

Visibility Contractors

Application to Mobile Robot Localization

Rémy GUYONNEAU

JOURNÉE MEA

LISA/LARIS - University of Angers (France)

Laurent HARDOUIN

Sébastien LAGRANGE



5th December 2013

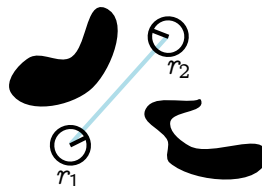


Introduction

- Visibility is studied and used in several fields
 - Computer graphics
 - Telecommunication
 - Robotics...
- Usually associated to bearing or ranging data
- We consider the visibility as a boolean information
 - Application to mobile robot localization

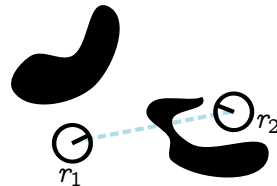
Introduction

- Visibility is studied and used in several fields
 - Computer graphics
 - Telecommunication
 - Robotics...
- Usually associated to bearing or ranging data
- We consider the visibility as a boolean information
 - Application to mobile robot localization



Introduction

- Visibility is studied and used in several fields
 - Computer graphics
 - Telecommunication
 - Robotics...
- Usually associated to bearing or ranging data
- We consider the visibility as a boolean information
 - Application to mobile robot localization



Outlines

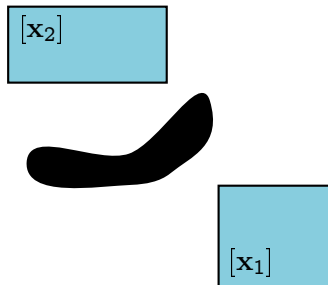
- 1 General Presentation
- 2 Visibility contractors
- 3 Multi-robot localization
- 4 Global localization
- 5 Conclusion

Outlines

- 1 General Presentation
- 2 Visibility contractors
- 3 Multi-robot localization
- 4 Global localization
- 5 Conclusion

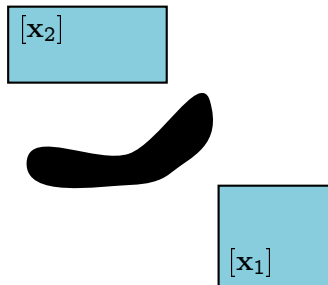
Objectives

- Developing a contractor associated to the constraint \mathbf{x}_1 sees \mathbf{x}_2



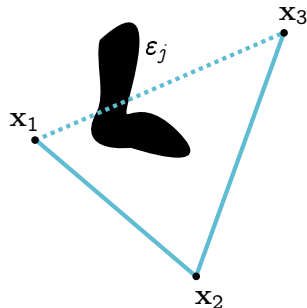
Objectives

- Developing a contractor associated to the constraint x_1 *sees* x_2
- Developing a contractor associated to the constraint x_1 *does not see* x_2



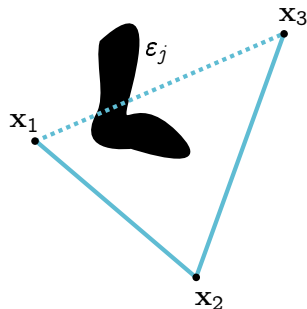
Visibility between two points

- $(\mathbf{x}_1 \mathbf{V} \mathbf{x}_2)_{\varepsilon_j} \Leftrightarrow \text{Seg}(\mathbf{x}_1, \mathbf{x}_2) \cap \varepsilon_j = \emptyset$
 - ε_j : connected subset of \mathbb{R}^n , with $\mathbf{x}_1 \notin \varepsilon_j$ and $\mathbf{x}_2 \notin \varepsilon_j$



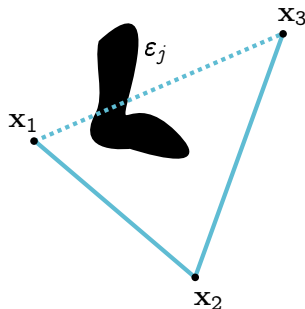
Visibility between two points

- $(\mathbf{x}_1 \mathbf{V} \mathbf{x}_2)_{\varepsilon_j} \Leftrightarrow \text{Seg}(\mathbf{x}_1, \mathbf{x}_2) \cap \varepsilon_j = \emptyset$
 - ε_j : connected subset of \mathbb{R}^n , with $\mathbf{x}_1 \notin \varepsilon_j$ and $\mathbf{x}_2 \notin \varepsilon_j$
 - Reflexive relation : $(\mathbf{x}_1 \mathbf{V} \mathbf{x}_1)_{\varepsilon_j}$
 - Symmetric relation : $(\mathbf{x}_1 \mathbf{V} \mathbf{x}_2)_{\varepsilon_j} \equiv (\mathbf{x}_2 \mathbf{V} \mathbf{x}_1)_{\varepsilon_j}$
 - Non-transitive relation : $(\mathbf{x}_1 \mathbf{V} \mathbf{x}_2)_{\varepsilon_j} \wedge (\mathbf{x}_2 \mathbf{V} \mathbf{x}_3)_{\varepsilon_j} \not\Rightarrow (\mathbf{x}_1 \mathbf{V} \mathbf{x}_3)_{\varepsilon_j}$



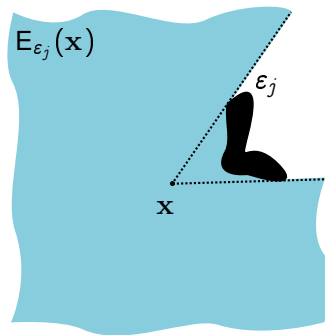
Visibility between two points

- $(\mathbf{x}_1 \mathbf{V} \mathbf{x}_2)_{\varepsilon_j} \Leftrightarrow \text{Seg}(\mathbf{x}_1, \mathbf{x}_2) \cap \varepsilon_j = \emptyset$
 - ε_j : connected subset of \mathbb{R}^n , with $\mathbf{x}_1 \notin \varepsilon_j$ and $\mathbf{x}_2 \notin \varepsilon_j$
 - Reflexive relation : $(\mathbf{x}_1 \mathbf{V} \mathbf{x}_1)_{\varepsilon_j}$
 - Symmetric relation : $(\mathbf{x}_1 \mathbf{V} \mathbf{x}_2)_{\varepsilon_j} \equiv (\mathbf{x}_2 \mathbf{V} \mathbf{x}_1)_{\varepsilon_j}$
 - Non-transitive relation : $(\mathbf{x}_1 \mathbf{V} \mathbf{x}_2)_{\varepsilon_j} \wedge (\mathbf{x}_2 \mathbf{V} \mathbf{x}_3)_{\varepsilon_j} \not\Rightarrow (\mathbf{x}_1 \mathbf{V} \mathbf{x}_3)_{\varepsilon_j}$
 - $\left((\mathbf{x}_1 \mathbf{V} \mathbf{x}_3)_{\varepsilon_j} \right)^c = (\mathbf{x}_1 \bar{\mathbf{V}} \mathbf{x}_3)_{\varepsilon_j}$



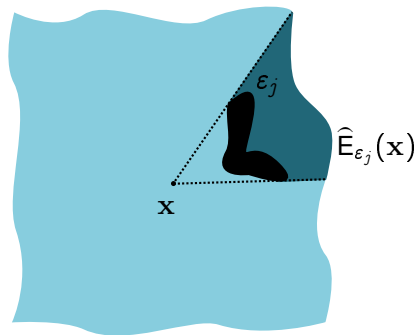
Visibility spaces of a point

- $E_{\varepsilon_j}(\mathbf{x}) = \{\mathbf{x}_i \in \mathbb{R}^n \mid (\mathbf{x}_i \vee \mathbf{x})_{\varepsilon_j}\}$



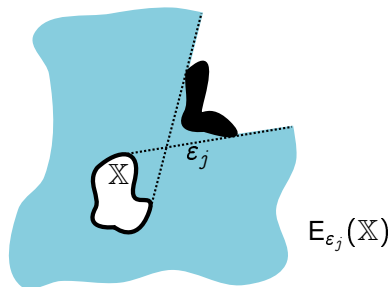
Visibility spaces of a point

- $E_{\varepsilon_j}(\mathbf{x}) = \{\mathbf{x}_i \in \mathbb{R}^n \mid (\mathbf{x}_i \vee \mathbf{x})_{\varepsilon_j}\}$
 - $\widehat{E}_{\varepsilon_j}(\mathbf{x}) = \{\mathbf{x}_i \in \mathbb{R}^n \mid (\mathbf{x}_i \bar{\vee} \mathbf{x})_{\varepsilon_j}\}$
- $\rightarrow (E_{\varepsilon_j}(\mathbf{x}))^c = \widehat{E}_{\varepsilon_j}(\mathbf{x})$



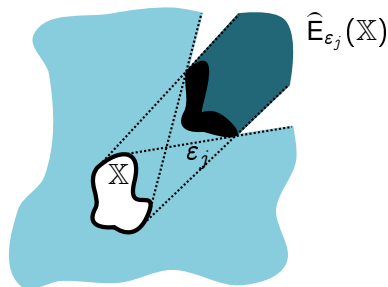
Visibility spaces of a set

- $E_{\varepsilon_j}(\mathbb{X}) = \{\mathbf{x}_i \in \mathbb{R}^n \mid \forall \mathbf{x} \in \mathbb{X}, (\mathbf{x}_i \vee \mathbf{x})_{\varepsilon_j}\}$



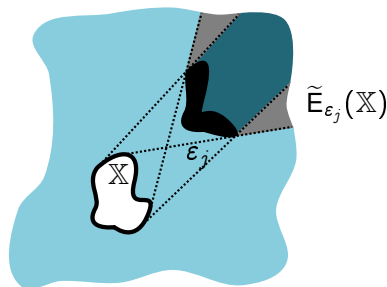
Visibility spaces of a set

- $E_{\varepsilon_j}(\mathbb{X}) = \{\mathbf{x}_i \in \mathbb{R}^n \mid \forall \mathbf{x} \in \mathbb{X}, (\mathbf{x}_i \vee \mathbf{x})_{\varepsilon_j}\}$
- $\hat{E}_{\varepsilon_j}(\mathbb{X}) = \{\mathbf{x}_i \in \mathbb{R}^n \mid \forall \mathbf{x} \in \mathbb{X}, (\mathbf{x}_i \bar{\vee} \mathbf{x})_{\varepsilon_j}\}$



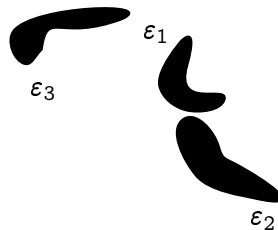
Visibility spaces of a set

- $E_{\varepsilon_j}(\mathbb{X}) = \{\mathbf{x}_i \in \mathbb{R}^n \mid \forall \mathbf{x} \in \mathbb{X}, (\mathbf{x}_i \vee \mathbf{x})_{\varepsilon_j}\}$
- $\widehat{E}_{\varepsilon_j}(\mathbb{X}) = \{\mathbf{x}_i \in \mathbb{R}^n \mid \forall \mathbf{x} \in \mathbb{X}, (\mathbf{x}_i \bar{\vee} \mathbf{x})_{\varepsilon_j}\}$
- $\widetilde{E}_{\varepsilon_j}(\mathbb{X}) = \{\mathbf{x}_i \in \mathbb{R}^n \mid \exists \mathbf{x}_1 \in \mathbb{X}, \exists \mathbf{x}_2 \in \mathbb{X}, (\mathbf{x}_i \vee \mathbf{x}_1)_{\varepsilon_j} \wedge (\mathbf{x}_i \bar{\vee} \mathbf{x}_2)_{\varepsilon_j}\}$



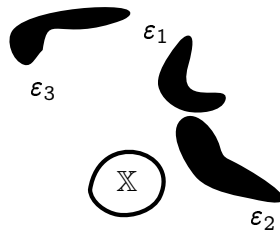
Consideration of an environment

- $\mathcal{E} = \bigcup_{j=1}^{n_O} \varepsilon_j$



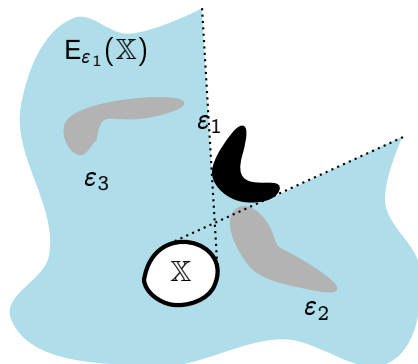
Consideration of an environment

- $\mathcal{E} = \bigcup_{j=1}^{n_O} \varepsilon_j$
- $E_{\mathcal{E}}(\mathbb{X}) = \bigcap_{j=1}^{n_O} E_{\varepsilon_j}(\mathbb{X})$



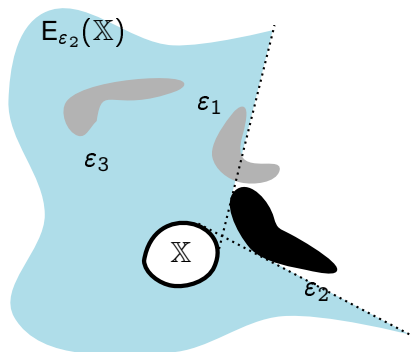
Consideration of an environment

- $\mathcal{E} = \bigcup_{j=1}^{n_O} \varepsilon_j$
- $E_{\mathcal{E}}(\mathbb{X}) = \bigcap_{j=1}^{n_O} E_{\varepsilon_j}(\mathbb{X})$



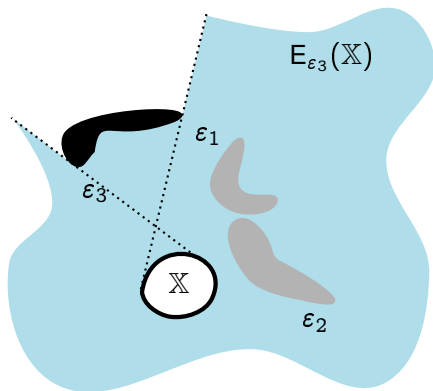
Consideration of an environment

- $\mathcal{E} = \bigcup_{j=1}^{n_O} \varepsilon_j$
- $E_{\mathcal{E}}(\mathbb{X}) = \bigcap_{j=1}^{n_O} E_{\varepsilon_j}(\mathbb{X})$



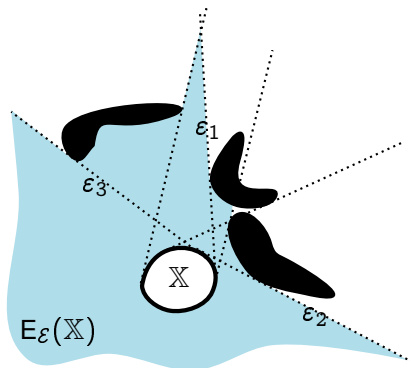
Consideration of an environment

- $\mathcal{E} = \bigcup_{j=1}^{n_O} \varepsilon_j$
- $E_{\mathcal{E}}(\mathbb{X}) = \bigcap_{j=1}^{n_O} E_{\varepsilon_j}(\mathbb{X})$



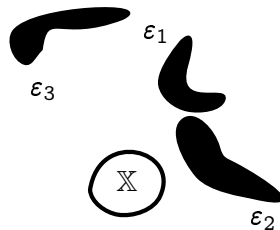
Consideration of an environment

- $\mathcal{E} = \bigcup_{j=1}^{n_O} \varepsilon_j$
- $E_{\mathcal{E}}(\mathbb{X}) = \bigcap_{j=1}^{n_O} E_{\varepsilon_j}(\mathbb{X})$



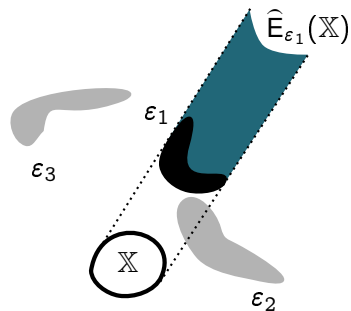
Consideration of an environment

- $\mathcal{E} = \bigcup_{j=1}^{n_O} \varepsilon_j$
- $E_{\mathcal{E}}(\mathbb{X}) = \bigcap_{j=1}^{n_O} E_{\varepsilon_j}(\mathbb{X})$
- $\hat{E}_{\mathcal{E}}(\mathbb{X}) \supseteq \bigcup_{j=1}^{n_O} \hat{E}_{\varepsilon_j}(\mathbb{X})$



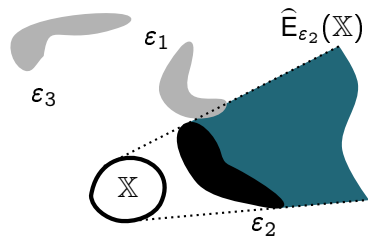
Consideration of an environment

- $\mathcal{E} = \bigcup_{j=1}^{n_O} \varepsilon_j$
- $E_{\mathcal{E}}(\mathbb{X}) = \bigcap_{j=1}^{n_O} E_{\varepsilon_j}(\mathbb{X})$
- $\hat{E}_{\mathcal{E}}(\mathbb{X}) \supseteq \bigcup_{j=1}^{n_O} \hat{E}_{\varepsilon_j}(\mathbb{X})$



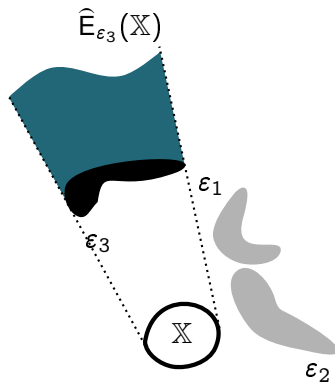
Consideration of an environment

- $\mathcal{E} = \bigcup_{j=1}^{n_O} \varepsilon_j$
- $E_{\mathcal{E}}(\mathbb{X}) = \bigcap_{j=1}^{n_O} E_{\varepsilon_j}(\mathbb{X})$
- $\hat{E}_{\mathcal{E}}(\mathbb{X}) \supseteq \bigcup_{j=1}^{n_O} \hat{E}_{\varepsilon_j}(\mathbb{X})$



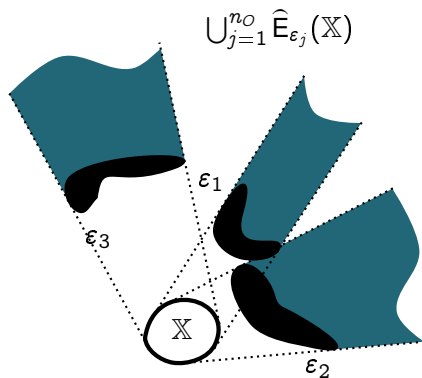
Consideration of an environment

- $\mathcal{E} = \bigcup_{j=1}^{n_O} \varepsilon_j$
- $E_{\mathcal{E}}(\mathbb{X}) = \bigcap_{j=1}^{n_O} E_{\varepsilon_j}(\mathbb{X})$
- $\hat{E}_{\mathcal{E}}(\mathbb{X}) \supseteq \bigcup_{j=1}^{n_O} \hat{E}_{\varepsilon_j}(\mathbb{X})$



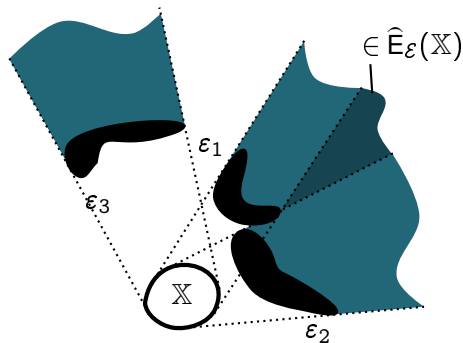
Consideration of an environment

- $\mathcal{E} = \bigcup_{j=1}^{n_O} \varepsilon_j$
- $E_{\mathcal{E}}(\mathbb{X}) = \bigcap_{j=1}^{n_O} E_{\varepsilon_j}(\mathbb{X})$
- $\hat{E}_{\mathcal{E}}(\mathbb{X}) \supseteq \bigcup_{j=1}^{n_O} \hat{E}_{\varepsilon_j}(\mathbb{X})$



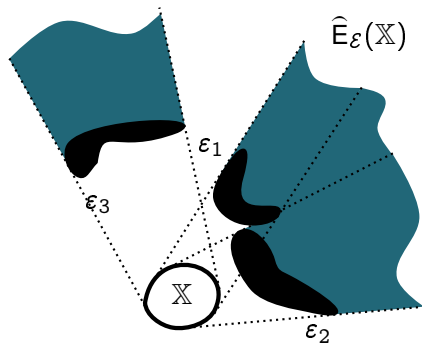
Consideration of an environment

- $\mathcal{E} = \bigcup_{j=1}^{n_O} \varepsilon_j$
- $E_{\mathcal{E}}(\mathbb{X}) = \bigcap_{j=1}^{n_O} E_{\varepsilon_j}(\mathbb{X})$
- $\hat{E}_{\mathcal{E}}(\mathbb{X}) \supseteq \bigcup_{j=1}^{n_O} \hat{E}_{\varepsilon_j}(\mathbb{X})$



Consideration of an environment

- $\mathcal{E} = \bigcup_{j=1}^{n_O} \varepsilon_j$
- $E_{\mathcal{E}}(\mathbb{X}) = \bigcap_{j=1}^{n_O} E_{\varepsilon_j}(\mathbb{X})$
- $\hat{E}_{\mathcal{E}}(\mathbb{X}) \supseteq \bigcup_{j=1}^{n_O} \hat{E}_{\varepsilon_j}(\mathbb{X})$

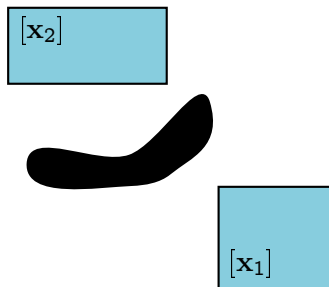


Objectives

- Developing a contractor associated to the constraint $(\mathbf{x}_1 \mathbf{V} \mathbf{x}_2)_{\mathcal{E}}$

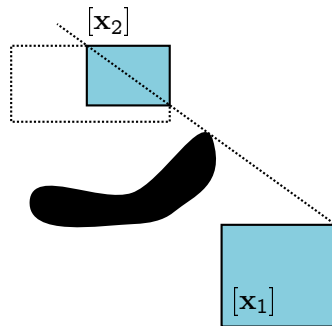
Objectives

- Developing a contractor associated to the constraint $(\mathbf{x}_1 \vee \mathbf{x}_2)_{\mathcal{E}}$
 - $\mathbf{x}_1 \in [\mathbf{x}_1]$ et $\mathbf{x}_2 \in [\mathbf{x}_2]$
 - $(\mathbf{x}_1 \vee \mathbf{x}_2)_{\mathcal{E}} \Rightarrow \mathbf{x}_2 \notin \hat{\mathbf{E}}_{\mathcal{E}}([\mathbf{x}_1])$ and $\mathbf{x}_1 \notin \hat{\mathbf{E}}_{\mathcal{E}}([\mathbf{x}_2])$



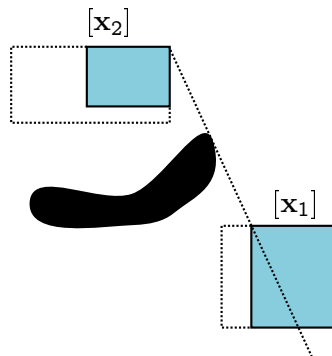
Objectives

- Developing a contractor associated to the constraint $(\mathbf{x}_1 \vee \mathbf{x}_2)_{\mathcal{E}}$
 - $\mathbf{x}_1 \in [\mathbf{x}_1]$ et $\mathbf{x}_2 \in [\mathbf{x}_2]$
 - $(\mathbf{x}_1 \vee \mathbf{x}_2)_{\mathcal{E}} \Rightarrow \mathbf{x}_2 \notin \hat{\mathbf{E}}_{\mathcal{E}}([\mathbf{x}_1])$ and $\mathbf{x}_1 \notin \hat{\mathbf{E}}_{\mathcal{E}}([\mathbf{x}_2])$



Objectives

- Developing a contractor associated to the constraint $(\mathbf{x}_1 \vee \mathbf{x}_2)_{\mathcal{E}}$
 - $\rightarrow \mathbf{x}_1 \in [\mathbf{x}_1]$ et $\mathbf{x}_2 \in [\mathbf{x}_2]$
 - $\rightarrow (\mathbf{x}_1 \vee \mathbf{x}_2)_{\mathcal{E}} \Rightarrow \mathbf{x}_2 \notin \hat{\mathbf{E}}_{\mathcal{E}}([\mathbf{x}_1])$ and $\mathbf{x}_1 \notin \hat{\mathbf{E}}_{\mathcal{E}}([\mathbf{x}_2])$

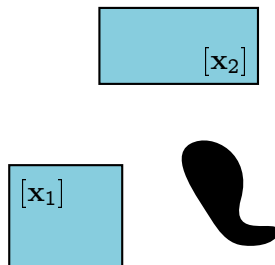


Objectives

- Developing a contractor associated to the constraint $(\mathbf{x}_1 \mathbf{V} \mathbf{x}_2)_{\mathcal{E}}$
- Developing a contractor associated to the constraint $(\mathbf{x}_1 \bar{\mathbf{V}} \mathbf{x}_2)_{\mathcal{E}}$

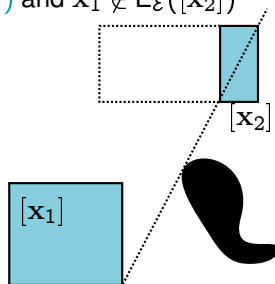
Objectives

- Developing a contractor associated to the constraint $(\mathbf{x}_1 \mathbf{V} \mathbf{x}_2)_\mathcal{E}$
- Developing a contractor associated to the constraint $(\mathbf{x}_1 \bar{\mathbf{V}} \mathbf{x}_2)_\mathcal{E}$
 - $\mathbf{x}_1 \in [\mathbf{x}_1]$ and $\mathbf{x}_2 \in [\mathbf{x}_2]$
 - $(\mathbf{x}_1 \bar{\mathbf{V}} \mathbf{x}_2)_\mathcal{E} \Rightarrow \mathbf{x}_2 \notin E_\mathcal{E}([\mathbf{x}_1])$ and $\mathbf{x}_1 \notin E_\mathcal{E}([\mathbf{x}_2])$



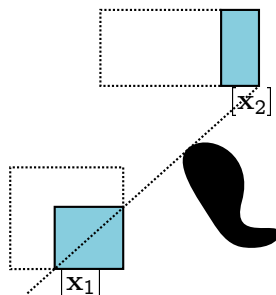
Objectives

- Developing a contractor associated to the constraint $(\mathbf{x}_1 \vee \mathbf{x}_2)_\mathcal{E}$
- Developing a contractor associated to the constraint $(\mathbf{x}_1 \bar{\vee} \mathbf{x}_2)_\mathcal{E}$
 - $\mathbf{x}_1 \in [\mathbf{x}_1]$ and $\mathbf{x}_2 \in [\mathbf{x}_2]$
 - $(\mathbf{x}_1 \bar{\vee} \mathbf{x}_2)_\mathcal{E} \Rightarrow \mathbf{x}_2 \notin E_\mathcal{E}([\mathbf{x}_1])$ and $\mathbf{x}_1 \notin E_\mathcal{E}([\mathbf{x}_2])$



Objectives

- Developing a contractor associated to the constraint $(\mathbf{x}_1 \vee \mathbf{x}_2)_\mathcal{E}$
- Developing a contractor associated to the constraint $(\mathbf{x}_1 \bar{\vee} \mathbf{x}_2)_\mathcal{E}$
 - $\mathbf{x}_1 \in [\mathbf{x}_1]$ and $\mathbf{x}_2 \in [\mathbf{x}_2]$
 - $(\mathbf{x}_1 \bar{\vee} \mathbf{x}_2)_\mathcal{E} \Rightarrow \mathbf{x}_2 \notin E_\mathcal{E}([\mathbf{x}_1])$ and $\mathbf{x}_1 \notin E_\mathcal{E}([\mathbf{x}_2])$

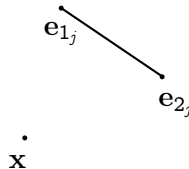


Outlines

- 1 General Presentation
- 2 Visibility contractors**
- 3 Multi-robot localization
- 4 Global localization
- 5 Conclusion

Visibility of a point

- With a segment as obstacle - $\varepsilon_j^s = \text{Seg}(e_{1j}, e_{2j})$



Visibility of a point

- With a segment as obstacle - $\varepsilon_j^s = \text{Seg}(\mathbf{e}_{1_j}, \mathbf{e}_{2_j})$

→ Visible space

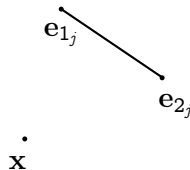
$$E_{\varepsilon_j^s}(\mathbf{x}) = \{\mathbf{x}_i \in \mathbb{R}^2 \mid$$

$$[\mathbf{x}_i \cup \mathbf{x}] \cap [\mathbf{e}_{1_j} \cup \mathbf{e}_{2_j}] = \emptyset \vee$$

$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - \mathbf{e}_{1_j} \mid \mathbf{e}_{2_j} - \mathbf{e}_{1_j}) > 0 \vee$$

$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - \mathbf{e}_{1_j} \mid \mathbf{x} - \mathbf{e}_{1_j}) > 0 \vee$$

$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - \mathbf{e}_{2_j} \mid \mathbf{x} - \mathbf{e}_{2_j}) < 0\}$$



Visibility of a point

- With a segment as obstacle - $\varepsilon_j^s = \text{Seg}(\mathbf{e}_{1j}, \mathbf{e}_{2j})$

→ Visible space

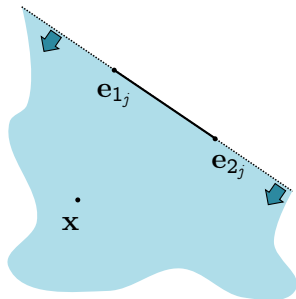
$$E_{\varepsilon_j^s}(\mathbf{x}) = \{\mathbf{x}_i \in \mathbb{R}^2 \mid$$

$$[\mathbf{x}_i \cup \mathbf{x}] \cap [\mathbf{e}_{1j} \cup \mathbf{e}_{2j}] = \emptyset \vee$$

$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - \mathbf{e}_{1j} \mid \mathbf{e}_{2j} - \mathbf{e}_{1j}) > 0 \vee$$

$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - \mathbf{e}_{1j} \mid \mathbf{x} - \mathbf{e}_{1j}) > 0 \vee$$

$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - \mathbf{e}_{2j} \mid \mathbf{x} - \mathbf{e}_{2j}) < 0\}$$



Visibility of a point

- With a segment as obstacle - $\varepsilon_j^s = \text{Seg}(\mathbf{e}_{1j}, \mathbf{e}_{2j})$

→ Visible space

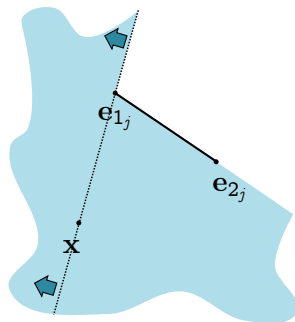
$$E_{\varepsilon_j^s}(\mathbf{x}) = \{\mathbf{x}_i \in \mathbb{R}^2 \mid$$

$$[\mathbf{x}_i \cup \mathbf{x}] \cap [\mathbf{e}_{1j} \cup \mathbf{e}_{2j}] = \emptyset \vee$$

$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - \mathbf{e}_{1j} \mid \mathbf{e}_{2j} - \mathbf{e}_{1j}) > 0 \vee$$

$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - \mathbf{e}_{1j} \mid \mathbf{x} - \mathbf{e}_{1j}) > 0 \vee$$

$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - \mathbf{e}_{2j} \mid \mathbf{x} - \mathbf{e}_{2j}) < 0\}$$



Visibility of a point

- With a segment as obstacle - $\varepsilon_j^s = \text{Seg}(\mathbf{e}_{1_j}, \mathbf{e}_{2_j})$

→ Visible space

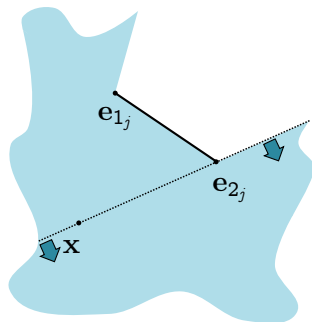
$$E_{\varepsilon_j^s}(\mathbf{x}) = \{\mathbf{x}_i \in \mathbb{R}^2 \mid$$

$$[\mathbf{x}_i \cup \mathbf{x}] \cap [\mathbf{e}_{1_j} \cup \mathbf{e}_{2_j}] = \emptyset \vee$$

$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - \mathbf{e}_{1_j} \mid \mathbf{e}_{2_j} - \mathbf{e}_{1_j}) > 0 \vee$$

$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - \mathbf{e}_{1_j} \mid \mathbf{x} - \mathbf{e}_{1_j}) > 0 \vee$$

$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - \mathbf{e}_{2_j} \mid \mathbf{x} - \mathbf{e}_{2_j}) < 0\}$$



Visibility of a point

- With a segment as obstacle - $\varepsilon_j^s = \text{Seg}(\mathbf{e}_{1j}, \mathbf{e}_{2j})$

→ Visible space

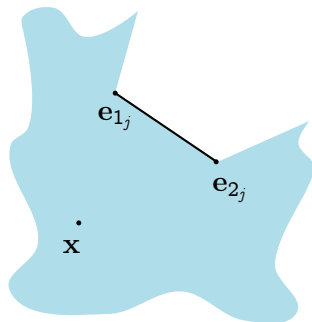
$$E_{\varepsilon_j^s}(\mathbf{x}) = \{\mathbf{x}_i \in \mathbb{R}^2 \mid$$

$$[\mathbf{x}_i \cup \mathbf{x}] \cap [\mathbf{e}_{1j} \cup \mathbf{e}_{2j}] = \emptyset \vee$$

$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - \mathbf{e}_{1j} \mid \mathbf{e}_{2j} - \mathbf{e}_{1j}) > 0 \vee$$

$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - \mathbf{e}_{1j} \mid \mathbf{x} - \mathbf{e}_{1j}) > 0 \vee$$

$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - \mathbf{e}_{2j} \mid \mathbf{x} - \mathbf{e}_{2j}) < 0\}$$



Visibility of a point

- With a segment as obstacle - $\varepsilon_j^s = \text{Seg}(e_{1j}, e_{2j})$
 - Visible space
 - Non-visible space

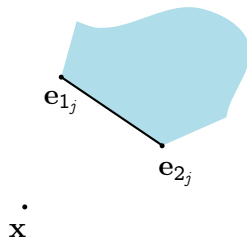
$$\widehat{E}_{\varepsilon_j^s}(\mathbf{x}) = \{\mathbf{x}_i \in \mathbb{R}^2 \mid$$

$$[\mathbf{x}_i \cup \mathbf{x}] \cap [e_{1j} \cup e_{2j}] \neq \emptyset \wedge$$

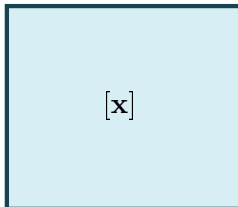
$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - e_{1j} \mid e_{2j} - e_{1j}) \leq 0 \wedge$$

$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - e_{1j} \mid \mathbf{x} - e_{1j}) \leq 0 \wedge$$

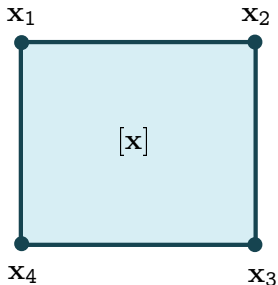
$$\zeta_{\mathbf{x}} \det(\mathbf{x}_i - e_{2j} \mid \mathbf{x} - e_{2j}) \geq 0\}$$



Visibility of a segment

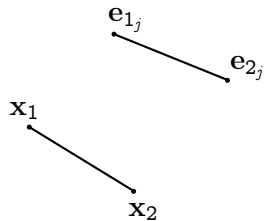


Visibility of a segment



Visibility of a segment

- With a segment as obstacle - $\varepsilon_j^s = \text{Seg}(e_{1j}, e_{2j})$



Visibility of a segment

- With a segment as obstacle - $\varepsilon_j^s = \text{Seg}(\mathbf{e}_{1j}, \mathbf{e}_{2j})$

→ Visible space - $E_{\varepsilon_j^s}(\text{Seg}(\mathbf{x}_1, \mathbf{x}_2))$

$$E_{\varepsilon_j^s}(\text{Seg}(\mathbf{x}_1, \mathbf{x}_2)) = \{\mathbf{x}_i \in \mathbb{R}^2 \mid$$

$$(\zeta_{x_1} = \zeta_{x_2}) \wedge \left(\zeta_{x_1} \det(\mathbf{x}_i - \mathbf{e}_{1j} \mid \mathbf{e}_{2j} - \mathbf{e}_{1j}) > 0 \vee$$

$$\zeta_{x_1} \det(\mathbf{x}_i - \mathbf{e}_{1j} \mid \mathbf{x}_1 - \mathbf{e}_{1j}) > 0 \wedge \zeta_{x_2} \det(\mathbf{x}_i - \mathbf{e}_{1j} \mid \mathbf{x}_2 - \mathbf{e}_{1j}) > 0 \vee$$

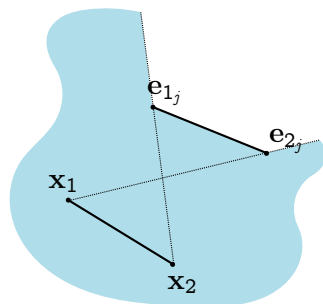
$$\zeta_{x_1} \det(\mathbf{x}_i - \mathbf{e}_{2j} \mid \mathbf{x}_1 - \mathbf{e}_{2j}) < 0 \wedge \zeta_{x_2} \det(\mathbf{x}_i - \mathbf{e}_{2j} \mid \mathbf{x}_2 - \mathbf{e}_{2j}) < 0 \right) \vee$$

$$(\zeta_{x_1} = -\zeta_{x_2}) \wedge \left($$

$$\left(\zeta_{e_1} \det(\mathbf{x}_i - \mathbf{e}_{1j} \mid \mathbf{x}_1 - \mathbf{e}_{1j}) > 0 \vee \zeta_{e_1} \det(\mathbf{x}_i - \mathbf{e}_{1j} \mid \mathbf{x}_2 - \mathbf{e}_{1j}) < 0 \right) \wedge$$

$$\left(\zeta_{e_2} \det(\mathbf{x}_i - \mathbf{e}_{2j} \mid \mathbf{x}_1 - \mathbf{e}_{2j}) > 0 \vee \zeta_{e_2} \det(\mathbf{x}_i - \mathbf{e}_{2j} \mid \mathbf{x}_2 - \mathbf{e}_{2j}) < 0 \right) \right) \vee$$

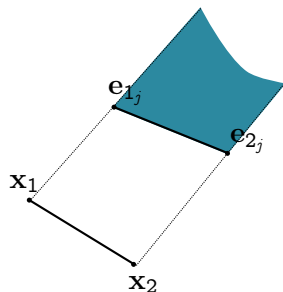
$$\left([\mathbf{x}_i \cup \mathbf{x}_1 \cup \mathbf{x}_2] \cap [\mathbf{e}_{1j} \cup \mathbf{e}_{2j}] = \emptyset \right) \}.$$



Visibility of a segment

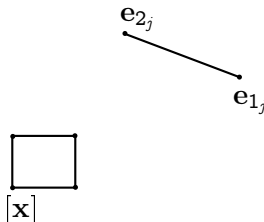
- With a segment as obstacle - $\varepsilon_j^s = \text{Seg}(e_{1j}, e_{2j})$
 - Visible space - $E_{\varepsilon_j^s}(\text{Seg}(x_1, x_2))$
 - Non-visible space - $\widehat{E}_{\varepsilon_j^s}(\text{Seg}(x_1, x_2))$

$$\widehat{E}_{\varepsilon_j^s}(\text{Seg}(x_1, x_2)) = \widehat{E}_{\varepsilon_j^s}(x_1) \cap \widehat{E}_{\varepsilon_j^s}(x_2)$$



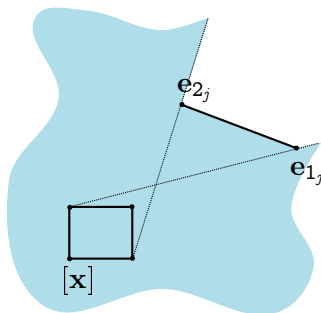
Visibility of a box

- With a segment as obstacle - $\varepsilon_j^s = \text{Seg}(e_{1j}, e_{2j})$



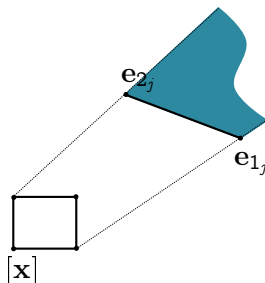
Visibility of a box

- With a segment as obstacle - $\varepsilon_j^s = \text{Seg}(e_{1j}, e_{2j})$
→ Visible space - $E_{\varepsilon_j^s}([\mathbf{x}])$



Visibility of a box

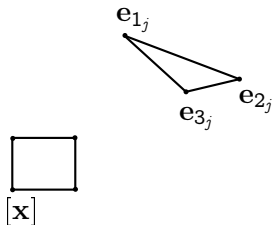
- With a segment as obstacle - $\varepsilon_j^s = \text{Seg}(e_{1j}, e_{2j})$
 - Visible space - $E_{\varepsilon_j^s}([\mathbf{x}])$
 - Non-visible space - $\widehat{E}_{\varepsilon_j^s}([\mathbf{x}])$



With a convex polygon as obstacle

- Convex polygon : set of segments

$$\rightarrow \varepsilon_j^p = \bigcup_{k=1}^{n_{P_j}} \varepsilon_k^s$$

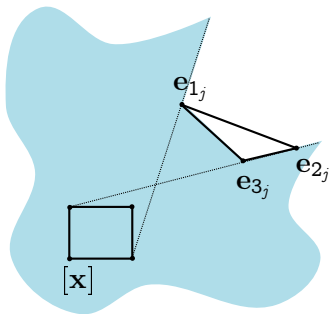


With a convex polygon as obstacle

- Convex polygon : set of segments

$$\rightarrow \varepsilon_j^p = \bigcup_{k=1}^{n_{P_j}} \varepsilon_k^s$$

$$\rightarrow E_{\varepsilon_j^p}([\mathbf{x}]) = \bigcap_{k=1}^{n_{P_j}} E_{\varepsilon_k^s}([\mathbf{x}])$$



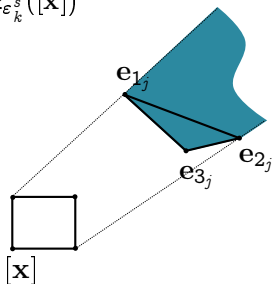
With a convex polygon as obstacle

- Convex polygon : set of segments

$$\rightarrow \varepsilon_j^p = \bigcup_{k=1}^{n_{P_j}} \varepsilon_k^s$$

$$\rightarrow E_{\varepsilon_j^p}([\mathbf{x}]) = \bigcap_{k=1}^{n_{P_j}} E_{\varepsilon_k^s}([\mathbf{x}])$$

$$\rightarrow \widehat{E}_{\varepsilon_j^p}([\mathbf{x}]) = \bigcup_{k=1}^{n_{P_j}} \widehat{E}_{\varepsilon_k^s}([\mathbf{x}])$$

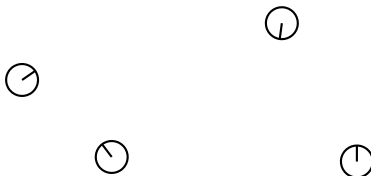


Outlines

- 1 General Presentation
- 2 Visibility contractors
- 3 Multi-robot localization**
- 4 Global localization
- 5 Conclusion

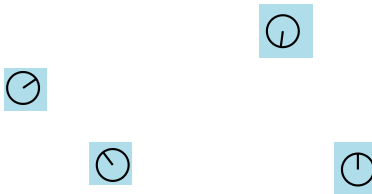
Problem presentation

- Team of robots



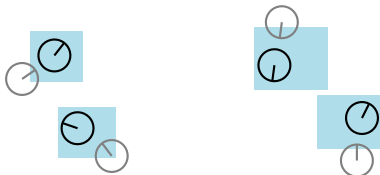
Problem presentation

- Team of robots
- Initial poses known



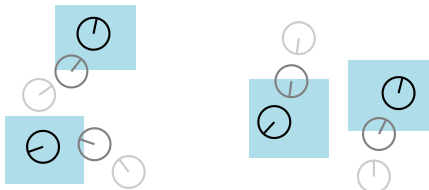
Problem presentation

- Team of robots
- Initial poses known



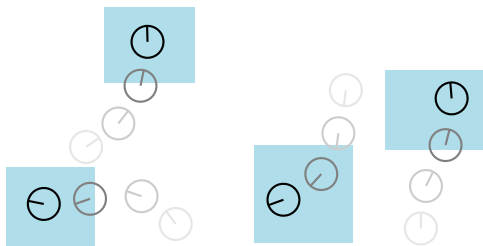
Problem presentation

- Team of robots
- Initial poses known



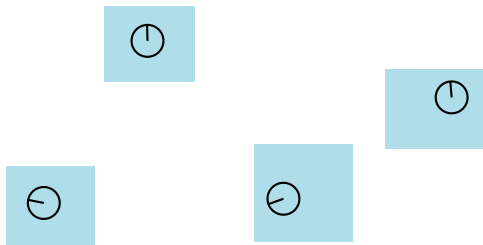
Problem presentation

- Team of robots
- Initial poses known



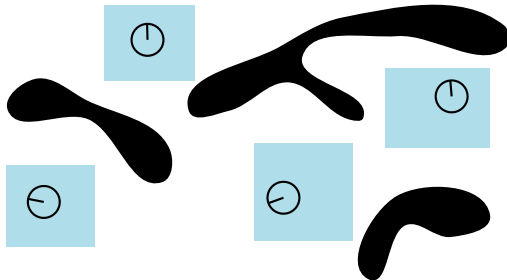
Problem presentation

- Team of robots
- Initial poses known
 - The robots are drifting



Problem presentation

- Team of robots
- Initial poses known
- Known environment



Problem presentation

- Team of robots
- Initial poses known
- Known environment
- Is it possible to avoid the drifting of the robot by using a boolean information : the visibility between the robots ?

The robots

- Bounded error context

$$\rightarrow \mathbf{q}_{i,0} \in [\mathbf{q}_{i,0}]$$

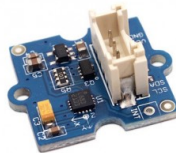
The robots

- Bounded error context
- Evaluation of the orientation by using a compass

$$\rightarrow \theta_{i,k} \in [\theta_{i,k}]$$



Compass CMPS10



Compass SEN12753P

The robots

- Bounded error context
- Evaluation of the orientation by using a compass
- Inter-robot communication
 - At each time step k each robot knows the position estimation of all the robots

The robots

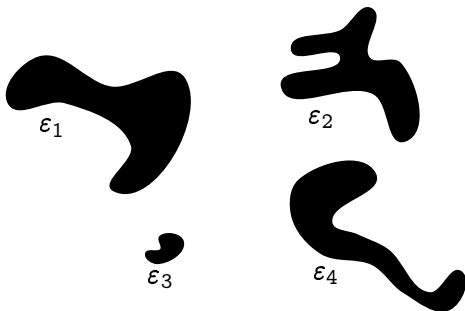
- Bounded error context
- Evaluation of the orientation by using a compass
- Inter-robot communication
- Boolean measurements
 - r_1 sees $r_2 \Leftrightarrow (\mathbf{x}_1 \mathbf{V} \mathbf{x}_2)_{\mathcal{E}}$
 - r_1 does not see $r_2 \Leftrightarrow (\mathbf{x}_1 \bar{\mathbf{V}} \mathbf{x}_2)_{\mathcal{E}}$
 - $\mathbf{z}_{i,k} = \{0, 1, \dots, 1\}$
 - 0 : the robot r_i does not see the first robot
 - 1 : the robot r_i sees the second robot
 - ...
 - 1 : the robot r_i sees the last robot

Environment characterisations

- Environment \mathcal{E}

→ $\mathcal{E} = \bigcup_{j=1}^{n_o} \varepsilon_j$

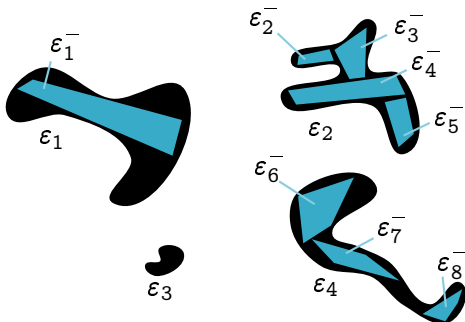
→ Sets of convex polygons



Environment characterisations

- Environment \mathcal{E}
- Inner characterisation \mathcal{E}^-

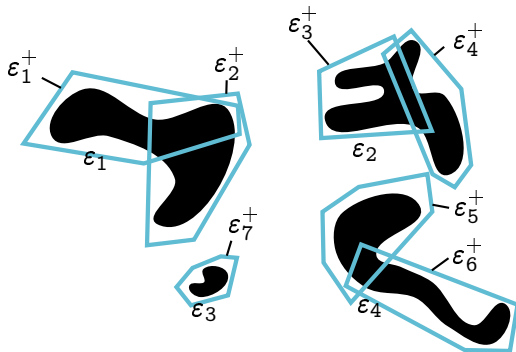
$$\rightarrow \mathcal{E}^- \subseteq \mathcal{E}$$



Environment characterisations

- Environment \mathcal{E}
- Inner characterisation \mathcal{E}^-
- Outer characterisation \mathcal{E}^+

$$\rightarrow \mathcal{E} \subseteq \mathcal{E}^+$$



Environment characterisations

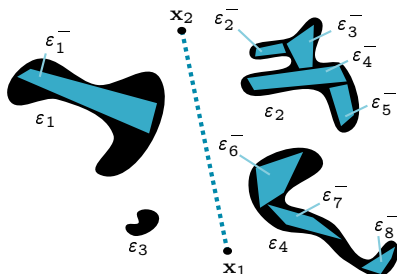
- Environment \mathcal{E}
- Inner characterisation \mathcal{E}^-
- Outer characterisation \mathcal{E}^+
- **Environment/Characterisations**

Environment characterisations

- Environment \mathcal{E}
- Inner characterisation \mathcal{E}^-
- Outer characterisation \mathcal{E}^+
- Environment/Characterisations

→ r_1 sees $r_2 \Rightarrow (\mathbf{x}_1 \vee \mathbf{x}_2)_{\mathcal{E}^-}$

- $\mathbf{x}_1 \in E_{\mathcal{E}^-}(\mathbf{x}_2)$, $\mathbf{x}_1 \notin \widehat{E}_{\mathcal{E}^-}(\mathbf{x}_2)$
- $\mathbf{x}_2 \in E_{\mathcal{E}^-}(\mathbf{x}_1)$, $\mathbf{x}_2 \notin \widehat{E}_{\mathcal{E}^-}(\mathbf{x}_1)$



Environment characterisations

- Environment \mathcal{E}
- Inner characterisation \mathcal{E}^-
- Outer characterisation \mathcal{E}^+
- Environment/Characterisations

→ r_1 sees $r_2 \Rightarrow (\mathbf{x}_1 \mathbf{V} \mathbf{x}_2)_{\mathcal{E}^-}$

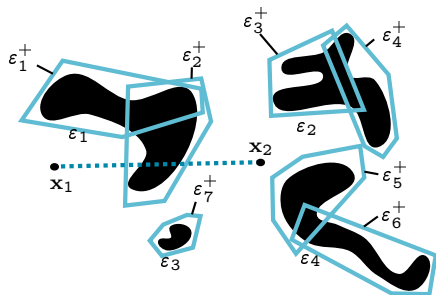
– $\mathbf{x}_1 \in \mathbf{E}_{\mathcal{E}^-}(\mathbf{x}_2)$, $\mathbf{x}_1 \notin \widehat{\mathbf{E}}_{\mathcal{E}^-}(\mathbf{x}_2)$

– $\mathbf{x}_2 \in \mathbf{E}_{\mathcal{E}^-}(\mathbf{x}_1)$, $\mathbf{x}_2 \notin \widehat{\mathbf{E}}_{\mathcal{E}^-}(\mathbf{x}_1)$

→ r_1 does not see $r_2 \Rightarrow (\mathbf{x}_1 \overline{\mathbf{V}} \mathbf{x}_2)_{\mathcal{E}^+}$

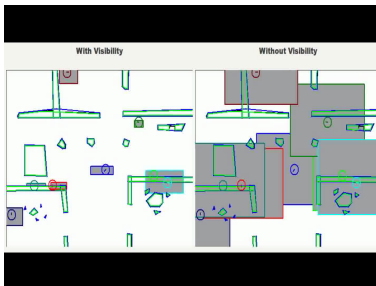
– $\mathbf{x}_1 \in \widehat{\mathbf{E}}_{\mathcal{E}^+}(\mathbf{x}_2)$, $\mathbf{x}_1 \notin \mathbf{E}_{\mathcal{E}^+}(\mathbf{x}_2)$

– $\mathbf{x}_2 \in \widehat{\mathbf{E}}_{\mathcal{E}^+}(\mathbf{x}_1)$, $\mathbf{x}_2 \notin \mathbf{E}_{\mathcal{E}^+}(\mathbf{x}_1)$



Results

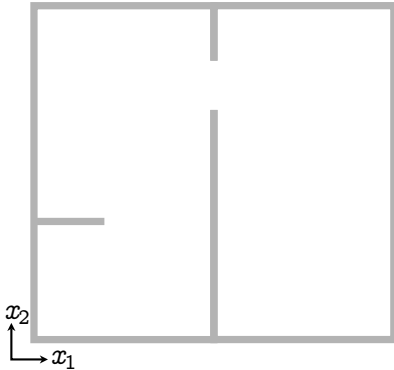
- Simulator



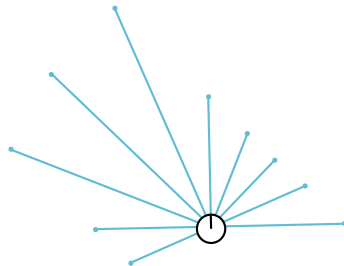
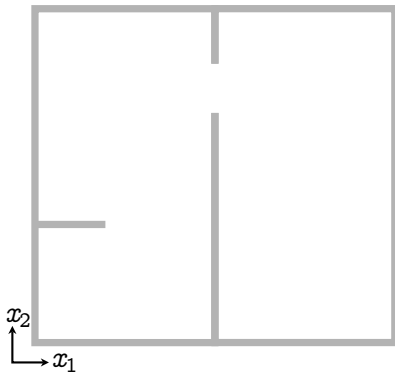
Outlines

- 1 General Presentation
- 2 Visibility contractors
- 3 Multi-robot localization
- 4 Global localization**
- 5 Conclusion

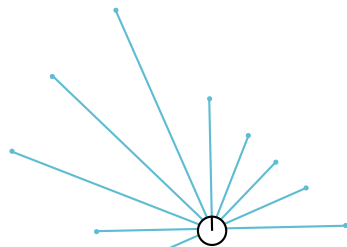
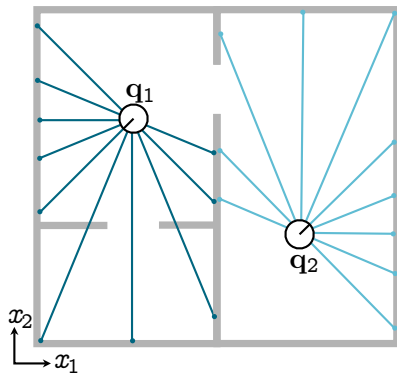
Considered problem



Considered problem

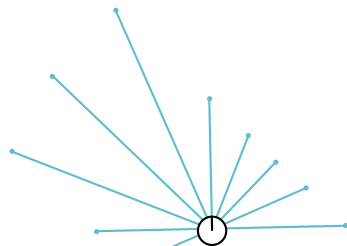
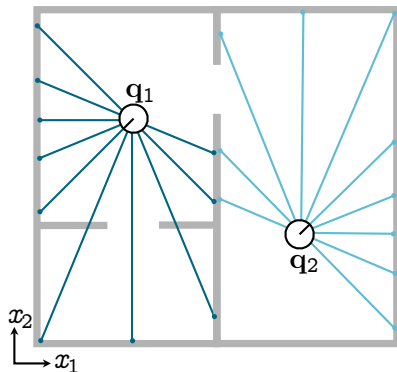


Considered problem



Two poses are consistent with the constraints

Considered problem



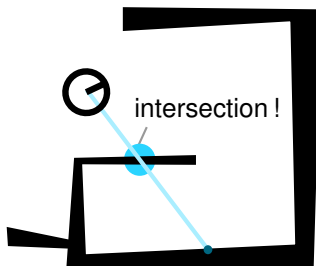
How to process the measurement intersection constraint ?

Considering an original constraint

- Presentation of the constraint

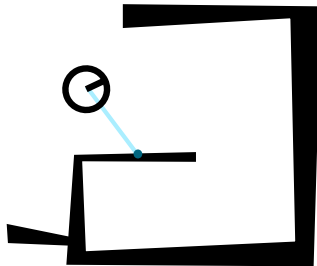
Considering an original constraint

- Presentation of the constraint
 - the measurement must not intersect the obstacle



Considering an original constraint

- Presentation of the constraint
 - the measurement must not intersect the obstacle

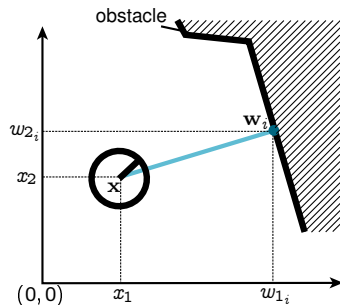


Considering an original constraint

- Presentation of the constraint
- Constraint formalisation

Considering an original constraint

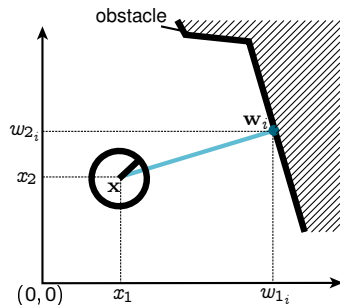
- Presentation of the constraint
- Constraint formalisation



→ Visibility relations between the robot and all the detected obstacles

Considering an original constraint

- Presentation of the constraint
- Constraint formalisation



→ Visibility relations between the robot and all the detected obstacles

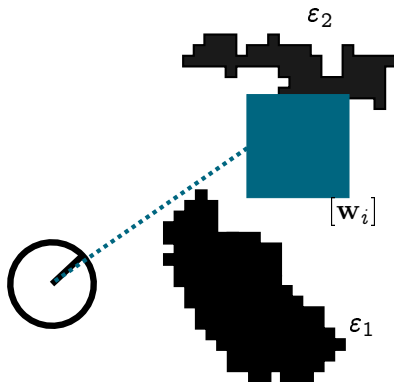
→ $\forall \varepsilon_j \in \mathcal{E}, \forall i, (\mathbf{x} \mathbf{V} \mathbf{w}_i)_{\varepsilon_j}$

Environment characterization

- Contraction over a visibility information
- Inner characterisation

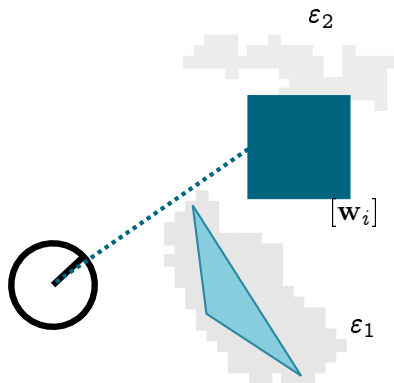
Environment characterization

- Contraction over a visibility information
- Inner characterisation



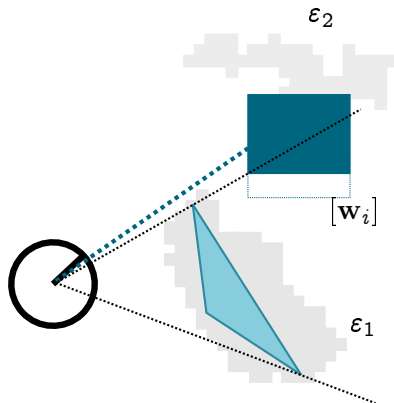
Environment characterization

- Contraction over a visibility information
- Inner characterisation



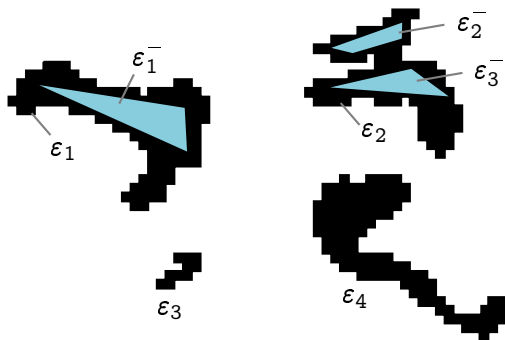
Environment characterization

- Contraction over a visibility information
- Inner characterisation

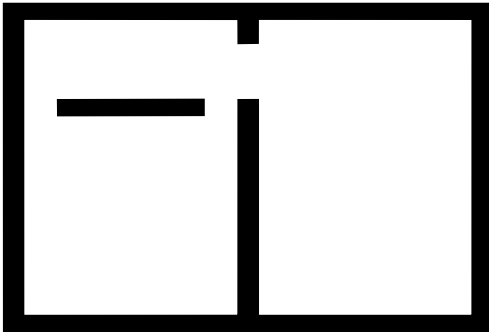


Environment characterization

- Contraction over a visibility information
- Inner characterisation
- Environment characterisation

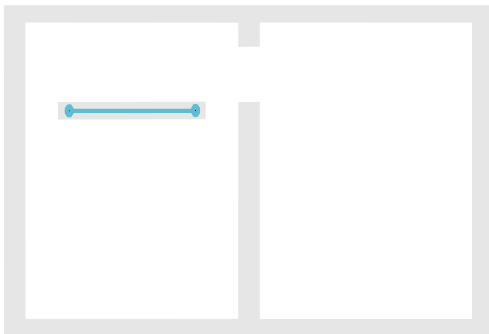


Example



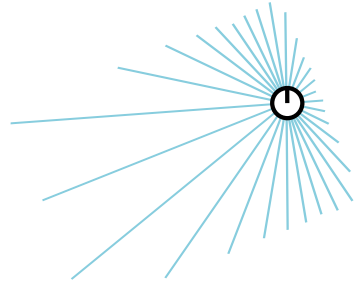
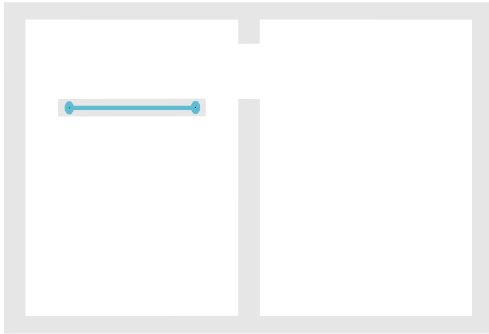
Considered environment

Example



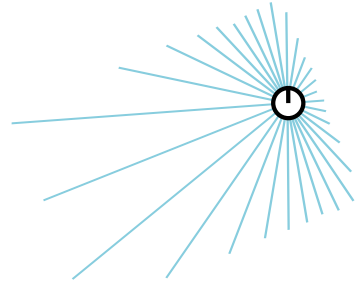
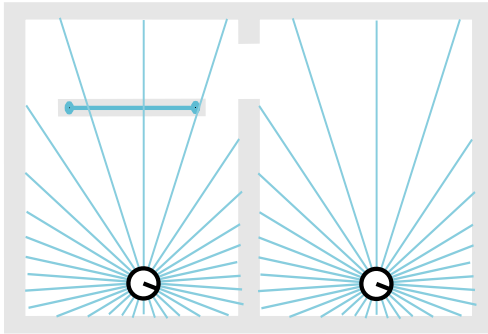
Inner characterisation

Example



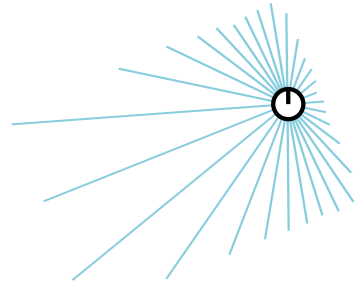
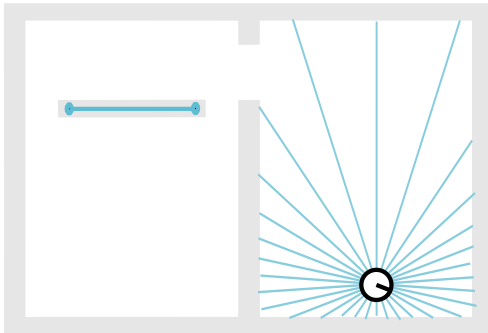
Considered measurements

Example



Result without visibility

Example



Result with visibility

Conclusion

- Original formalisation of the visibility information
- Optimal visibility contractors
- Two application to mobile robot localization
 - Multi-robot localization
 - Avoid the drifting of the robots
 - Results are critical over the number of robots and the environment topology
 - The contractors can be added to classical localization approach
 - Original measurement intersection constraint
 - Improvement of the localization results
 - Avoidance of symmetries
 - Those contractors could be useful to other application