsilon_{r,t}))] \\ & + \pi^a(\varepsilon_{r,t-1}) [\pi^{cd}(\varepsilon_{r,t})\Phi(h_{t+1}^{cd}) (1 + p_r \mathfrak{E}(\varepsilon_{r,t}) + \mu^{ocdb}(\varepsilon_{r,t})) \\ & + (1 - \pi^{cd}(\varepsilon_{r,t}))\Phi(h_{t+1}^a) (1 + p_r \mathfrak{E}(\varepsilon_{r,t}) + \mu^{ocdg}(\varepsilon_{r,t}))] \}^{-1} \quad (\text{A.39}) \end{aligned}$$

B The health risk function

In this section, we discuss the specification of the health function. First we denote $F^i(\varepsilon_r)$ the sigmoid function related the probability, for the agent of type i ($i = a, o$), to be sick according to the value of the energy efficiency ε_r . We define $F^i(\varepsilon_r)$ as:

$$F^i(\varepsilon_r) \equiv \frac{1}{1 + 0.01 \left(\frac{a}{\varepsilon_r^{i,max}} \right)^\iota} - \frac{1}{1 + 0.01 \left(\frac{a}{\varepsilon_r} \right)^\iota} \quad (\text{B.1})$$

where $\varepsilon_r^{i,max}$ is the upper-bound of ε_r above which agent of type i can not be sick (here we assumed for adults $\varepsilon_r^{a,max} = 0.8$ then above 0.8 energy efficiency is high enough to prevent adults to be sick due to energy poverty). As a consequence $F^i(\varepsilon_r)$ is decreasing in ε_r and is negative when $\varepsilon_r > \varepsilon_r^{i,max}$. Parameters $a > 0$ and $\iota > 0$ defined the curvature of the sigmoid and the inflection point. We choose here $a = 3$ and $\iota = 2.5$ for both types of agents.

We also assume that under a lower bound, denoted by $\varepsilon_r^{i,min}$, the probability of becoming sick due to energy poverty is maximal (but lower than unity) and independent from ε_r . This probability is then obtained when $\varepsilon_r = \varepsilon_r^{i,min}$ and is therefore given by $F^i(\varepsilon_r^{i,min})$. As a consequence, for type i agents, the health risk function associated to bad energy efficiency is defined by:

$$\pi_h^i = \min [F^i(\varepsilon_r), F^i(\varepsilon_r^{i,min})] \quad (\text{B.2})$$

Agents of type i face a “non-energy” induced health risk, denoted by $s^i \in [0, 1[$ which adds to the energy induced health risk, and when energy efficiency is higher than $\varepsilon_r^{i,max}$ the only health risk that agents face is s^i . As a consequence, the probability to be sick is limited by s^i for the higher values of ε_r and by unity for the lower values of ε_r , such that:

$$\pi^i(\varepsilon_r) \equiv \min [\max [\pi_h^i + s^i, s^i], 1] \quad (\text{B.3})$$