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# Optimal accelerated test plan: optimization procedure using Genetic Algorithm

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**Abstract:** This paper describes an optimization procedure using Genetic Algorithm to define an optimal accelerated test plan considering an economic approach. We introduce a general framework to obtain plans of optimal accelerate tests with a specific objective, such as cost. The objective is to minimize the costs involved in testing without reducing the quality of the data obtained. The optimal test plans are defined by considering prior knowledge of reliability, including the reliability function and its scale and shape parameters, and the appropriate model to characterize the accelerated life. This information is used in Bayesian inference to optimize the test plan. To perform optimization, a specific genetic algorithm is described and applied to obtain the best test plan. This procedure is then illustrated on a numerical example.

**Keywords:** Accelerated testing; Bayesian inference; parameter estimation; Maximum A Posteriori; optimization; cost; robustness; Genetic algorithm.

## 1 Introduction

(ALTs) are widely used in reliability studies. Because many modern high-reliability components are expected to perform their proper functions for a very long time, simply testing these components under use conditions will usually yield little useful information about reliability within practical time and cost constraints. Accelerating variables, such as temperature, are often applied to obtain failures more rapidly. The resulting data at the higher stresses are used to estimate, through extrapolation with an appropriate acceleration model, the life distribution of the component at specified use conditions (See Figure 1).

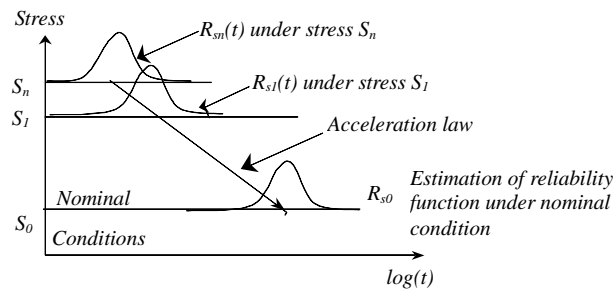


Figure 1- Principle of ALT test

Careful planning of an ALT is important to allow the most efficient use of limited resources, namely, time, number of test units, and the test facilities. Typically, ALT plans specify the levels of the accelerating variable and the quantity of available test units for these levels. With certain planning criteria, such as the estimation precision of a particular characteristic of the life distribution at use conditions, optimization can be used to find optimum test plan. Optimum test plans provide insight needed to obtain good practical test plans [1,5,6,9]. Moreover, a test plan needs to be developed to obtain appropriate and sufficient information in order to accurately estimate reliability performance at operating conditions, significantly reduce test times and costs and achieve other objectives. One of the first decisions to be made when designing a reliability verification test is to determine how many units to test. If many units are tested, the duration of the test will be short. With this approach, prototype costs will be high, and development time costs will be low. If few units are tested, the duration of the test will be longer and prototype costs will be low, but development time costs will be high. The second decision is to determine stress level and

corresponding censoring times to perform an accurate precision on reliability estimation. On these considerations, the optimal test plan has to be design on a global cost criterion including test cost and warranty cost.

ALTs are often conducted to estimate the life distribution at the use conditions. The statistical error of the estimate depends upon the test plan. Obviously, it is desirable to device the optimal test plans that minimizes the error [8,9]. For a constant stress test, the test plans determine the stress levels, the number of test units allocated to each stress level, and other variables. In this work, we study the model by an evaluation of parameters using maximum likelihood and Bayesian methods. We estimate accelerated life model parameters allowing to assess the reliability function under operating conditions from only accelerated life data [5]. We provide an overview of the application of Bayesian inference to accelerated life testing (ALT) models with estimation by Maximum of A Posteriori (MAP) method in the case of constant stress levels. This work presents the approach in Bayesian estimation of parameters of models SVA. It helps reduce the confidence intervals by providing prior knowledge. The approach was applied to parametric models by studying the classical estimates and MAP.

In this paper, an optimization test plan is proposed integrating the Bayesian inference and an objective function based on economical formulation. The proposed method consists of 5 subsequent steps:

➤ **Reliability target**

The first step consists of defining the concept of a reliability target, its scope, and the regulatory standards to be respected. Work at this stage is critical because it will influence all subsequent activities. Various metrics are used to characterize the reliability of products, such as MTTF,  $L_{10}$ , or probability of failure for the warranty period. The verification consists of evaluating the risk of not reaching the reliability target using a point estimate and confidence interval.

➤ **Prior knowledge on product**

The Bayesian inference can be used to include all available knowledge. For most of the cases, typical values for the parameters can be found. The values and associated confidence intervals may be considered either as results of expertise using references (reliability handbooks such as FIDES [3]), as values associated with older/similar products, or as results of expert opinion(s). Moreover, in FIDES [3], acceleration modes and activation energy through influencing factors with respect to failure modes are given.

➤ **Test plan to optimize**

The kind of accelerated test plan is fixed at the beginning of study. Due to the better development of accelerated test model for constant stress, and well development of data analysis for reliability estimation, we consider constant stress testing. In practice, constant stress testing are the most common because of simplicity of stress application and accuracy on reliability estimation. So the number of stress levels and sample size are fixed. For designing test plan, the choice of an appropriate stress is importance, as well as to identify the appropriate limits (or stress levels); because the first and most obvious benefit of accelerated life testing is the time savings, which is based on the decrease in test duration due to the increased stress levels. The choice of two levels is statistically optimum for the estimate of the probability and three levels allow the linearity of stress transfer. Meeker, Nelson and Yang used three accelerated constant stress levels for obtaining the best compromise test plan.

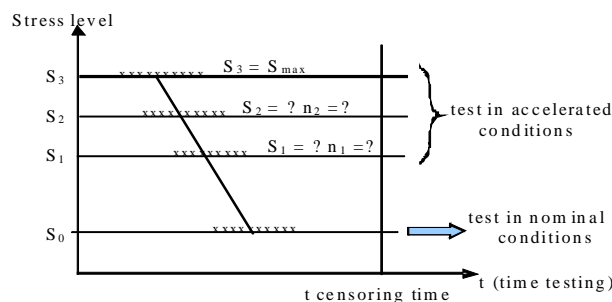


Figure 2- Optimal test plan

According to Figure 2, a simple test plan with three accelerated constant stress levels  $S_i$  ( $i=1,2,3$ ) is studied by considering the sample size  $n$ , the proportion allocation  $\pi_i$  of sample size and the censoring time  $\tau_i$  at stress level  $S_i$ . The sample size  $n$  and the maximum stress  $S_3$  are fixed. The low, middle stress level and the proportion allocation  $\pi_1$ ,  $\pi_2$  of sample size at stress level  $S_1$ ,  $S_2$  are unknown.

In Bayesian statistics, the uncertainty about the unknown parameters is quantified used probability so that the unknown parameters are regarded as random variables. The decision variables of test plan optimization are chosen from among the test plan parameters. It is assumed that the reliability function  $R_{S_0}$  belongs to a class of functions depending only on the parameters of location  $\gamma$  and shape  $\beta$  [2].

### ➤ Objective function

The objective of the accelerated testing plan optimization is to minimize the global cost as defined by the costs of testing and operation. The evaluation of operation cost includes the difference between the reliability target and its estimation considering a risk  $\alpha$ . This term allows us to introduce a robustness analysis according through an objective function.

The principal optimization variables are:

- lower and middle stress level  $S_1$  and  $S_2$ ,
- proportion allocation  $\pi_1$  and  $\pi_2$  of sample size respectively at stress level  $S_1$  and  $S_2$ .

Additional optimization variables can be the censoring time  $\tau_i$  at stress level  $S_i$ , but there are generally predetermined by test schedule and other industrial constraints.

### ➤ Optimization procedure

To obtain the best test plan, we propose an optimization procedure using the genetic algorithm. The proposed method will be illustrated by a numerical example.

## 2 Estimation in parametric ALT Model using Bayesian inference

We consider the parametric ALT model has been described in [1]. After selecting the model, in order to provide estimates for the model's parameters, we apply maximum-likelihood estimation as point estimators. The maximum likelihood estimators return a single point estimate for a given data set. In contrast, the Bayesian posterior is an entire distribution over the parameter space. We can turn this in to a point estimate by taking some measure of central tendency, such as the conditional mean of the parameter given the data. In Bayesian Inference by Maximum of A Posteriori, the Bayesian approach is based on the concept of subjective probability depending on the degree of belief in the occurrence of an event [4]. This is not a point value, which is estimated, but the probability distribution of the random variable (probability of non-functioning), the degree of belief that each probability value can be true. In Bayesian statistics, the uncertainty about the unknown parameters is quantified used probability so that the unknown parameters are regarded as random variables.

To simplify the parametric model, we develop the methodology with one accelerating variable and a linear relationship between the location parameter and the stress level. Without adding complexity, the methodology can be generalized to multiple accelerating variables with linear relationship.

It is assumed that the survival function  $R(t)$  belongs to a class of functions depending only on the parameters of scale  $\eta$  and shape  $\beta$  [0]:

$$R_{S_0}(t) = R_0 \left( \left( \frac{t}{\eta} \right)^\beta \right), (\eta, \beta > 0) \quad (1)$$

Several models, such as Weibull and lognormal, are just particular cases of the above form  $R_0(t) = e^{-t}$ ,  $R_0(t) = 1 - \Phi(\ln t)$  respectively as detailed in [12].

In this section, we assume, for a particular case of a constant stress with one accelerating

variable, that the logarithm of scale parameter  $\eta$  follow a linear function of transformed stress  $S$  as:

$$\ln(\eta) = \gamma_0 + \gamma_1 S$$

For our particular case of constant stress,  $S$ , with one accelerating variable, the reliability function the equation (1) becomes:

$$R_S(t) = R_{S_0}(e^{\gamma_0 + \gamma_1 S} t) \quad (2)$$

The notations  $R(u) = R_0(e^u)$ ,  $u \in \mathfrak{R}$ ,  $u = \ln(t)$ ,  $\gamma = (\gamma_0, \gamma_1)$ , allow us to rewrite the equation (2) as:

$$R_S(t) = R\left(\frac{\ln t - \gamma^T S}{1/\beta}\right) \quad (3)$$

The likelihood function can be written as:

$$L(T | \gamma, \beta) = \prod_{i=1}^k \prod_{j=1}^{n_i} \left\{ \left[ \beta \lambda \left( \frac{T_{ij} - \gamma^T S^{(i)}}{1/\beta} \right) \right]^{\delta_{ij}} R\left(\frac{T_{ij} - \gamma^T S^{(i)}}{1/\beta}\right) \right\} \quad (4)$$

Note:  $T_{ij}$  is the life time observed or censored of the  $j^{\text{th}}$  unit from  $i^{\text{th}}$  stress level group.

We consider a prior information on unknown parameters modeled by the functions  $\pi(\gamma_0)$ ,  $\pi(\gamma_1)$ ,  $\pi(\beta)$ . We assume that the variables  $(\gamma_0, \gamma_1, \beta)$  are independent and the joint prior distribution can be defined as:

$$\pi(\gamma_0, \gamma_1, \beta) = \pi_{\gamma_0}(\gamma_0) \times \pi_{\gamma_1}(\gamma_1) \times \pi_{\beta}(\beta) \quad (5)$$

The choice of the form of  $\pi$  depends on degree of knowledge on parameter  $\gamma_0$ ,  $\gamma_1$  or  $\beta$ .

The continuous form of Bayes theorem for the random variable  $\theta$  over the  $\Omega$  domain, having  $t_i$ ,  $i = 1..n$  as test results, is:

$$\pi_{apo}(\theta / t_1, \dots, t_n) = \frac{L(t_1, \dots, t_n / \theta) \pi(\theta)}{\int_{\Omega} L(t_1, \dots, t_n / \theta) \pi(\theta) d\theta} \quad (6)$$

with  $\pi(\theta)$  the mathematical form, which formalizes the prior information.

With regard to the aspect of reversing in statistics, we consider a probability density as  $\pi_{apo}(\gamma_0, \gamma_1, \beta)$ . As consequence, the ML theory can be applied. So a search of values that maximizes the  $\pi_{apo}(\gamma_0, \gamma_1, \beta)$  and the covariance matrix associated to these estimators will be searched. MAP method considers the a posteriori density function  $\pi_{apo}(\gamma_0, \gamma_1, \beta | T)$  and the punctual estimators of unknown parameters  $(\gamma_0, \gamma_1, \beta)$  are estimated so that they maximize:

$$(\hat{\gamma}_0, \hat{\gamma}_1, \hat{\beta}) = \text{Arg max}(\pi_{apo}(\gamma_0, \gamma_1, \beta | T)) \quad (7)$$

By differentiating after the variables  $(\gamma_0, \gamma_1, \beta)$  of the function  $\ln[\pi_{apo}(\gamma_0, \gamma_1, \beta)]$ , the MAP estimators  $(\hat{\gamma}_0, \hat{\gamma}_1, \hat{\beta})$  can be obtained by solving the equation system [1]:

$$\frac{\partial \ln[\pi_{apo}(\gamma_0, \gamma_1, \beta)]}{\partial \gamma_i} = 0 \quad (i = 1, 2), \quad \frac{\partial \ln[\pi_{apo}(\gamma_0, \gamma_1, \beta)]}{\partial \beta} = 0 \quad (8)$$

Fisher information applies to the function that describes the information on the parameters,  $\pi_{apo}(\gamma_0, \gamma_1, \beta | T)$  [1, 11] becomes:

$$\begin{aligned}
I^{MAP}(\gamma_0, \gamma_1, \beta) &= E \left[ \left( \frac{\partial \log \pi_{apo}(\gamma_0, \gamma_1, \beta | T)}{\partial \gamma_0 \partial \gamma_1 \partial \beta} \right)^2 \right]_{\hat{\gamma}, \hat{\beta}} \\
&= E \left[ \left( \frac{\partial \log L(T | \gamma_0, \gamma_1, \beta)}{\partial \gamma_0 \partial \gamma_1 \partial \beta} \right)^2 \right]_{\hat{\gamma}, \hat{\beta}} + E \left[ \left( \frac{\partial \log \pi(\gamma_0, \gamma_1, \beta)}{\partial \gamma_0 \partial \gamma_1 \partial \beta} \right)^2 \right]_{\hat{\gamma}, \hat{\beta}}
\end{aligned} \tag{9}$$

So, the estimator of the reliability function  $\hat{R}_{S_0}$  is defined by:

$$\hat{R}_{S_0}(t) = R \left( \frac{\ln(t) - \hat{\gamma}^T S^{(0)}}{1 / \hat{\beta}} \right) \tag{10}$$

The parameters  $\hat{\gamma}_1, \hat{\gamma}_0, \hat{\beta}$  and  $I^{MAP}(\hat{\gamma}_0, \hat{\gamma}_1, \hat{\beta})$  are obtained by Monte Carlo simulation [5].

### 3 Optimization Problem and Simulation

#### 3.1 Principle of method

In the proposed method, we define an optimal accelerated testing plan, considering an objective function based on economic approach. Bayesian inference is used for optimizing the test plan, and taking into account the uncertainty on parameters. Thus we will have a robust optimal testing plan. We propose an optimization procedure using the genetic algorithm for obtaining the best test plan.

#### 3.2 Objective function

The objective of the accelerated testing plan optimization is to minimize the global cost as defined by the costs of testing and operation. In designing the test plan, we define a contractual reliability metric target as probability of failure  $p_{target}$  for warranty period (the operation time  $t_{operation}$ ).

The global cost is defined by:

$$C_{global} = C_{testing} + C_{operation} \tag{11}$$

where

$$\begin{aligned}
C_{testing} &= \text{fixed testing cost} + n \times \text{unit price} + \tau_1 \times \text{cost per testing hour} \\
&+ \sum_i \left( \frac{n_i}{Nb} \right)^{\gamma_{testing}} [\text{Fixed cost per batch} + \tau_i \times \text{cost per testing hour per batch}]
\end{aligned}$$

$Nb$  represents the maximum number of units per batch and  $\gamma_{testing}$  represents the critical index for increasing the number of batch per stress level.

$$\begin{aligned}
C_{operation} &= \text{fixed operation cost} + (p_{operation} - p_{target}) \times \text{product population unit cost} \\
&+ \text{cost of brand image loss} \times (p_{operation} - p_{target})^{\gamma_{operation}}
\end{aligned}$$

And

$p_{target}$  represents the target as probability of failure for the warranty period and  $\gamma_{operation}$  represents the critical index for brand image loss.

$p_{operation}$  represents the upper bound of the unilateral confidence interval for the risk  $\alpha$  of estimated probability of failure during the operation time for the warranty period (see Figure 3).

The verification consists of evaluating the risk of failing to reach the reliability target in terms of the point estimate and confidence interval.

The point estimate is defined by:

$$\hat{p} = 1 - e^{-\left( t / e^{(\gamma_0 + \gamma_1 S)} \right)^\beta} \tag{12}$$

and confidence interval on  $\hat{p}$  is determined by the fisher information matrix of MAP estimators.

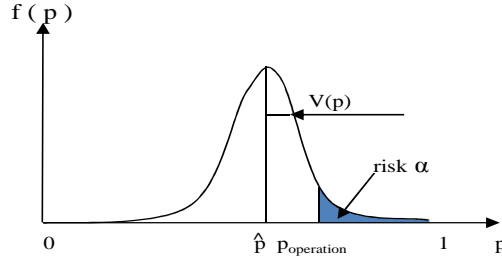


Figure 3- Probability function of operation failure  $f(p)$

By considering estimator  $\hat{R}$  as a regular function of  $(\gamma, \beta)$  we will define:

$$Q_{S_0}(t) = \ln \frac{R_{S_0}(t)}{1 - R_{S_0}(t)} \quad \hat{Q}_{S_0}(t) = \ln \frac{\hat{R}_{S_0}(t)}{1 - \hat{R}_{S_0}(t)} \quad (13)$$

The function  $Q_{S_0}$  takes values in  $\mathcal{R}$ , therefore the speed of convergence of  $\hat{Q}_{S_0}$  to the limit law is greater than that of  $\hat{R}_{S_0}$  to its limit law.

We obtain that the law

$$\frac{\hat{Q}_{S_0}(t) - Q_{S_0}(t)}{\hat{\sigma}_{Q_0}} \quad (14)$$

is approximated by standard normal distribution  $N(0,1)$  with:

$$\begin{aligned} (\hat{\sigma}_{Q_0})^2 &= \left( \frac{\partial \hat{Q}_{S_0}}{\partial \hat{\gamma}_0}, \dots, \frac{\partial \hat{Q}_{S_0}}{\partial \hat{\gamma}_m}, \frac{\partial \hat{Q}_{S_0}}{\partial \hat{\beta}} \right) \left[ I^{MAP}(\gamma, \beta) \right]^{-1} \left( \frac{\partial \hat{Q}_{S_0}}{\partial \hat{\gamma}_0}, \dots, \frac{\partial \hat{Q}_{S_0}}{\partial \hat{\gamma}_m}, \frac{\partial \hat{Q}_{S_0}}{\partial \hat{\beta}} \right) \\ &= \left( \frac{\hat{\beta}^2 R'(R^{-1}(\hat{R}_{S_0}(t)))}{\hat{R}_{S_0}(t)(1 - \hat{R}_{S_0}(t))} \right)^2 \times \left[ \left( \frac{1}{\hat{\beta}^4} \right) \sum_{l=0}^m \sum_{c=0}^m s_{0l} s_{0c} I_{MAP}^{lc}(\hat{\gamma}, \hat{\beta}) - 2 \left( \frac{1}{\hat{\beta}^2} \right) \hat{\gamma}^T S^{(0)} \sum_{l=0}^m s_{0l} I_{MAP}^{l,m+1}(\hat{\gamma}, \hat{\beta}) + (\hat{\gamma}^T S^{(0)})^2 I_{MAP}^{m+1,m+1}(\hat{\gamma}, \hat{\beta}) \right] \end{aligned} \quad (15)$$

$I_{MAP}^{l,c}$  represents the terms of the matrix  $[I^{MAP}(\gamma_0, \gamma, \beta)]^{-1}$ .

The upper bound of approximate unilateral confidence interval  $(1 - \alpha)$  for  $\hat{R}_{S_0}(t)$  is

$$\left( 1 + \frac{1 - \hat{R}_{S_0}(t)}{\hat{R}_{S_0}(t)} \exp \left\{ -\hat{\sigma}_{Q_0} \omega_{1-\alpha} \right\} \right)^{-1} \quad (16)$$

with  $\omega_\alpha$  the  $\alpha$ -quantile of standard normal distribution  $N(0,1)$ .

The estimation of probability  $\hat{p}_{operation}$ , is derived from:

$$\hat{p}_{operation} = 1 - \hat{R}_{S_0}(t_{operation}) \quad (17).$$

The operation cost is defined by considering risk  $\alpha$  in terms of  $\hat{p}_{operation}$ . This term allows us to introduce a robustness analysis according to objective function.  $p_{operation}$  is function of  $\pi_1, \pi_2, S_1, S_2$  and  $\tau_1, \tau_2, \tau_3$  and the optimization model can be written as follows:

$$\text{Min}_{\pi_i, S_i, \tau_i} (C_{global}) \quad (18)$$

subject to  $\pi_i \in ]0,1[$ ,  $i = 1, 2$  and  $\pi_1 + \pi_2 < 1$ ,  $S_3 \leq S_2 \leq S_1 \leq S_0$ ,  $\tau_3 \leq \tau_2 \leq \tau_1 \leq \tau_{max}$ .

### 3.3 Optimization procedure and efficiency

According to estimate of  $p_{operation}$  in (17), the integration problem is intractable and numerical methods that are used to find the best test plan. Several optimization algorithms can be applied, including Least-mean-square and simulated annealing. In this paper, we propose an optimization procedure using Genetic Algorithm (GA) to define test plan and Monte-Carlo simulation in order to the numerically estimate  $p_{operation}$  that leads to global cost as fitness function. The reasons of this choice are that GA is a quite simple and efficient technique to be applied with heterogeneous inputs, especially in research of an optimum accelerated life test plan [6]. As well as Hamada and al. has shown how GAs can be used to find near- optimal Bayesian experimental designs. Their methodology is easy to implement and allows a practical approach for designing even more complicated experiments. The near symmetry of the resulting best designs may suggest a symmetrical design which may indeed be optimal. In summary, they believe that GAs provide a useful addition to the statistical practitioner's toolkit for designing experiments [7]. Moreover, GA allows us to search the optimum with more freedom inputs.

### 3.4 Genetic algorithm application

In GA, the individuals are generated randomly in order to construct a population. After estimating the fitness of individuals, parents are selected from the population according to the fitness value. Offspring are generated from the parents by using genetic operators such as the mutation or crossover. For convenience, we restrict the definition of population to continuous independent variables. So, each element of the population is defined by a combination of factors ( $\pi_1, \pi_2, S_1, S_2, \tau_1, \tau_2, \tau_3$ ) with constraints explained in formula (18). The fitness function corresponds to the global cost detailed in (11) and the programming is performed by GA MATLAB function with the specific selection, crossover and mutation operators. These genetic operators are used within the context of an elitist GA by describing the construction of an initial population of solutions and subsequent populations of solutions obtained. This condition guarantees that the best (most fit) solution in population of the  $k$  generation are not lost in population  $k+1$  [7]. In the original GA, each new population completely replaces the previous one, thus it is possible that solution in new population is worse than the best solution in previous population. Consequently, very good solutions can be lost forever [7].

To improve the performance of GA, specially the efficiency of crossover and mutation algorithm, adaptive GAs are often used in reliability-related optimization studies as [Zhisheng Y. and al \[10\]](#).

Adaptive GAs are capable of reducing the population scale and reducing the simulation time. New adjusting method using the mean and standard deviation of the population is employed on mutation-first and crossover-first adaptive GA. According to [10] the new mutation-first GA is more efficient, we apply it with decreasing mutation ratio and increasing crossover ratio proportionally to the number of generations.

For a mutation, the last two elements are replaced by two other ones chosen at random infeasible bound of constraints. If the mutation does not lead to good plans, they will be attributed a bad fitness and thus a low probability to be chosen later. We also decrease the probability that mutation occurs on each factor as the number of generations increases as the evolutionary phenomenon known as “punctuated equilibrium”.

For a crossing, two parents  $P_1$  and  $P_2$  are chosen with probability proportional to their efficiency. They generate two infants  $E_1$  and  $E_2$  defined by the mean of each gene. Thus, the better the plans  $P_1$  and  $P_2$  are, the more often similar infants are generated. As soon as there is hardly any evolution, it can be assumed that a local minimum of the final error is obtained.

## 4 Numerical example and simulation data

To illustrate our method, we will compare our solution consequences on a well-known example



obtained by Yang [8] for the Weibull distribution with one accelerating variable. We also show the results of simple comparison studies between GAs and the response surface methodology.

This example consider an electronic module for pump control that normally operates at 45°C. To estimate its reliability at the use condition, 50 units are to be tested at three elevated temperatures. The high one is 105°C, which is 5°C lower than the maximum allowable temperature. The censoring times are fixed for low, middle and high stress levels respectively at 1080, 600 and 380 hours. The unequal censoring times are considered to be fixed due to industrial constraints (test schedule and total test time fixed).

#### 4.1 Yang's compromise test plans

In order to find the best compromise test plans, Yang fixed the proportion  $\pi_2$  to  $(1-\pi_1)/3$  and the transformed stress level  $S_2$  to  $(S_1+S_3)/2$ . From the use of reliability handbook and historical data, the parameters  $\gamma_0, \gamma_1, \beta$  of linear function of transformed stress  $S$  are preestimated respectively by -15.8, 8100.8 and 1.5. Yang determined the best compromise test plan that minimizes the variance of the MLE of the mean log life at the use stress level [8] based on standardized variance formulation given in [9].

#### 4.2 Comparisons and results

To complete our approach, we define the interval values of prior knowledge for  $\gamma_0, \gamma_1, \beta$  respectively by [-17, -13], [-10100, -6100] and [1.3, 1.7]. According to that, we consider a prior knowledge following normal distribution on independent variables  $\gamma_0, \gamma_1, \beta$  obtained by the moments method. The parameters for evaluating the testing cost and operation cost are given in Table 1.

Table 1- Values of parameters for evaluating the Testing Cost and the operation Cost

Parameters of Testing Cost		Parameters of operation Cost	
Testing unit price	50 €	P target	0.01
Testing fixed cost	1000 €	Sales population	100 units
Cost per hour	10 €	Warranty time	150 hours
Cost per batch	1000 €	Replace unit price	100 €
Cost per hour per batch	5 €	After sales cost	10.000 €
Max units number per batch	12	Brand image loss	100 €
$\gamma_{\text{testing}}$	1.3	$\gamma_{\text{operation}}$	1.1

The table 2 gives the results by proposed approach (GAs) and Yang's test plan with the same constraints. The results show that the GAs optimization generates the similar results.

One of the principal added values using GA optimization is to provide test plan that allow adding more freedom variables without adding complexity and times consuming explosion. In this example, the test plan can be improved by adding  $\pi_2$  and  $S_2$  as optimization variables. The table 2 gives the results by proposed approach in "releasing" constraints on  $\pi_2$  and  $S_2$ . The result shows a different optimum considering global cost evaluation instead of reliability estimation accuracy.

Table 2- Comparison of GA and Yang's results

	Global cost evaluation	Group	1	2	3
Yang best compromise test plan	4.3364e+004	Number of Test Units	34	5	11
		Temperature (°C)	74	89	105
GAs best compromise test plan with Yang constraints	4.3354e+004	Number of Test Units	34	5	11
		Temperature (°C)	74.54	88.98	105
GAs best compromise test plan	3.2265e+004	Number of Test Units	9	7	34
		Temperature (°C)	75.34	87.93	105

## 5 Conclusion

In this paper, we introduced an optimization procedure using the genetic algorithm to obtain optimal accelerated test plans considering a cost objective. GA is a simple and efficient technique to be applied with heterogeneous inputs, especially in research of an optimum accelerated life test plan [6]. Moreover, this GA procedure allows us to improve best compromise test plans by searching the optimum with more freedom variables on ALT plan.

The cost objective function depends on the parameters of test plan that includes batch proportion, stress level and censoring times. Generally, the censoring times are fixed in test plan definition. To define these parameters, an optimization procedure is developed to minimize an economical function (testing and operational costs) with taking into account the uncertainties on input data by the Bayesian inference. The optimization process can be also applying to conduct a test. It will allows to verify the compatibility of results with prior knowledge and reduce the censoring time in case of "Good results" while keeping the same level of risk.

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