

Introduction

- \rightarrow Visibility is studied and used in several fields
 - Computer graphics
 - Telecommunication
 - Robotics...
- ightarrow Usually associated to bearing or ranging data
- ightarrow We consider the visibility as a boolean information
 - Application to mobile robot localization

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Introduction

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 - Application to mobile robot localization



Outlines

- General Presentation
- 2 Visibility contractors
- 3 Multi-robot localization
- 4 Global localization
- **5** Conclusion

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Outlines

General Presentation

- 2 Visibility contractors
- 3 Multi-robot localization
- 4 Global localization
- **5** Conclusion

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General Presentation	Multi-robot localization	
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Objectives		

• Developing a contractor associated to the constraint x_1 sees x_2



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General Presentation	Multi-robot localization	
Objectives		

- Developing a contractor associated to the constraint x₁ sees x₂
- Developing a contractor associated to the constraint \mathbf{x}_1 does not see \mathbf{x}_2



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Visibility between two points

•
$$(\mathbf{x}_1 \forall \mathbf{x}_2)_{\varepsilon_j} \Leftrightarrow Seg(\mathbf{x}_1, \mathbf{x}_2) \cap \varepsilon_j = \emptyset$$

 $\to \ arepsilon_j$: connected subset of \mathbb{R}^n , with $\mathbf{x}_1
ot\in arepsilon_j$ and $\mathbf{x}_2
ot\in arepsilon_j$



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Visibility between two points

•
$$(\mathbf{x}_1 \mathsf{V} \mathbf{x}_2)_{arepsilon_j} \Leftrightarrow Seg(\mathbf{x}_1, \mathbf{x}_2) \cap arepsilon_j = \emptyset$$

- $\rightarrow \varepsilon_j$: connected subset of \mathbb{R}^n , with $\mathbf{x}_1 \not\in \varepsilon_j$ and $\mathbf{x}_2 \not\in \varepsilon_j$
- \rightarrow Reflexive relation : $(\mathbf{x}_1 V \mathbf{x}_1)_{\varepsilon_i}$
- \rightarrow Symmetric relation : $(\mathbf{x}_1 V \mathbf{x}_2)_{\varepsilon_j} \equiv (\mathbf{x}_2 V \mathbf{x}_1)_{\varepsilon_j}$
- \rightarrow Non-transitive relation : $(\mathbf{x}_1 \vee \mathbf{x}_2)_{\varepsilon_i} \wedge (\mathbf{x}_2 \vee \mathbf{x}_3)_{\varepsilon_i} \not\Rightarrow (\mathbf{x}_1 \vee \mathbf{x}_3)_{\varepsilon_i}$



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$$\rightarrow \left((\mathbf{x}_1 \mathbf{V} \mathbf{x}_3)_{\varepsilon_j} \right)^c = (\mathbf{x}_1 \overline{\mathbf{V}} \mathbf{x}_3)_{\varepsilon_j}$$



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Visibility spaces of a point

• $\mathsf{E}_{\varepsilon_j}(\mathbf{x}) = \{\mathbf{x}_i \in \mathbb{R}^n \mid (\mathbf{x}_i \mathsf{V} \mathbf{x})_{\varepsilon_j}\}$



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Visibility spaces of a point

•
$$\mathsf{E}_{\varepsilon_j}(\mathbf{x}) = \{\mathbf{x}_i \in \mathbb{R}^n \mid (\mathbf{x}_i \vee \mathbf{x})_{\varepsilon_j}\}$$

• $\widehat{\mathsf{E}}_{\varepsilon_j}(\mathbf{x}) = \{\mathbf{x}_i \in \mathbb{R}^n \mid (\mathbf{x}_i \nabla \mathbf{x})_{\varepsilon_j}\}$
 $\rightarrow (\mathsf{E}_{\varepsilon_j}(\mathbf{x}))^c = \widehat{\mathsf{E}}_{\varepsilon_j}(\mathbf{x})$



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Visibility spaces of a set

• $\mathsf{E}_{\varepsilon_j}(\mathbb{X}) = \{\mathbf{x}_i \in \mathbb{R}^n \mid \forall \mathbf{x} \in \mathbb{X}, (\mathbf{x}_i \lor \mathbf{x})_{\varepsilon_j}\}$



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Visibility spaces of a set

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- $\widehat{\mathsf{E}}_{\varepsilon_j}(\mathbb{X}) = \{\mathbf{x}_i \in \mathbb{R}^n \mid \forall \mathbf{x} \in \mathbb{X}, (\mathbf{x}_i \overline{\mathsf{V}} \mathbf{x})_{\varepsilon_j}\}$
- $\widetilde{\mathsf{E}}_{\varepsilon_j}(\mathbb{X}) = \{\mathbf{x}_i \in \mathbb{R}^n \mid \exists \mathbf{x}_1 \in \mathbb{X}, \exists \mathbf{x}_2 \in \mathbb{X}, (\mathbf{x}_i \mathsf{V} \mathbf{x}_1)_{\varepsilon_j} \land (\mathbf{x}_i \overline{\mathsf{V}} \mathbf{x}_2)_{\varepsilon_j}\}$



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• $\mathcal{E} = \bigcup_{j=1}^{n_O} \varepsilon_j$ • $\mathsf{E}_{\mathcal{E}}(\mathbb{X}) = \bigcap_{j=1}^{n_O} \mathsf{E}_{\varepsilon_j}(\mathbb{X})$



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- $\widehat{\mathsf{E}}_{\mathcal{E}}(\mathbb{X}) \supseteq \bigcup_{j=1}^{n_O} \widehat{\mathsf{E}}_{\varepsilon_j}(\mathbb{X})$



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General Presentation	Multi-robot localization	
Objectives		

• Developing a contractor associated to the constraint $(x_1 V x_2)_{\mathcal{E}}$

General Presentation	Visibility contractors	Multi-robot localization	Global localization	Conclusion
Objectives				

• Developing a contractor associated to the constraint $(x_1 V x_2)_{\mathcal{E}}$

$$\begin{array}{l} \rightarrow \ \mathbf{x}_1 \in [\mathbf{x}_1] \text{ et } \mathbf{x}_2 \in [\mathbf{x}_2] \\ \rightarrow \ (\mathbf{x}_1 \mathsf{V} \mathbf{x}_2)_{\mathcal{E}} \Rightarrow \mathbf{x}_2 \not\in \widehat{\mathsf{E}}_{\mathcal{E}}([\mathbf{x}_1]) \text{ and } \mathbf{x}_1 \notin \widehat{\mathsf{E}}_{\mathcal{E}}([\mathbf{x}_2]) \end{array}$$



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General Presentation	Visibility contractors	Multi-robot localization	Global localization	Conclusion
Objectives				

Developing a contractor associated to the constraint (x₁Vx₂)_E

 $\begin{array}{l} \rightarrow \ \mathbf{x}_1 \in [\mathbf{x}_1] \text{ et } \mathbf{x}_2 \in [\mathbf{x}_2] \\ \rightarrow \ (\mathbf{x}_1 \mathsf{V} \mathbf{x}_2)_{\mathcal{E}} \Rightarrow \mathbf{x}_2 \not \in \widehat{\mathsf{E}}_{\mathcal{E}}([\mathbf{x}_1]) \text{ and } \mathbf{x}_1 \not \in \widehat{\mathsf{E}}_{\mathcal{E}}([\mathbf{x}_2]) \end{array}$



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acricit		visionity contractors		Chobal localization	
Ob	jectives				
	Develop	ing a contractor as	ssociated to the con	straint $(\mathbf{x}_1V\mathbf{x}_2)_\mathcal{E}$	



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 $[\mathbf{x}_1]$

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General Presentation	Multi-robot localization	
Objectives		

- Developing a contractor associated to the constraint $(x_1 V x_2)_{\mathcal{E}}$
- Developing a contractor associated to the constraint $(x_1 \overline{V} x_2)_{\mathcal{E}}$

General Presentation	Multi-robot localization	
Objectives		

- Developing a contractor associated to the constraint $(x_1 V x_2)_{\mathcal{E}}$
- Developing a contractor associated to the constraint $(\mathbf{x}_1 \overline{\mathsf{V}} \mathbf{x}_2)_{\mathcal{E}}$

$$ightarrow \, \mathbf{x}_1 \in [\mathbf{x}_1] ext{ and } \mathbf{x}_2 \in [\mathbf{x}_2]$$

 $\rightarrow \ \big(\mathbf{x}_1 \overline{\mathsf{V}} \mathbf{x}_2 \big)_{\mathcal{E}} \Rightarrow \mathbf{x}_2 \not\in \mathsf{E}_{\mathcal{E}}([\mathbf{x}_1]) \text{ and } \mathbf{x}_1 \not\in \mathsf{E}_{\mathcal{E}}([\mathbf{x}_2])$





General Presentation		Multi-robot localization		
Objectives				
Develop	ing a contractor as	sociated to the con	straint $(\mathbf{x}_1 \mathbf{V} \mathbf{x}_2)_{\mathbf{c}}$	

• Developing a contractor associated to the constraint $(x_1 \overline{V} x_2)_{\mathcal{E}}$



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General Presentation	Multi-robot localization	
Objectives		

- Developing a contractor associated to the constraint $(x_1 V x_2)_{\mathcal{E}}$
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$$\begin{array}{l} \rightarrow \ \mathbf{x}_1 \in [\mathbf{x}_1] \text{ and } \mathbf{x}_2 \in [\mathbf{x}_2] \\ \rightarrow \ (\mathbf{x}_1 \overline{\mathsf{V}} \mathbf{x}_2)_{\mathcal{E}} \Rightarrow \mathbf{x}_2 \notin \mathsf{E}_{\mathcal{E}}([\mathbf{x}_1]) \text{ and } \mathbf{x}_1 \notin \mathsf{E}_{\mathcal{E}}([\mathbf{x}_2]) \end{array}$$



Outlines

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Visibility of a point

• With a segment as obstacle - $\varepsilon_j^s = Seg(\mathbf{e}_{1_j}, \mathbf{e}_{2_j})$



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General Presentation	Visibility contractors	Multi-robot localization		Conclusion		
Visibility of a point						
• With a	segment as obsta	acle - $arepsilon_j^s = Seg(\mathbf{e}_{1_j})$	$,\mathbf{e}_{2_{j}})$			

$$\begin{split} \mathsf{E}_{\varepsilon_{j}^{s}}(\mathbf{x}) =& \{\mathbf{x}_{i} \in \mathbb{R}^{2} \mid \\ & [\mathbf{x}_{i} \cup \mathbf{x}] \cap [\mathbf{e}_{1_{j}} \cup \mathbf{e}_{2_{j}}] = \emptyset \lor \\ & \zeta_{x} \det(\mathbf{x}_{i} - \mathbf{e}_{1_{j}} \mid \mathbf{e}_{2_{j}} - \mathbf{e}_{1_{j}}) > 0 \lor \\ & \zeta_{x} \det(\mathbf{x}_{i} - \mathbf{e}_{1_{j}} \mid \mathbf{x} - \mathbf{e}_{1_{j}}) > 0 \lor \\ & \zeta_{x} \det(\mathbf{x}_{i} - \mathbf{e}_{2_{j}} \mid \mathbf{x} - \mathbf{e}_{2_{j}}) < 0 \rbrace \end{split}$$



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Visibility of a point						
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	Visibility contractors	Multi-robot localization				
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General Presentation	Visibility contractors	Multi-robot localization		Conclusion		
Visibility of a point						
• With a	segment as obsta	acle - $arepsilon_j^s = Seg(\mathbf{e}_{1_j})$	$,\mathbf{e}_{2_{j}})$			

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General Presentation	Visibility contractors	Multi-robot localization		Conclusion		
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	Visibility contractors	Multi-robot localization			
Visibility of a	a point				
• With a	• With a segment as obstacle - $arepsilon_j^s = Seg(\mathbf{e}_{1_j},\mathbf{e}_{2_j})$				
\rightarrow	Visible space				

 \rightarrow Non-visible space

$$\begin{split} \widehat{\mathsf{E}}_{\varepsilon_{j}^{s}}(\mathbf{x}) = & \{\mathbf{x}_{i} \in \mathbb{R}^{2} \mid \\ & [\mathbf{x}_{i} \cup \mathbf{x}] \cap [\mathbf{e}_{1_{j}} \cup \mathbf{e}_{2_{j}}] \neq \emptyset \land \\ & \zeta_{x} \det(\mathbf{x}_{i} - \mathbf{e}_{1_{j}} | \mathbf{e}_{2_{j}} - \mathbf{e}_{1_{j}}) \leq 0 \land \\ & \zeta_{x} \det(\mathbf{x}_{i} - \mathbf{e}_{1_{j}} | \mathbf{x} - \mathbf{e}_{1_{j}}) \leq 0 \land \\ & \zeta_{x} \det(\mathbf{x}_{i} - \mathbf{e}_{2_{j}} | \mathbf{x} - \mathbf{e}_{2_{j}}) \geq 0 \} \end{split}$$



Visibility of a segment



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Visibility of a segment



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Visibility of a segment

• With a segment as obstacle - $arepsilon_j^s = Seg(\mathbf{e}_{1_j},\mathbf{e}_{2_j})$



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General Presentation	Visibility contractors	Multi-robot localization	Global localization	Conclusion
Visibility of a	a segment			
• With a	segment as obst	acle - $arepsilon_j^s = Seg(\mathbf{e}_{1_j}$	$,\mathbf{e}_{2_{j}})$	
\rightarrow	Visible space - E	$S_{i}^{s}(Seg(\mathbf{x}_{1},\mathbf{x}_{2}))$		
$E_{\varepsilon_{j}^{s}}(\mathit{Seg}(\mathbf{x_{1}},\mathbf{x_{2}})) = \{$	$\mathbf{x}_i \in \mathbb{R}^2$	-		
$(\zeta_{x_1} = \zeta_{x_2}) \land (\zeta_{x_1} d$	$let(\mathbf{x}_i - \mathbf{e}_{1_j} \mathbf{e}_{2_j} - \mathbf{e}_{1_j}) >$	0 ∨		
$\zeta_{x_1} \det(\mathbf{x}_i - \mathbf{e}_{1_j} \mathbf{x}_1 - \mathbf$	$-\mathbf{e}_{1_j}) > 0 \land \zeta_{x_2} \det(\mathbf{x}_i - \mathbf{x}_j)$	$\mathbf{e}_{1_j} \mathbf{x}_2 - \mathbf{e}_{1_j} \rangle > 0 \lor$		
$\zeta_{x_1} \det(\mathbf{x}_i - \mathbf{e}_{2_j} \mathbf{x}_1 - \mathbf{e}_{2_j})$	$-\mathbf{e}_{2_j}) < 0 \land \zeta_{x_2} \det(\mathbf{x}_i - \mathbf{x}_j)$	$\mathbf{e}_{2_j} \mathbf{x}_2 - \mathbf{e}_{2_j} \rangle < 0 $ $) \vee$	$\mathbf{e}_{1_{j}}$	
$(\zeta_{x_1} = -\zeta_{x_2}) \land ($				e _{2j}
$\begin{pmatrix} \zeta_{e_1} \det(\mathbf{x}_i - \mathbf{e}_{1_j} \mathbf{x}_1) \\ (\zeta_{e_1} \det(\mathbf{x}_i - \mathbf{e}_{2_j} \mathbf{x}_1) \end{pmatrix}$	$-\mathbf{e}_{1_j} > 0 \lor \zeta_{e_1} \det(\mathbf{x}_i)$	$-\mathbf{e}_{1_j} \mathbf{x}_2 - \mathbf{e}_{1_j} \rangle < 0 \end{pmatrix} \land$	\mathbf{x}_1	
$\left(\begin{bmatrix} \mathbf{x}_i \\ \mathbf{x}_i \end{bmatrix} \mathbf{x}_1 \\ \left(\begin{bmatrix} \mathbf{x}_i \\ \mathbf{x}_1 \end{bmatrix} \mathbf{x}_2 \end{bmatrix} \right) \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix} = \left[\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} \right] \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \end{bmatrix}$	$ e_{2j}\rangle > 0 \lor \zeta_{e_2} \operatorname{det}(\mathbf{x}_i)$	$e_{2_j} \times 2 e_{2_j} < 0)) \vee$		
([, 0] 0	$j = 2_j = 2_j $		\mathbf{x}_2	

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	Visibility contractors	Multi-robot localization				
Visibility of a segment						
• With a	a segment as obsta	acle - $arepsilon_j^s = Seg(\mathbf{e}_{1_j}$	$,\mathbf{e}_{2_{j}})$			
\rightarrow	Visible space - E_{ε}	$_{j}^{s}(\mathit{Seg}(\mathbf{x}_{1},\mathbf{x}_{2}))$				
\rightarrow	Non-visible space	$- \widehat{E}_{arepsilon_{j}^{s}}(\mathit{Seg}(\mathbf{x}_{1},\mathbf{x}_{2}))$)			
$\widehat{E}_{arepsilon_{j}^{s}}(\mathit{Seg}(\mathbf{x}$	$(\mathbf{x}_1,\mathbf{x}_2)) = \widehat{E}_{arepsilon_j^s}(\mathbf{x}_1)$	$\cap \widehat{E}_{arepsilon_{j}^{s}}(\mathbf{x}_{2})$				
			eı	P _{2i}		
			x1	7 '		

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 \mathbf{x}_2

• With a segment as obstacle - $arepsilon_j^s = Seg(\mathbf{e}_{1_j}, \mathbf{e}_{2_j})$



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General Presentation	Visibility contractors	Multi-robot localization	Global localization	Conclusion
Visibility of a	a box			

• With a segment as obstacle - $arepsilon_j^s = Seg(\mathbf{e}_{1_j},\mathbf{e}_{2_j})$

$$ightarrow$$
 Visible space - $\mathsf{E}_{arepsilon_{j}^{s}}([\mathbf{x}])$



General Presentation	visibility contractors	Multi-robot localization	Giobal localization	Conclusion
Visibility of a	box			

- With a segment as obstacle $\varepsilon_j^s = Seg(\mathbf{e}_{1_j}, \mathbf{e}_{2_j})$
 - \rightarrow Visible space $\mathsf{E}_{\varepsilon_i^s}([\mathbf{x}])$
 - ightarrow Non-visible space $\widehat{\mathsf{E}}_{arepsilon_{j}^{s}}([\mathbf{x}])$



With a convex polygon as obstacle

Convex polygon : set of segments

$$ightarrow \ arepsilon_{j}^{p} = igcup_{k=1}^{n_{P_{j}}} arepsilon_{k}^{s}$$



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With a convex polygon as obstacle

Convex polygon : set of segments

$$\begin{array}{l} \rightarrow \ \varepsilon_{j}^{p} = \bigcup_{k=1}^{n_{P_{j}}} \varepsilon_{k}^{s} \\ \rightarrow \ \mathsf{E}_{\varepsilon_{j}^{p}}([\mathbf{x}]) = \bigcap_{k=1}^{n_{P_{j}}} \mathsf{E}_{\varepsilon_{k}^{s}}([\mathbf{x}]) \end{array}$$



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With a convex polygon as obstacle

Convex polygon : set of segments



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Outlines

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- 4 Global localization
- **5** Conclusion

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Team of robots



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- Team of robots
- Initial poses known



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- Team of robots
- Initial poses known
 - ightarrow The robots are drifting



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- Team of robots
- Initial poses known
- Known environment



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- Team of robots
- Initial poses known
- Known environment
- Is it possible to avoid the drifting of the robot by using a boolean information : the visibility between the robots ?

Bounded error context

 $ightarrow \mathbf{q}_{i,0} \in [\mathbf{q}_{i,0}]$

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- Bounded error context
- · Evaluation of the orientation by using a compass
 - $\rightarrow \ \theta_{i,k} \in [\theta_{i,k}]$





Compass CMPS10

Compass SEN12753P

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- Bounded error context
- Evaluation of the orientation by using a compass
- Inter-robot communication
 - \rightarrow At each time step k each robot knows the position estimation of all the robots

- Bounded error context
- Evaluation of the orientation by using a compass
- Inter-robot communication
- Boolean measurements

$$ightarrow \ r_1 \ {\sf sees} \ r_2 \Leftrightarrow ({f x}_1{\sf V}{f x}_2)_{{\cal E}}$$

$$ightarrow r_1$$
 does not see $r_2 \Leftrightarrow (\mathbf{x}_1 \overline{\mathsf{V}} \mathbf{x}_2)_\mathcal{E}$

$$ightarrow \mathbf{z}_{i,k} = \{0, 1, \cdots, 1\}$$

- 0 : the robot r_i does not see the first robot
- 1 : the robot r_i sees the second robot
- • •
- 1 : the robot r_i sees the last robot

Environment characterisations

• Environment \mathcal{E}

$$ightarrow \, \mathcal{E} = igcup_{j=1}^{n_O} oldsymbol{arepsilon}_j$$

ightarrow Sets of convex polygons



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Environment characterisations

- Environment \mathcal{E}
- Inner characterisation \mathcal{E}^-

$$\rightarrow \mathcal{E}^{-} \subseteq \mathcal{E}$$



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Environment characterisations

- Environment *E*
- Inner characterisation \mathcal{E}^-
- Outer characterisation \mathcal{E}^+



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- Environment *E*
- Inner characterisation \mathcal{E}^-
- Outer characterisation \mathcal{E}^+
- Environment/Characterisations

- Environment *E*
- Inner characterisation \mathcal{E}^-
- Outer characterisation \mathcal{E}^+
- Environment/Characterisations
- $ightarrow \ r_1 \ { extsf{sees}} \ r_2 \Rightarrow (\mathbf{x}_1 { extsf{V}} \mathbf{x}_2)_{\mathcal{E}^-}$

$$- \mathbf{x}_1 \in \mathsf{E}_{\mathcal{E}^-}(\mathbf{x}_2), \mathbf{x}_1 \not\in \widehat{\mathsf{E}}_{\mathcal{E}^-}(\mathbf{x}_2)$$

$$- \mathbf{x}_2 \in \mathsf{E}_{\mathcal{E}^-}(\mathbf{x}_1), \mathbf{x}_2 \not\in \mathsf{E}_{\mathcal{E}^-}(\mathbf{x}_1)$$



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- Environment \mathcal{E}
- Inner characterisation \mathcal{E}^-
- Outer characterisation \mathcal{E}^+
- Environment/Characterisations
- $ightarrow \ r_1 \ { extsf{sees}} \ r_2 \Rightarrow (\mathbf{x}_1 { extsf{V}} \mathbf{x}_2)_{\mathcal{E}^-}$
 - $\mathbf{x}_1 \in \mathsf{E}_{\mathcal{E}^-}(\mathbf{x}_2), \, \mathbf{x}_1 \not\in \widehat{\mathsf{E}}_{\mathcal{E}^-}(\mathbf{x}_2) \\ \mathbf{x}_2 \in \mathsf{E}_{\mathcal{E}^-}(\mathbf{x}_1), \, \mathbf{x}_2 \notin \widehat{\mathsf{E}}_{\mathcal{E}^-}(\mathbf{x}_1)$
 - $\rightarrow r_1 \text{ does not see } r_2 \Rightarrow (\mathbf{x}_1 \overline{\mathbf{V}} \mathbf{x}_2)_{\mathcal{E}^+}$
 - $\mathbf{x}_1 \in \widehat{\mathsf{E}}_{\mathcal{E}^+}(\mathbf{x}_2), \, \mathbf{x}_1 \notin \mathsf{E}_{\mathcal{E}^+}(\mathbf{x}_2) \\ \mathbf{x}_2 \in \widehat{\mathsf{E}}_{\mathcal{E}^+}(\mathbf{x}_1), \, \mathbf{x}_2 \notin \mathsf{E}_{\mathcal{E}^+}(\mathbf{x}_1)$



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Results

Simulator



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Outlines

- General Presentation
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Two poses are consistent with the constraints

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How to process the measurement intersection constraint?

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• Presentation of the constraint

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- Presentation of the constraint
 - ightarrow the measurement must not intersect the obstacle



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- Presentation of the constraint
 - ightarrow the measurement must not intersect the obstacle



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- Presentation of the constraint
- Constraint formalisation

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ightarrow Visibility relations between the robot and all the detected obstacles

 $(0, \overline{0})$

 x_1

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Visibility Contractors - Application to mobile robot localization

 \overline{w}_{1_i}



 $(0, \overline{0})$

 x_1

Visibility relations between the robot and all the detected obstacles

 $\rightarrow \forall \varepsilon_i \in \mathcal{E}, \forall i, (\mathbf{x} \forall \mathbf{w}_i)_{\varepsilon_i}$

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Visibility Contractors - Application to mobile robot localization

 w_{1_i}

- Contraction over a visibility information
- Inner characterisation

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- Contraction over a visibility information
- Inner characterisation



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- Contraction over a visibility information
- Inner characterisation



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- Contraction over a visibility information
- Inner characterisation



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- Contraction over a visibility information
- Inner characterisation
- Environment characterisation



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Considered environment

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Inner characterisation

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Considered measurements

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Result without visibility

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Result with visibility

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Conclusion

- ightarrow Original formalisation of the visibility information
- \rightarrow Optimal visibility contractors
- \rightarrow Two application to mobile robot localization
 - Multi-robot localization
 - Avoid the drifting of the robots
 - Results are critical over the number of robots and the environment topology
 - The contractors can be added to classical localization approach
 - Original measurement intersection constraint
 - Improvement of the localization results
 - Avoidance of symmetries
 - Those contractors could be useful to other application

Conclusion