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## To cite this version:

Andrey Komarov, Foued Amrani, Alexander Dmitriev, Konstantin Komarov, Dmitry Meshcheriakov, et al.. Dispersive-wave mechanism of interaction between ultrashort pulses in passive mode-locked fiber lasers. Physical Review A : Atomic, molecular, and optical physics [1990-2015], 2012, 85 (1), pp.013802. 10.1103/PhysRevA.85.013802 . hal-03187688

**HAL Id: hal-03187688**

**https://univ-angers.hal.science/hal-03187688v1**

Submitted on 1 Apr 2021

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# Dispersive-wave mechanism of interaction between ultrashort pulses in passive mode-locked fiber lasers

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(Received 8 November 2011; published 3 January 2012)

On the basis of numerical simulation, it is found that powerful long-distance soliton wings can be formed by dispersive waves which are emitted by solitons because of lumped elements in a laser cavity. We analyze peculiarities of the interaction of two solitons through such wings in lasers with lumped saturable absorbers. Various sets of bound steady states of a two-soliton molecule are demonstrated. The relation between the spectral sidebands and the dispersive-wave wings of a soliton is found. The periodic changes in a soliton's profile during its pass through the laser cavity are studied.

DOI: 10.1103/PhysRevA.85.013802 PACS number(s): 42.65.Tg, 42.65.Sf, 42.60.Fc

## I. INTRODUCTION

Passive mode-locked fiber lasers are widely used in many areas of science, technology, and engineering. The important advantage of these lasers is the great diversity of their generation regimes. A special place among lasing regimes is occupied by multiple-pulse passive mode locking. A large number of publications is devoted to this area of research. The authors of [1] experimentally demonstrated that ultrashort pulses in a laser cavity are created and annihilated one by one and that a large hysteresis occurs in the number of pulses with varying pump power. All intracavity pulses have identical parameters (peak intensity, pulse duration, chirp, and so on). This phenomenon has been named the effect of quantization of the intracavity radiation into identical solitons. A theory of these phenomena was presented in [2]. For passive mode-locked fiber lasers with nonlinear polarization rotation, the corresponding theory was developed in [3].

The type of intersoliton interaction plays a crucial role in the established type of multiple-pulse operation. Depending on the properties of the interaction, the ultrashort pulses can either space themselves equally along a laser cavity, group into a tight bunch, or be randomly distributed. In the experimental paper [4] it was demonstrated that a set of lasing solitons (about several hundreds) can form complexes analogous to various aggregate states of matter: a soliton gas, a liquid, a glass, a soliton crystal, a polycrystal. The elementary unit of any complex is a pair of interacting solitons. Bound steady states of two solitons were investigated in [5–10]. Steady states with phase differences for the peak amplitudes of the solitons equal to $\pi$, $\pi/2$, and 0 were predicted. The phase differences $\pi/2$ and $\pi$ were experimentally observed [8,10]. Stable bound states of vibrating two-soliton molecules are also possible [11]. Hysteresis in the dependence of such states on pumping was predicted in [12].

The possibility of realization of strong bonds between interacting solitons ($\sim$10% of an individual soliton's energy) was numerically found in [13] for an erbium fiber laser with lumped nonlinear losses by use of a nonlinear polarization rotation technique. In this case a pair of interacting solitons forms a highly stable two-soliton molecule with a set of quantum bound energy levels corresponding to various stable steady states. For odd steady states the phase difference of the interacting solitons equals approximately $\pi$. For even ones it is about 0. The spectrum of a single pulse has powerful sidebands which indicates a possible role of dispersive waves emitted by solitons in the formation of strong intersoliton bonds. The generation of spectral sidebands is a well-known phenomenon [14]. The soliton circulating in the laser cavity periodically experiences perturbations caused by lumped nonlinear losses and various intracavity components. After each perturbation the soliton emits a dispersive wave. A constructive interference between these waves forms powerful spectral sidebands and powerful long-distance soliton wings. Such wings result in a long-distance interaction and thus allow the formation of bound steady states of interacting solitons. Previous theoretical and experimental investigations demonstrated that the interaction of solitons through such sidebands results in a quantization of intersoliton separations for the soliton pair in a fiber laser [15]. Nonlinear losses in fiber lasers with nonlinear polarization rotation are essentially lumped. This complicates the analysis of the role of the dispersive waves in the interaction of pulses: any change of the lumped nonlinear losses with the purpose of changing the intensity of dispersive waves results simultaneously in a change of parameters of the pulse.

Great attention has actually been given to the study of passive mode locking of fiber lasers with saturable absorbers based on various nanomaterials (media with quantum wells, nanotubes, graphene, etc.) [16–19]. These lasers are described by simpler physical models than fiber lasers based on nonlinear polarization rotation. To understand the role of dispersive waves in the formation of soliton wings, we have used the following methodology. It is well known that spatially uniformly distributed nonlinear losses do not result in dispersive waves. In our analysis we use a laser model with a mixture of distributed and lumped saturable absorbers. By varying the lumped and distributed nonlinear losses so that

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the total nonlinear losses remain unchanged, we considerably change the intensity of dispersive waves with a conservation of parameters of the central part of the soliton. This simple model provides an effective way to elucidate the nature of the formation of powerful soliton wings by dispersive waves. In this paper we study the interaction of solitons through dispersive waves due to a lumped saturable absorber and investigate the lasing regimes realized under such an interaction. In Sec. II we present a laser model for the study of the dispersive-wave mechanism of formation of powerful soliton wings. Numerical simulations and discussion of the obtained results are given in Sec. III. The basic conclusions are made in Sec. IV.

## II. MODEL

The investigated laser is schematically represented in Fig. 1. It consists in a unidirectional ring cavity including a gain fiber medium and a saturable absorber. The equation describing the evolution of a field in the fiber has the following form [13,20]:

$$\frac{\partial E}{\partial \zeta} = (D_r + iD_i)\frac{\partial^2 E}{\partial \tau^2} + (G + iq|E|^2 - \sigma_0 - \sigma_d)E, \qquad (1)$$

where $E(\zeta,\tau)$ is the electric field amplitude, $\tau$ is the time coordinate expressed in units of $\delta t = \sqrt{|\beta_2|L/2}$ (here $\beta_2$ is the second-order group-velocity dispersion for the fiber and $L$ is the fiber length), $\zeta$ is the normalized propagation distance (the number of passes of radiation through the laser cavity), $D_r$ and $D_i$ are the frequency dispersions for the gain or loss and for the refractive index, respectively, $\sigma_0$ is the linearly distributed loss coefficient, $\sigma_d$ is the nonlinear distributed loss, and $q$ is the Kerr nonlinearity. The term $G$ describes the saturable amplification determined by the total energy of the intracavity radiation: $G = a/(1 + b\int I d\tau)$, where the integration is carried out over the whole round-trip period, $a$ is the pumping parameter, $b$ is the saturation parameter, and $I = |E|^2$. The evolution of a field in the lumped saturable absorber with the nonlinear losses $\sigma_l$ is described by the equation

$$\frac{\partial E}{\partial \zeta} = -\sigma_l E. \qquad (2)$$

The distributed and lumped nonlinear losses are described with the use of a model of two-level absorber atoms [21]:

$$\sigma_l = \frac{\eta\sigma_{nl}}{1 + pI}, \qquad (3)$$

$$\sigma_d = \frac{(1-\eta)\sigma_{nl}}{1 + pI}, \qquad (4)$$

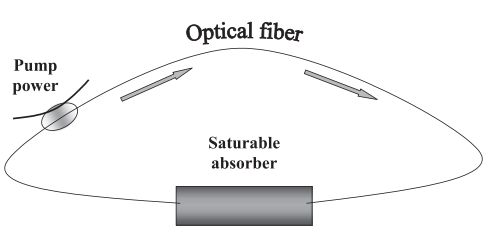


FIG. 1. Schematic representation of the studied passive mode-locked fiber laser.

where $p$ is the saturation parameter for nonlinear losses, $\eta$ is the fraction of two-level atoms which forms the lumped nonlinear losses, $(1-\eta)$ is the fraction of two-level atoms which forms the distributed nonlinear losses in the fiber, and $\sigma_{nl}$ is the total unsaturated nonlinear losses. Varying the parameter $\eta$ from 1 to 0 changes the fraction of the lumped nonlinear losses from 1 to 0 and correspondingly the fraction of the distributed ones from 0 to 1. As this takes place, the total nonlinear losses remain unchanged and, as a result, the central part of an individual soliton does not change significantly. However, its wings change drastically. In the frame of this model only the nonlinear losses can be lumped. All other characteristics of the intracavity components are uniformly distributed along the laser cavity. Numerical simulations have been performed for typical parameters of an Er-doped fiber laser with the anomalous dispersion of group velocity $\beta_2$ [4].

## III. NUMERICAL SIMULATIONS AND DISCUSSION

Figure 2 shows the distances between two interacting solitons in stable steady states. These distances have been obtained by numerical simulations of transient processes and established states of a laser operation with various initial conditions. By varying the initial conditions we obtained various established states of interacting solitons forming a two-soliton molecule. The location of the first soliton on the time axis in a stable steady state of such a molecule is denoted by the white circle. The obtained locations of the second soliton are denoted by black squares for odd steady states of the two-soliton molecule and by gray circles for even ones. Here the nonlinear losses are totally lumped ($\eta = 1$). Figure 2(a) demonstrates the intersoliton separations for sets of bound steady states with alternation of the parity. Greater separation corresponds to smaller bound energy. Such a set of steady states of a two-soliton molecule was obtained by numerical simulation of a fiber laser using a nonlinear polarization rotation technique [13]. The odd and even steady states are described correspondingly by odd and even field functions $E(\tau) = \mp E(-\tau)$. The phase difference for the peak amplitudes of the solitons equals approximately $\pi$ for odd states and 0 for even states. With varying laser parameters

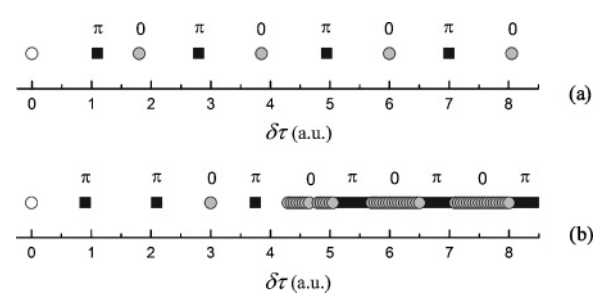


FIG. 2. Distances between two bound solitons in stable stationary states. The white circle corresponds to the first soliton and the black squares and gray circles to the second soliton with the intersoliton phase differences $\pi$ and 0, respectively. (a) The rigorous alternation of parity (of 0 and $\pi$ states). $\eta = 1$, $a = 0.55$, $D_r = 0.02$, $D_i = 0.1$, $q = 1.5$, $p = 1$, $\sigma_0 = 0.01$, and $\sigma_{nl} = 0.8$. (b) Breakdown of parity alternation ($\delta\tau \approx 1$–2) and occurrence of bands ($\delta\tau > 4$). $\eta = 1$, $a = 0.5$, and $D_r = 0.01$.

we observed a breakdown of the parity alternation. This breakdown is shown in Fig. 2(b) with $\delta\tau \approx 1$–2. This is due to the fact that the first even state becomes unstable. We can see also the transformation of fixed intersoliton separations into bands ($\tau > 4$). In this case the intersoliton interaction results in a fixed phase relation but unfixed intersoliton separations. We have checked the stability of the bound states by slightly changing the separation of the two solitons and then looking at the temporal evolution. With variation of the laser parameters we observed various modifications of the breakdown of the parity alternation. We also observed a simultaneous instability for both types of steady state. In this case the interaction between solitons results in their attraction or repulsion.

To find the established intersoliton distances we used the following procedure. For some chosen initial intersoliton distance equal to several pulse durations, the laser generation created one of the stable steady states with a well-defined interpulse interval. The time of the transient process was equal to about 100 passes of the field through the resonator. The stationary solitons obtained were used as initial pulses, but with a different distance between them. The size of the displacement of the pulses was varied with a small step. After the transient process the system can either come back to the initial stable steady state or switch to the next stable one with another interpulse interval. Thus, by scanning the initial values of the interpulse intervals, we obtained all the established intersoliton distances shown in Fig. 2. During the transient process and the realization of steady states we detected the temporal and spectral profiles of the pulses and their phase differences. The very large time slot of observation of the obtained stationary states ($10^5$ passes of the field through the resonator) permits us to say that these states are stable. Instabilities of the even and odd stationary states that arise due to a change of laser parameters are manifested by monotonic change in the intersoliton intervals and phase differences, or monotonically increasing oscillations of these values. During the transient evolution such increasing oscillations transform themselves into a monotonic change of the corresponding values with a departure from the initial stationary state.

The large number of stable steady states and established intersoliton separations shown in Fig. 2 arises from interactions of the ultrashort pulses through their powerful long-distance wings. The observed powerful sidebands in a single-soliton spectrum point out the important role of dispersion waves in the formation of the soliton wings. In the frame of the model described by Eqs. (1)–(4), the dispersive waves are emitted by solitons only because of the lumped saturable absorber. Figure 3 demonstrates the change in a soliton wing formed by dispersion waves with a variation of a fraction of the lumped nonlinear losses $\eta$ (the total nonlinear losses remain unchanged). With decreasing lumped fraction $\eta$ the soliton wings quickly weaken. The parameters of the central part of the soliton remain almost unchanged. Figure 4 demonstrates the corresponding change in the spectral sideband structure of the single soliton. This means that the narrow spectral sidebands are related to the soliton wings.

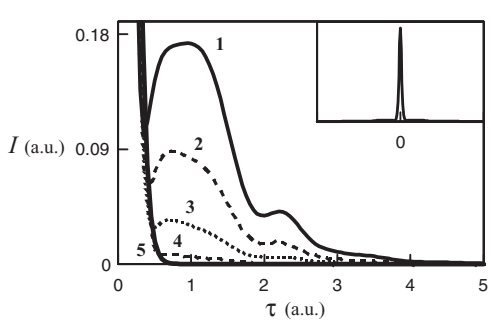


FIG. 3. Soliton wing intensity $I(\tau)$ with varying lumped fractions of the saturable absorber, $\eta$: (1) $\eta = 1$, (2) $\eta = 0.75$, (3) $\eta = 0.50$, (4) $\eta = 0.25$, and (5) $\eta = 0$. The other parameters are the same as in Fig. 2(a).

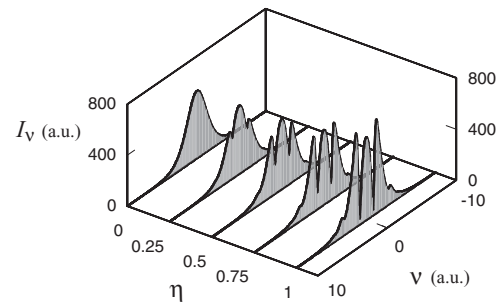


FIG. 4. Spectrum of single soliton with varying value of the lumped fraction of the saturable absorber $\eta$. The parameters are the same as in Fig. 2(a).

Figure 5 shows the change in the binding energy $\delta J$ with varying parameter $\eta$ for the first six bound steady states shown in Fig. 2(a):

$$\delta J_n = \frac{(J_n - J_\infty)}{J_p}, \tag{5}$$

where $J_n$ is the energy of two solitons in bound steady states, $J_\infty$ is the energy of two far separated noninteracting solitons, and $J_p$ is the energy of one soliton. With decreasing lumped part of the total saturable absorber, $\eta$, the intensity of the dispersive waves decreases. This decrease results in a decrease of the intensity of the long-distance soliton wings which are formed by these waves. As a result, the binding energy of the interacting solitons decreases also. Without lumped losses, we have checked that neither strong interaction between solitons nor a set of a large number of bound states (see Fig. 2) becomes possible.

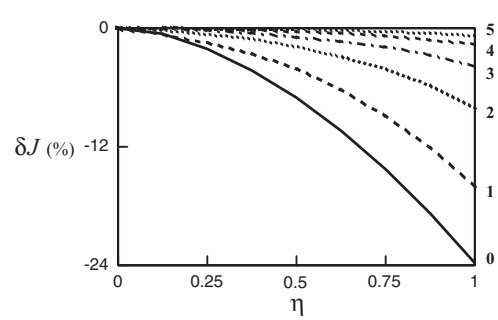


FIG. 5. Binding energy $\delta J$ for first six steady states [see Fig. 2(a)] with varying values of the lumped fraction of the saturable absorber, $\eta$.

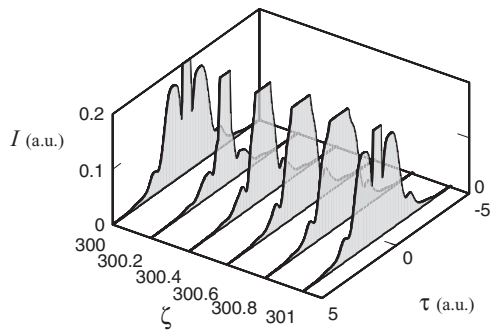


FIG. 6. Periodic change in a soliton pedestal during one pass through the laser cavity. The parameters are the same as in Fig. 2(a).

We have also investigated the dynamics of formation of powerful wings through dispersive waves. The main features of the formation are the following. In a medium with a focusing Kerr nonlinearity and with an anomalous dispersion there exist stable self-localized waves in the form of a stationary pulse (conservative soliton). In a stationary state the nonlinear mechanism shortening the pulse and the dispersive mechanism lengthening it counterbalance each other. The stationary pulse duration and shape are determined by an equilibrium of these processes. If any perturbation changes the pulse then the disturbed pulse transits into a stationary state through relaxation oscillations with a loss of the excess energy through the emission of dispersion waves. The oscillation frequency is determined by the rate of chirp formation resulting from nonlinearity and dispersion of the refractive index. The frequency dispersion of gains and losses results only in some additional peculiarities of the described process. An ultrashort pulse propagating in a ring laser cavity experiences periodically a shock perturbation when it passes through a lumped saturable absorber. Thus, the investigated system is analogous to a damped oscillator submitted to an external periodic force. In the established regime the structure of the radiation is reproduced after each pass of the field through the laser cavity. However, along a fiber the soliton parameters (peak intensity, duration, chirp, etc.) oscillate. Figure 6 shows the periodic change in the soliton pedestal during a single pass through the laser cavity in the established operation. One can see the oscillation in the duration of the central part of the soliton.

For the pump $a \approx 0.40$ and $a \approx 0.55$ [the others parameters correspond to those used in Fig. 2(b)], the oscillation period equals respectively 1 and $1/2$ cavity round-trip period. Under such resonance the single-soliton operation becomes unstable and transforms into the regime of two and three bound solitons, respectively. The pedestal structures differ essentially for the pump above and below threshold values.

In Fig. 6 one can see the dispersive waves leaving from the soliton. These waves form the powerful long-distance soliton wings which result in spectral sidebands. In the case of normal dispersion, conservative solitons and their oscillations are not realized and consequently both powerful long-distance pulse wings and powerful spectral sidebands are not realized.

Figure 7 shows the change in phase evolution along the soliton in established operation after each round-trip period. In the case of Fig. 7(a) the dispersive waves are emitted only because of the lumped nonlinear losses. In the case of

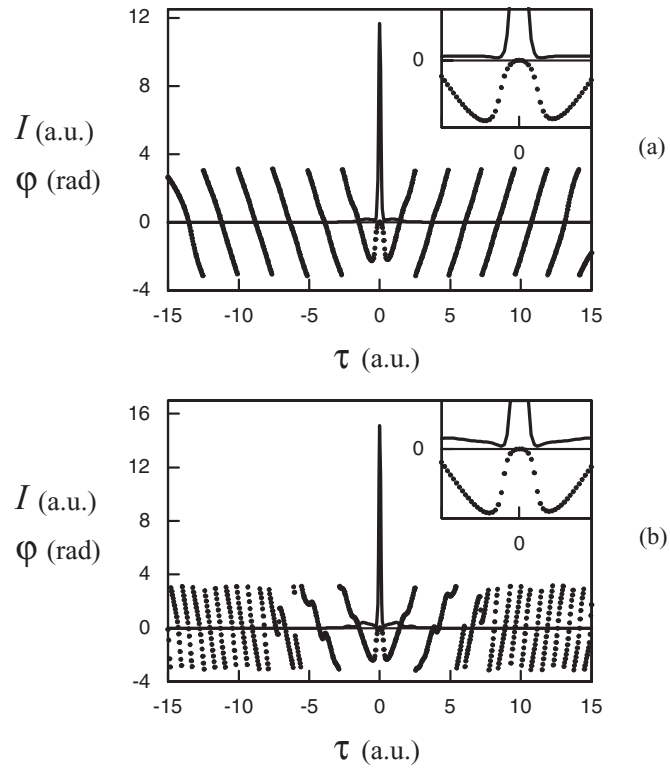


FIG. 7. Phase change $\varphi(\tau)$ along the wings of the soliton with the intensity distribution $I(\tau)$. The upper right inset shows the changes of the phase (the bottom curve) and intensity (the top curve) in a vicinity of the center of the soliton in the increased scale. (a) The parameters are the same as in Fig. 2(a). (b) The lumped linear losses are equal to 0.1. The other parameters are the same as in Fig. 2.

Fig. 7(b) there exist two mechanisms inducing dispersive waves: nonlinear and linear losses. As a result, the phase distribution in the second case appears more complex. In established stationary operation the phase distribution is reproducible at any point of the wing after each round-trip period (to within the same constant for all points). In this sense the pictures shown in Fig. 7 are stationary. However, at any point of the pulse the phase is changed when the pulse propagates along a fiber. In a round-trip period the total phase change is a multiple of $2\pi$ (that is, $\delta\varphi = 2\pi n$, where $n$ is an integer). Each fragment of the wing is determined by a fixed $n$ which in turn determines the distance between neighboring in-phase points in the soliton wing. In the case of Fig. 7(b), for the central part of the pulse ($0 < \tau < 1$) the integer $n$ equals zero. For the near wing ($1 < \tau < 5$) $n = 1$. For the far wing ($6 < \tau < 15$) $n = 3$.

In this paper we have studied the formation of dispersive waves due to lumped nonlinear losses. There exist also other mechanisms resulting in dispersive waves, for example, a change of the fiber dispersion in lasers with several compound fibers. The study of these mechanisms is left for further research.

## IV. CONCLUSION

On the basis of numerical simulation we have found that the main mechanism of formation of powerful long-

distance soliton wings in a laser with a lumped saturable absorber is connected with dispersive waves emitted by the soliton because of its interaction with lumped losses. These wings result in a strong interaction between solitons leading to formation of a soliton molecule with a large set of energy levels and stable steady states. The alternation of the parity for steady states of neighboring energy levels can be broken by varying the laser parameters. The powerful soliton wings due to the dispersive waves result also in sidebands in the spectrum of an individual soliton.

## ACKNOWLEDGMENTS

The work is supported by the Agence Nationale de la Recherche (Contract No. ANR-2010-BLANC-0417-01-SOLICRISTAL) and by the RF President's Grant No. MK-2602.2011.2.

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