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Hervé Leblond, Philippe Grelu, Dumitru Mihalache. Non-SVEA models for supercontinuum generation. 2020. hal-02572727

**HAL Id: hal-02572727**

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Preprint submitted on 13 May 2020

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# Non-SVEA models for supercontinuum generation

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<sup>3</sup>Horia Hulubei National Institute for Physics and Nuclear Engineering, and Academy of Romanian Scientists, Bucharest, Romania

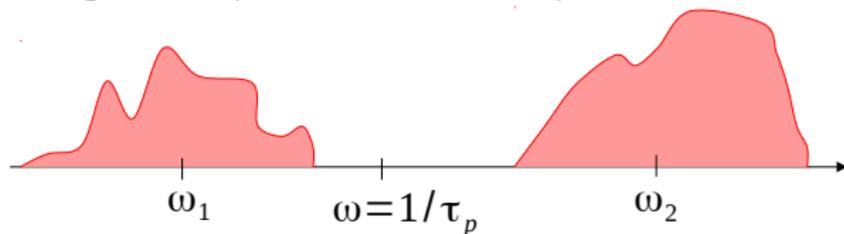
- 1 The mKdV-sG equation
- 2 Supercontinuum generation
  - Stokes side
  - Self-phase modulation
  - High harmonics generation
- 3 FCP solitons
  - The sG model
  - The mKdV model

# Introduction

- The shortest laser pulses:  
a duration of a few optical cycles.
- The slowly varying envelope approximation ( SVEA ) is not valid any more.
- We look for simplified models accounting for Few-Cycle Pulse propagation

# A transparent medium

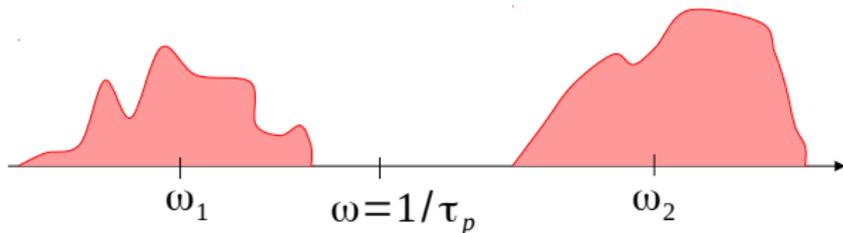
- The general spectrum of a transparent medium



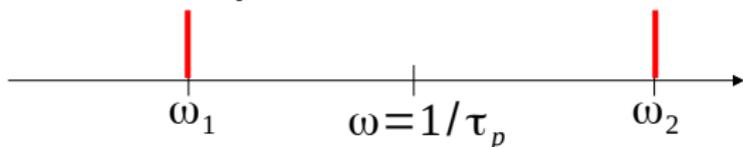
- A simple model: A two-component medium, each component is described by a two-level model
- We assume that the FCP duration  $\tau_p$  is such that  $\omega_1 \ll (1/\tau_p) \ll \omega_2$

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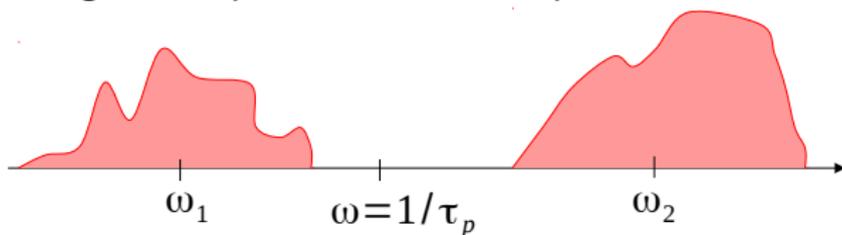
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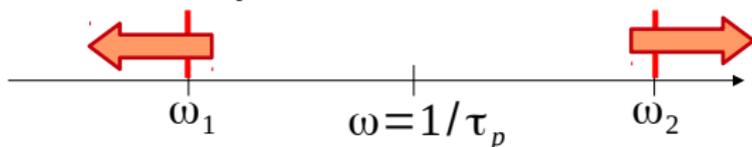
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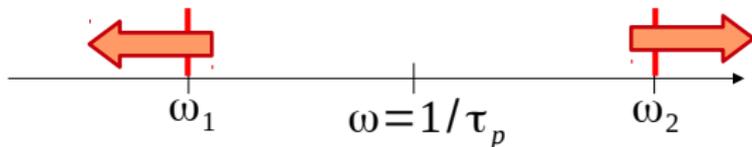
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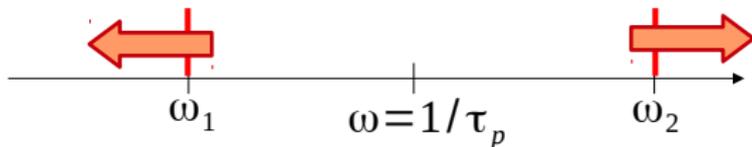
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 $\Rightarrow$  Long-wave approximation

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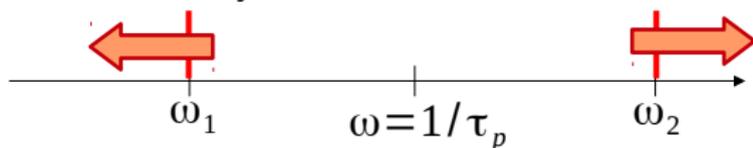
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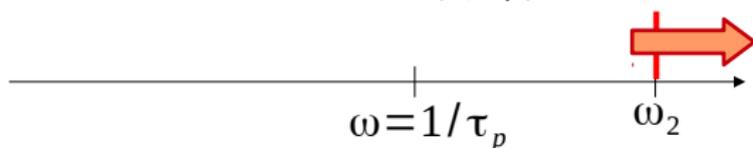
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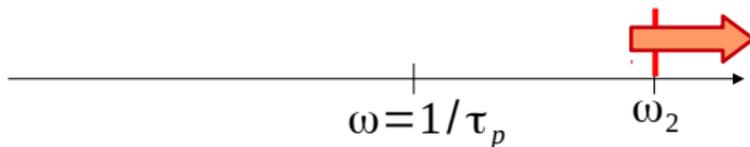
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⇒ Long-wave approximation

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⇒ Long-wave approximation

modified Korteweg-de Vries (mKdV) equation

$$\frac{\partial E}{\partial \zeta} = \frac{1}{6} \frac{d^3 k}{d\omega^3} \bigg|_{\omega=0} \frac{\partial^3 E}{\partial \tau^3} - \frac{6\pi}{nc} \chi^{(3)}(\omega; \omega, \omega, -\omega) \bigg|_{\omega=0} \frac{\partial E^3}{\partial \tau}$$

# A transparent medium

- In a first stage, the two components are treated separately
- IR transition only, with  $\omega_1 \ll (1/\tau_p)$   
 $\Rightarrow$  Short-wave approximation

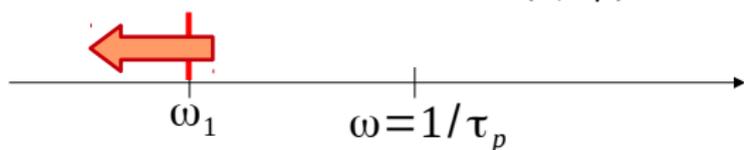
- sine-Gordon (sG) equation:  $\frac{\partial^2 \psi}{\partial z \partial t} = c_1 \sin \psi$

with  $c_1 = \frac{w_\infty}{w_r}$  : normalized initial population difference

and  $\frac{\partial \psi}{\partial t} = \frac{E}{E_r}$  : normalized electric field

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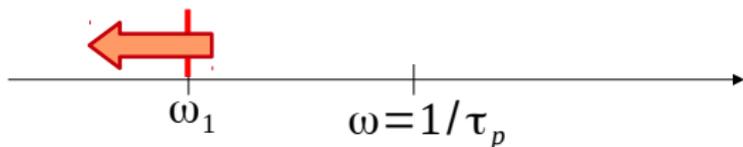
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- Then the two approximations are brought together to yield a general model:

The mKdV-sG equation

$$\frac{\partial^2 \psi}{\partial z \partial t} + c_1 \sin \psi + c_2 \frac{\partial}{\partial t} \left( \frac{\partial \psi}{\partial t} \right)^3 + c_3 \frac{\partial^4 \psi}{\partial t^4} = 0$$

- $\frac{\partial u}{\partial z} + c_1 \sin \int^t u + c_2 \frac{\partial u^3}{\partial t} + c_3 \frac{\partial^3 u}{\partial t^3} = 0$

with  $u = \frac{\partial \psi}{\partial t} = \frac{E}{E_r}$  : normalized electric field

- Integrable by inverse scattering transform in some cases:

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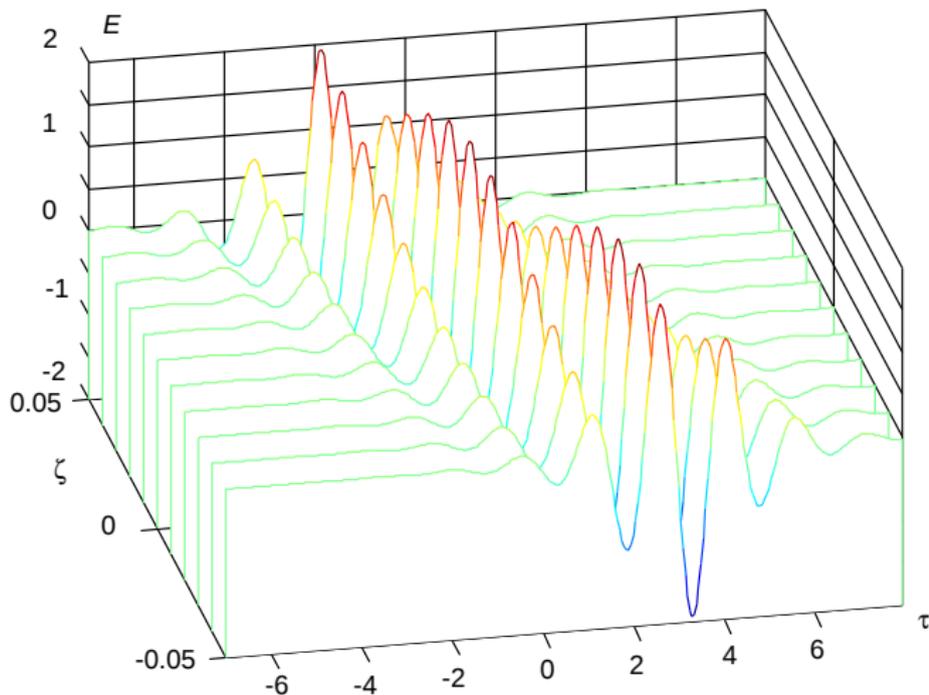
The mKdV-sG equation

- $$\frac{\partial u}{\partial z} + c_1 \sin \int^t u + c_2 \frac{\partial u^3}{\partial t} + 2c_2 \frac{\partial^3 u}{\partial t^3} = 0$$

with  $u = \frac{\partial \psi}{\partial t} = \frac{E}{E_r}$  : normalized electric field

- Integrable by inverse scattering transform in some cases:  
 mKdV, sG, and  $c_3 = 2c_2$ .

- The breather: a few-cycle pulse



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## Evidence for supercontinuum generation

- The mKdV equation:

$$\frac{\partial u}{\partial z} + c_2 \frac{\partial u^3}{\partial t} + c_3 \frac{\partial^3 u}{\partial t^3} = 0$$

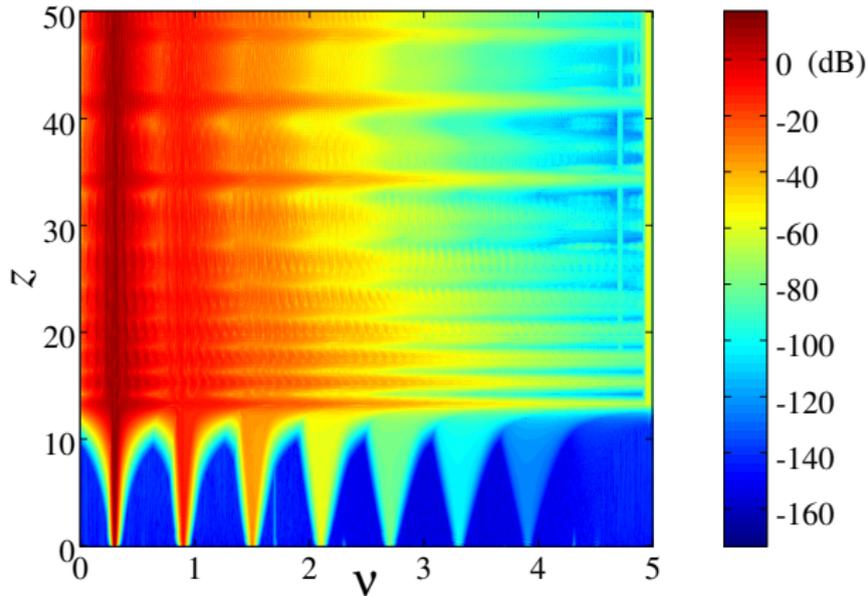
- Input is a Gaussian pulse:

$$u(0, t) = A \sin(\omega t) e^{-t^2/\tau^2}.$$

- Normalized (dimensionless),  $c_2 = c_3 = 1$

- Optical period  $\frac{2\pi}{\omega}$  and pulse duration  $\tau$ :  
 same as a 100 fs long pulse,  $\lambda = 1\mu\text{m}$ .

- Numerical resolution.....



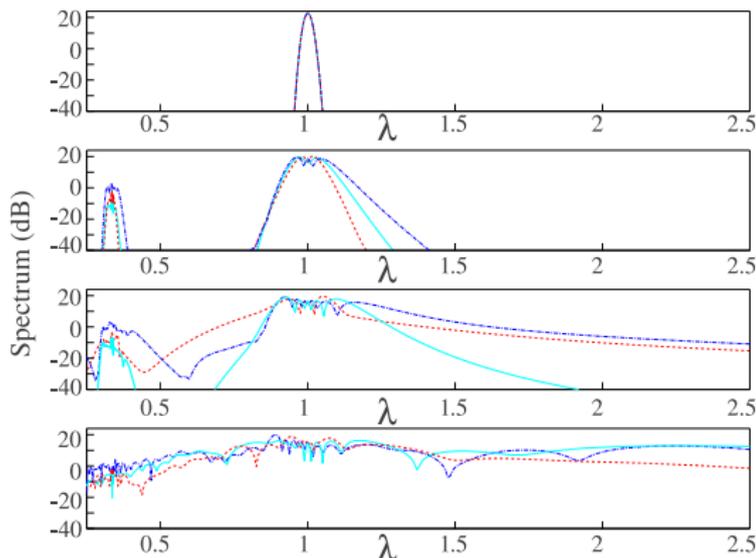
mKdV, input with  $FWHM = 100$ ,  $\nu = 0.3$ ,  $A = 0.7$ .

- A very broad spectrum is reached quickly.

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- Question: generation of frequencies lower than  $\omega$ ?
- We solve numerically the mKdV, sG and mKdV-sG models starting with a 100 fs pulse,  $\lambda = 1\mu\text{m}$   
→ Compare the evolution of the spectrum

- Comparison between mKdV, sG, and mKdV-sG models



For  $z = 0, 6, 12,$  and  $16$ .

- sG extends first towards Stokes side,  
 mKdV extends first towards anti-Stokes.
- mKdV-sG model lies in between.

- Recall that mKdV accounts for UV transitions, and sG for IR transitions.
- Usual Raman broadening yields extension of the spectrum towards low frequencies at the beginning of the process and is due to IR transitions.
- Although Raman effect itself is not taken into account by sG model, the corresponding spectral broadening is.

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- A quasi monochromatic wave

$$u = U(z,t)e^{i(kz-\omega t)} + cc + u_1(z,t), \quad (1)$$

$U$ : fundamental wave amplitude;  $u_1$ : small correction.

- Neglect dispersion, and disregard third harmonic generation:

$$U = \frac{A}{2} \exp \left[ i \left( \frac{c_1}{2\omega^3} + 3\omega c_2 \right) \frac{A^2}{4} z \right]. \quad (2)$$

- Self phase modulation.  
→ broadening of the spectrum  
with typical oscillations of the spectral density.

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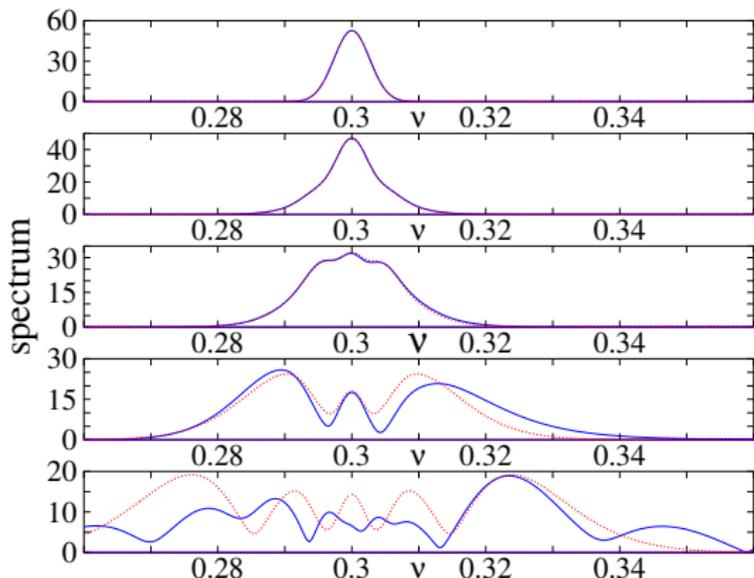
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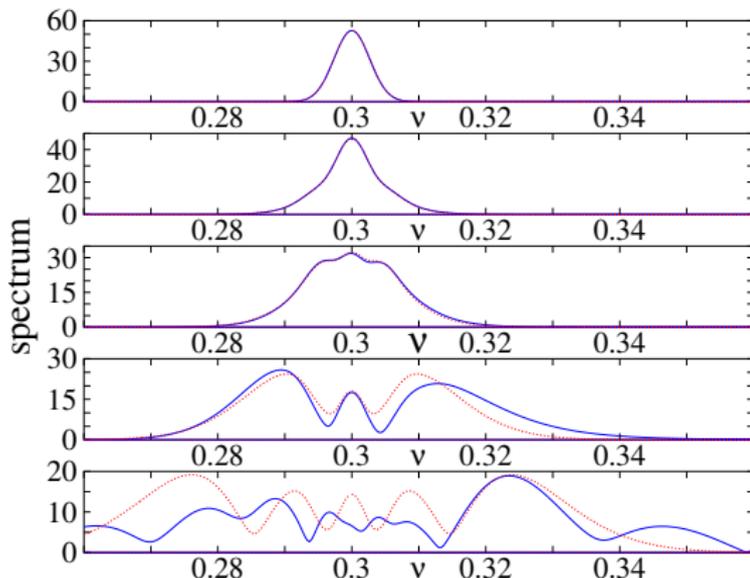
- Numerical **mKdV** vs analytical **self-phase modulation**, Eq. (2)



$FWHM = 100$ ,  $\nu = 0.3$ ,  $A = 0.7$ , for  $z = 0, 2, 4, 10, 20$ .

- Then **actual broadening** becomes asymmetric, while **analytic formula** remains symmetric.
- The very beginning of the broadening process

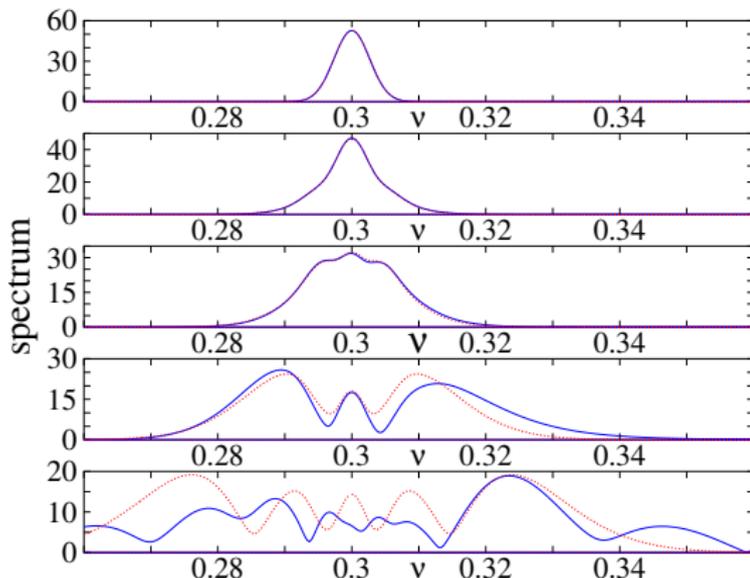
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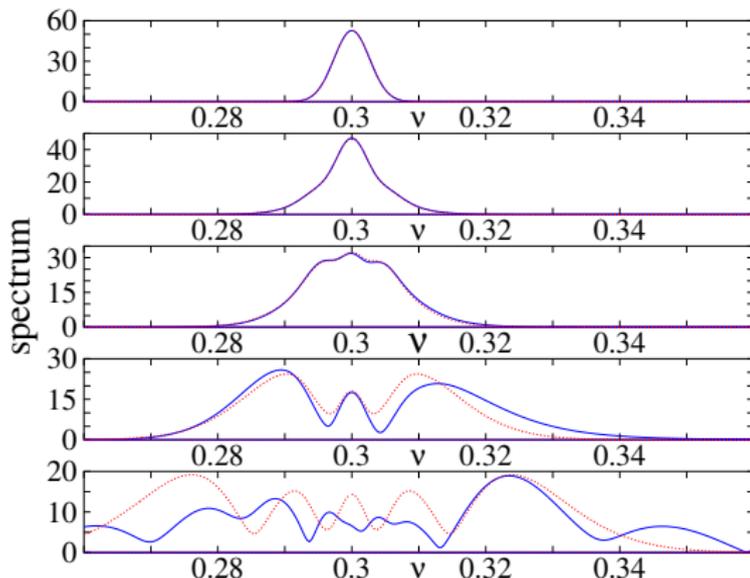
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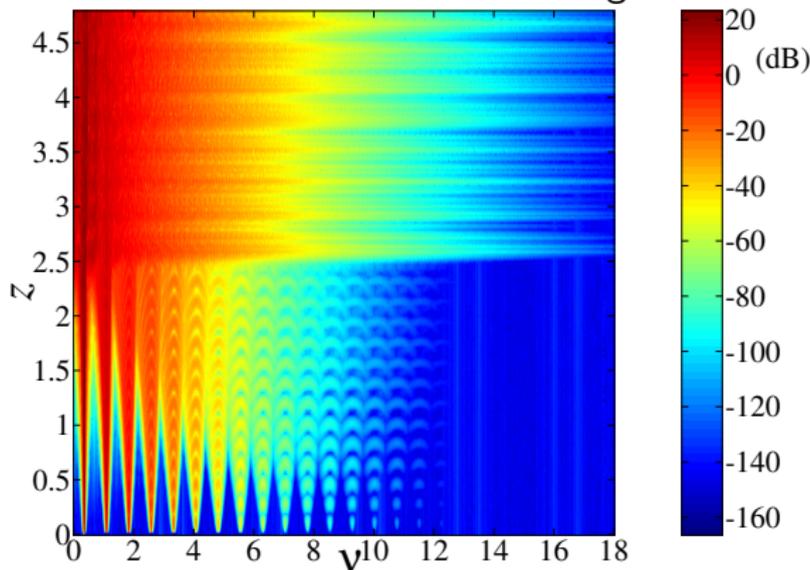
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- The very beginning of the broadening process is due to self phase modulation.

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- A lot of high harmonics are created and involved in the process
- The SVEA does not account for high harmonic generation



- Up to 15 harmonics can be seen  
 $FWHM = 80$ ,  $\nu = 0.375$ , and  $A = 5$ , sG model.

# Spectral width of harmonics

- Assume a Gaussian profile  $u = Ae^{\frac{-t^2}{\tau^2}} e^{-i\omega_0 t}$ .
- Its Fourier transform is  $\hat{u} = \frac{2\sqrt{\pi}}{A\tau} e^{\frac{-(\omega-\omega_0)^2 \tau^2}{4}}$ .
- The  $n$ th harmonic is then  $u^n = Ae^{\frac{-nt^2}{\tau^2}} e^{-in\omega_0 t}$ ,  
 and its width is  $\frac{\tau}{\sqrt{n}}$ .
- Consequently its Fourier transform is  

$$\widehat{(u^n)} = \frac{2\sqrt{n\pi}}{A\tau} e^{\frac{-(\omega-n\omega_0)^2 \tau^2}{4n}}$$
.
- Hence the spectral width of the  $n$ th harmonic is  $\frac{2\sqrt{n}}{\tau}$ ;  
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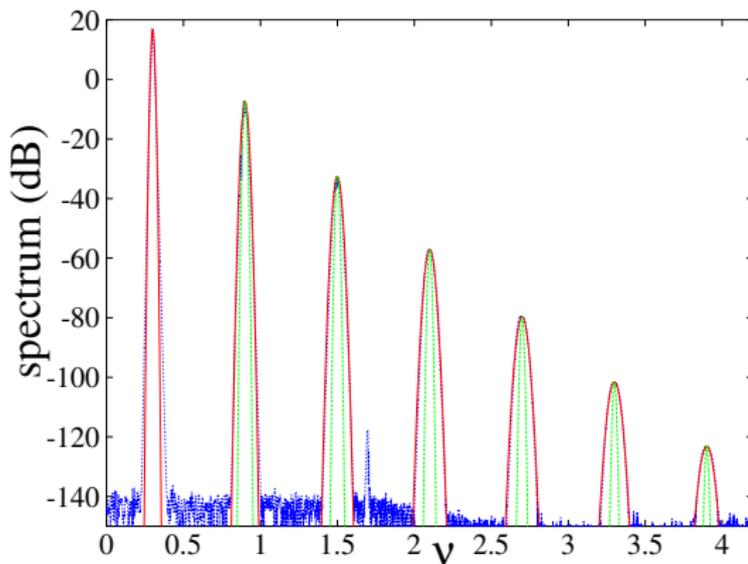
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- Spectral widths of the harmonics.  
 mKdV compared to **Gaussian** with width increasing as  $\sqrt{n}$



$FWHM = 100$ ,  $\nu = 0.3$ , and  $A = 0.7$ , for  $z = 2$ .

- Spectral broadening due to parametric interaction between the sidebands of the harmonics and the ones of the fundamental.
- $\omega_0$ : fundamental frequency. Assume a sideband  $\omega_0 + \delta\omega$ ,
- the third harmonics contains the sideband  $3(\omega_0 + \delta\omega)$ .
- It may interact with the fundamental  $\pm\omega_0$ :  
→  $3(\omega_0 + \delta\omega) - \omega_0 - \omega_0 = \omega_0 + 3\delta\omega$
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→  $3(\omega_0 + \delta\omega) - \omega_0 - \omega_0 = \omega_0 + 3\delta\omega$
- Hence  $\omega_0 + \delta\omega$  creates  $\omega_0 + 3\delta\omega$ :  
spectrum broadens

- Spectral broadening due to parametric interaction between the sidebands of the harmonics and the ones of the fundamental.
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- The sG equation,

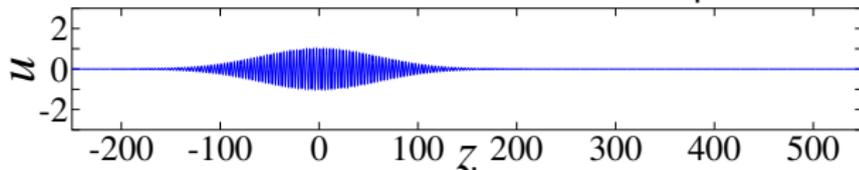
$$\frac{\partial u}{\partial z} + c_1 \sin \int^z u d\tau' = 0$$

$c_1 = 50$ ,  $A = 1$  ( a change in  $c_1$  results only in a change of the unit along the z-axis).

- Input is a Gaussian pulse:

$$u(0,t) = A \sin(\omega t) e^{-t^2/\tau^2}.$$

- A few FCP solitons form and tend to separate



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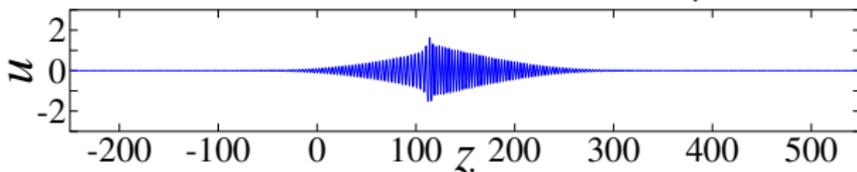
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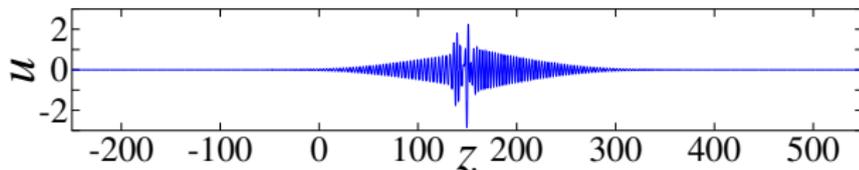
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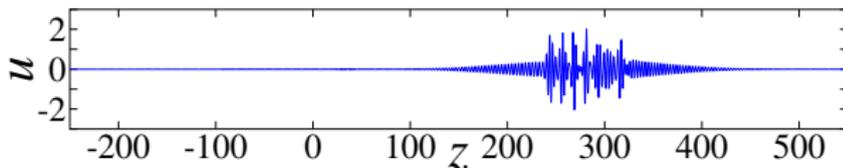
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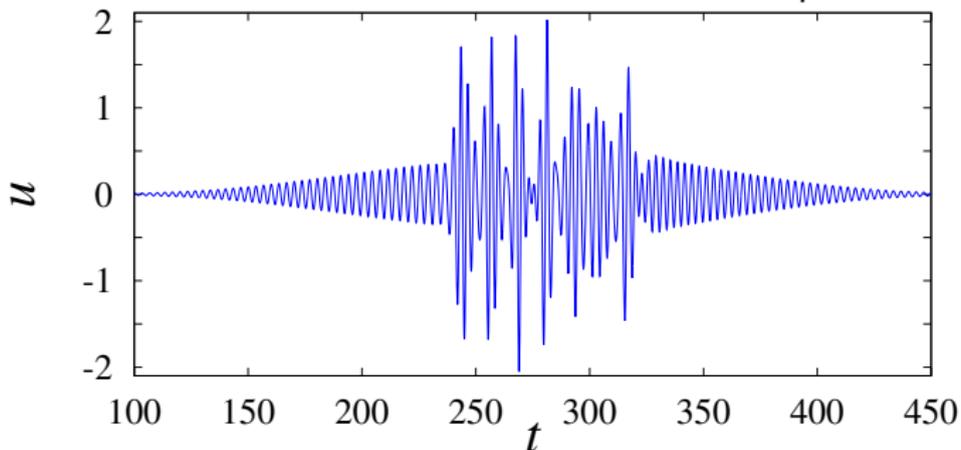
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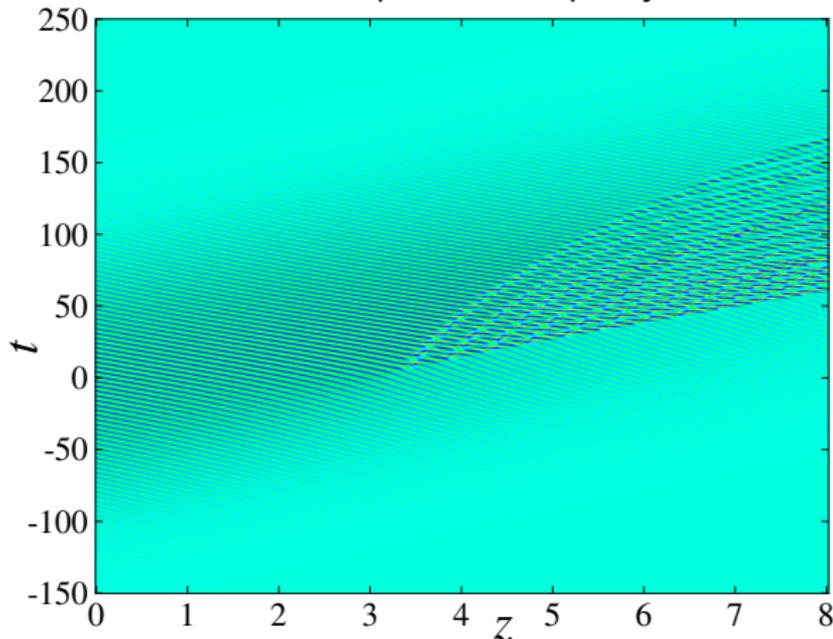
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- FCP solitons don't separate completely, but interact
- $A = 1$  a few FCP solitons form and tend to separate

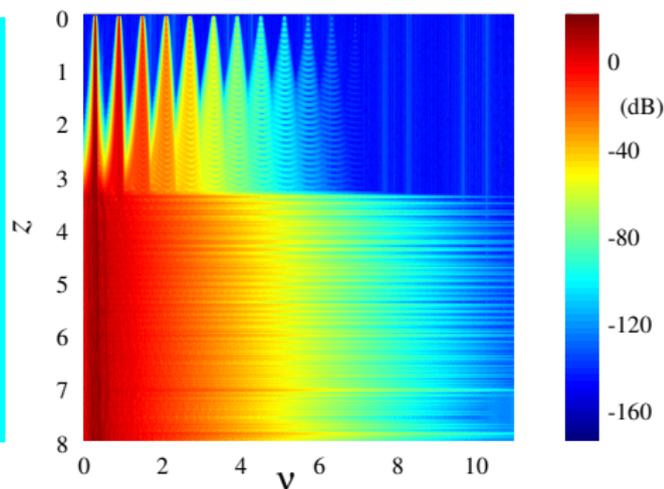
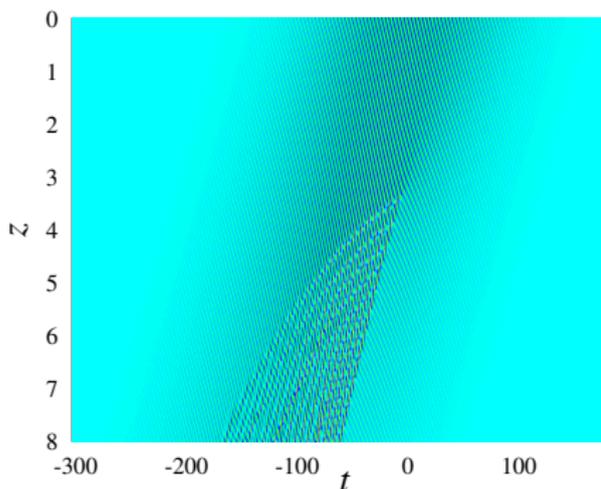


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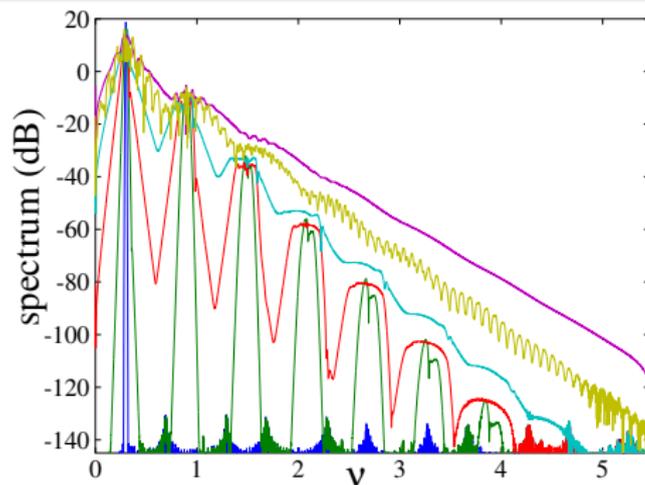
Initial pulse with  $FWHM = 100$ ,  $\nu = 0.3$ , and  $A = 2.5$

- FCP solitons form, don't separate, but interact  $\rightarrow$  ultrabroad supercontinuum



Initial pulse with  $FWHM = 100$ ,  $\nu = 0.3$ , and  $A = 2.5$ , according to the sG model.

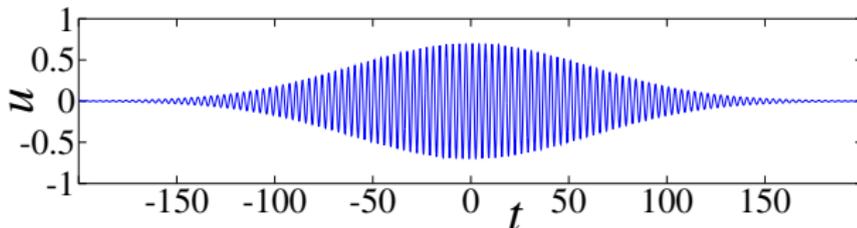
# Evolution of the spectrum according to sG



- **Input** as above,  $z = 0$
- **Harmonic** generation,  $z = 4$
- **Broadening** of harmonics,  $z = 8$
- Arising of the **first soliton**,  $z = 9.2$
- The **second soliton** is just formed,  $z = 11.6$
- **Final** ultrabroad spectrum,  $z = 20$

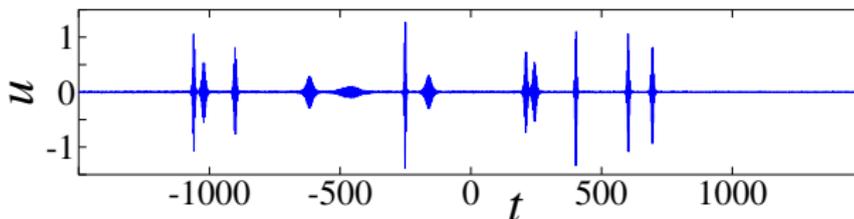
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- At high amplitude level ( $A \sim 2$ ) an initial 100 fs pulse splits into a set of FCP solitons, which are mKdV breathers



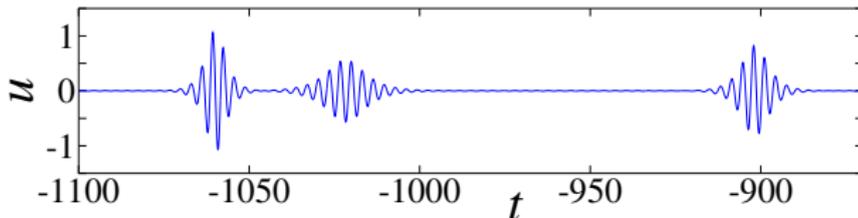
- The FCPs form in a first stage until  $z \simeq 12$  then slowly go away one from the other.
- A huge spectral broadening occurs when they form.

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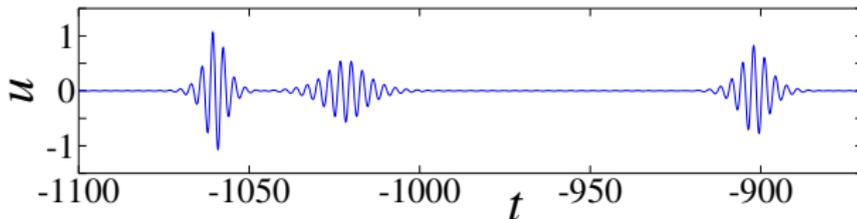
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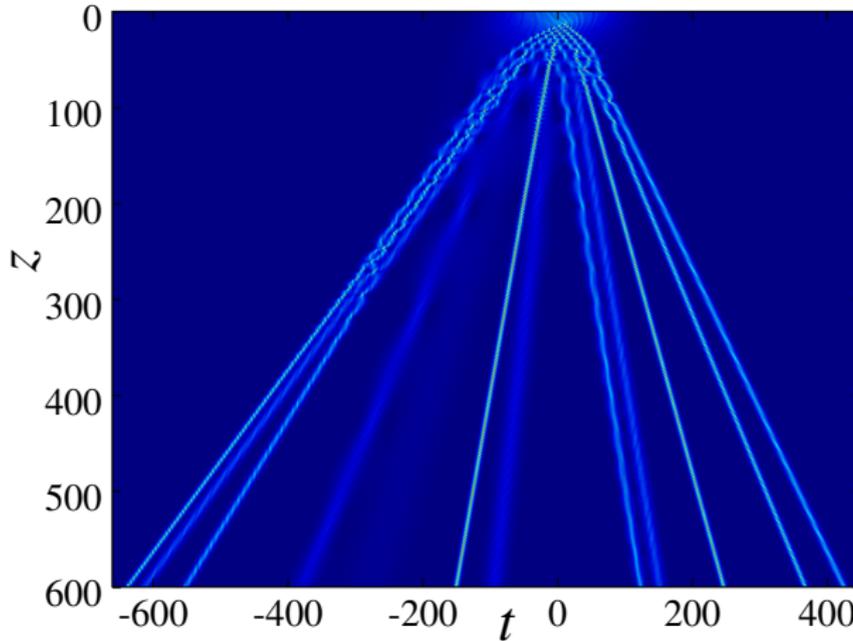


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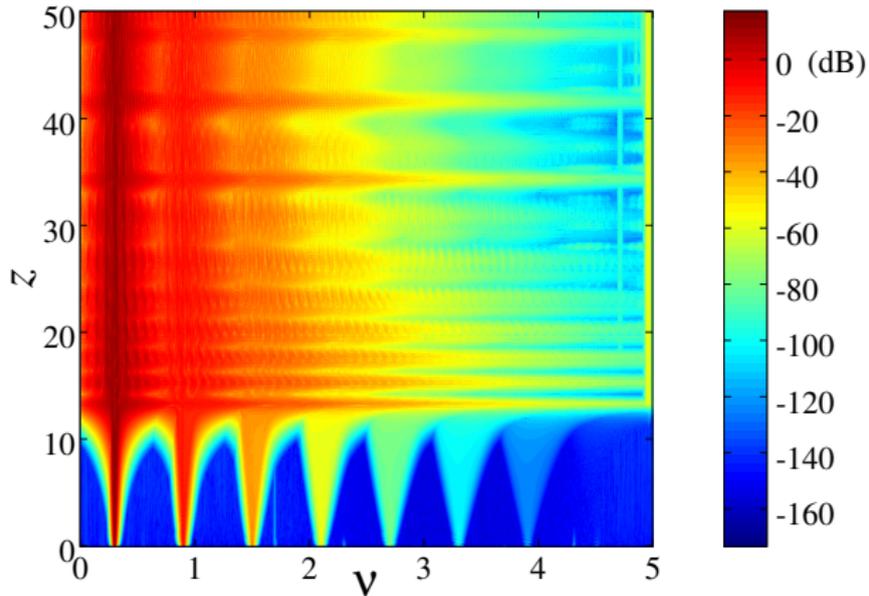
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- A huge spectral broadening occurs when they form.



Initial pulse with

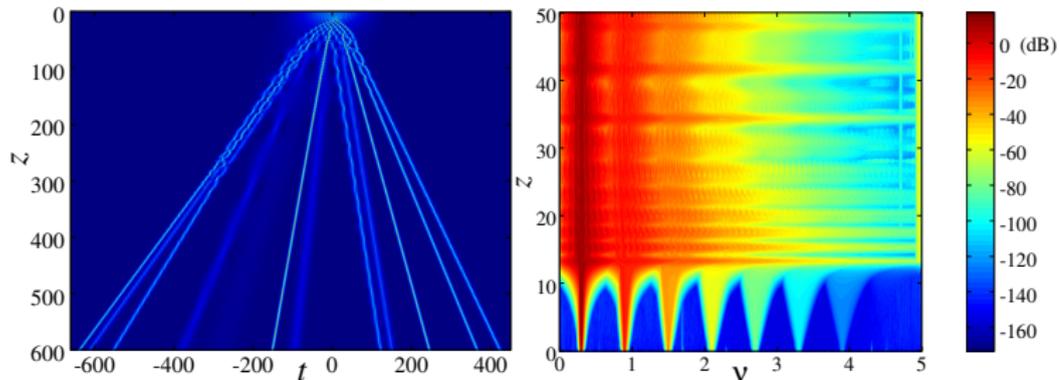
$FWHM = 100$ ,  $\nu = 0.3$ , and  $A = 0.7$ , according to mKdV.

- The input pulse splits into a set of FCP solitons



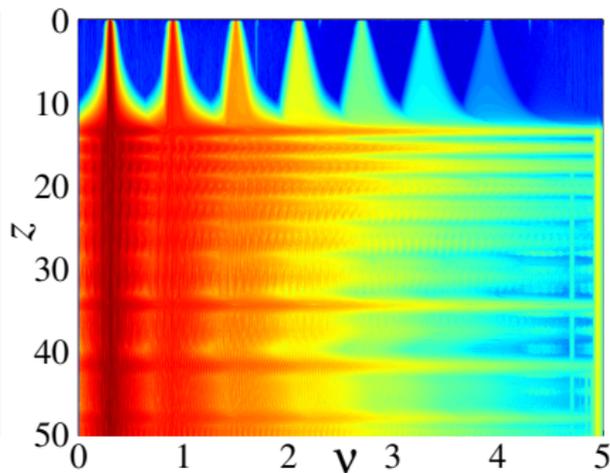
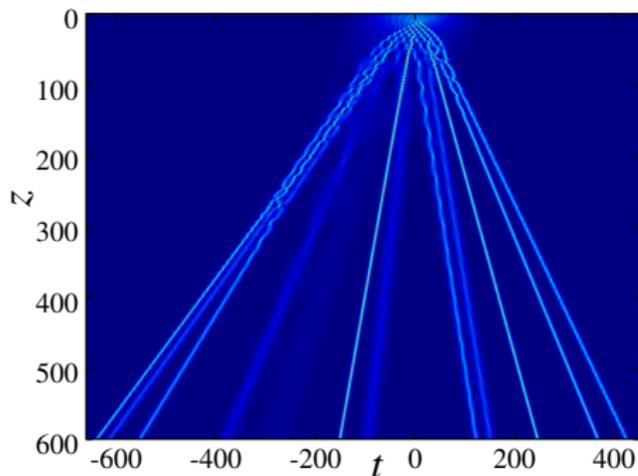
mKdV, input with  $FWHM = 100$ ,  $\nu = 0.3$ ,  $A = 0.7$ .

- A very broad spectrum is reached quickly.



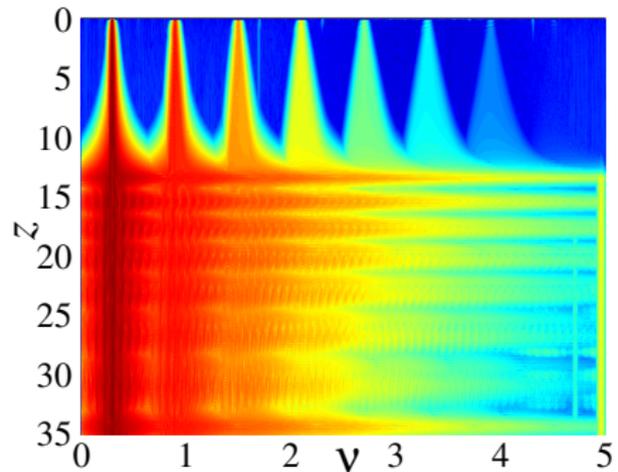
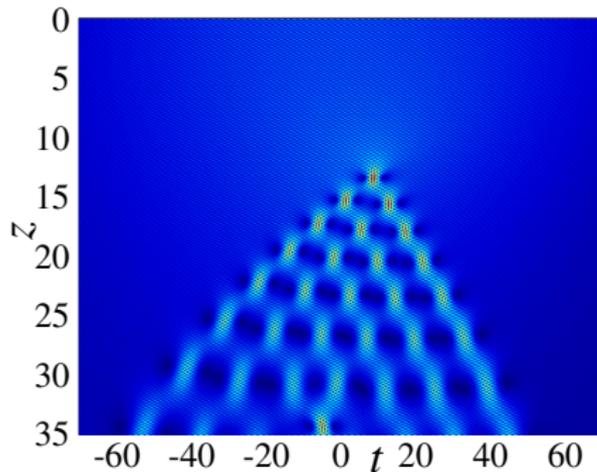
mKdV, input with  $FWHM = 100$ ,  $\nu = 0.3$ ,  $A = 0.7$ .

- Compare temporal profile and spectrum



mKdV, input with  $FWHM = 100$ ,  $\nu = 0.3$ ,  $A = 0.7$ .

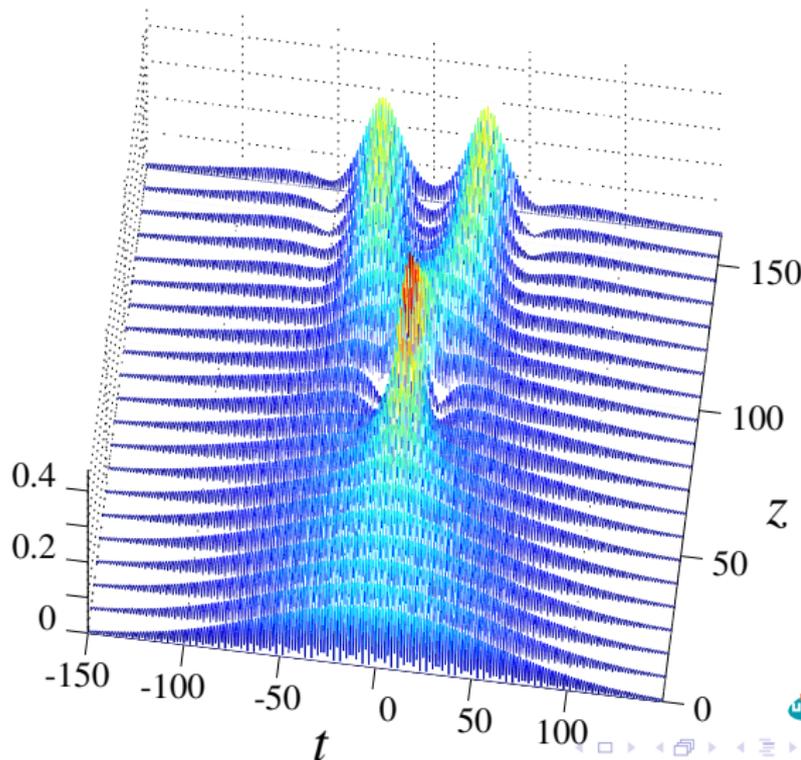
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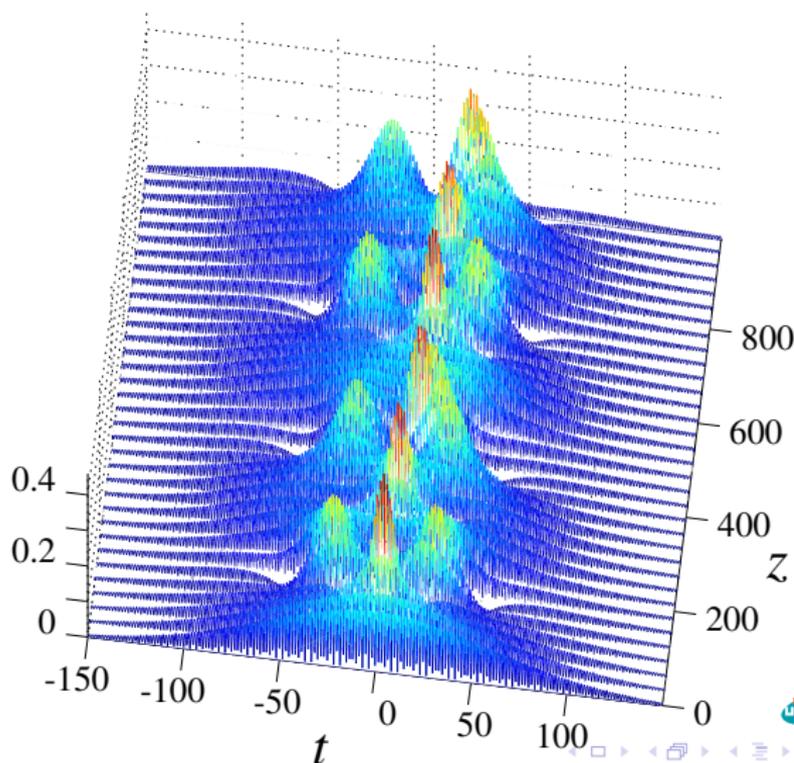
mKdV, input with  $FWHM = 100$ ,  $\nu = 0.3$ ,  $A = 0.7$ .

- The spectrum extends when the solitons separate

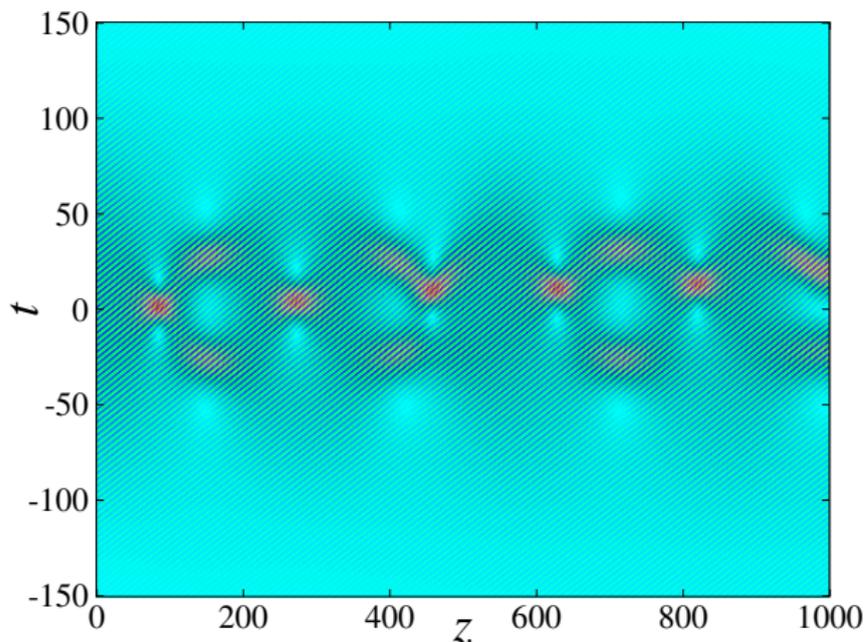
- At a lower amplitude ( $A \sim 0.1, 0.2$ ), it may happen, that two solitons only form



- At a lower amplitude ( $A \sim 0.1, 0.2$ ), they interact with each other

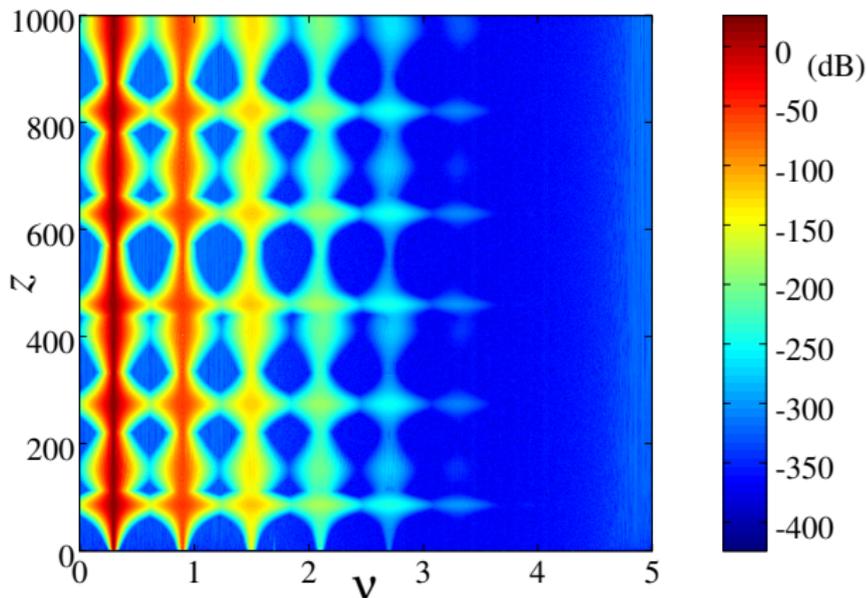


- At a lower amplitude ( $A \sim 0.1, 0.2$ ),  
 and form a kind of superbreather



Initial pulse with  $FWHM = 100$ ,  $\nu = 0.3$ , and  $A = 0.17$ .

- Then the spectral width oscillates

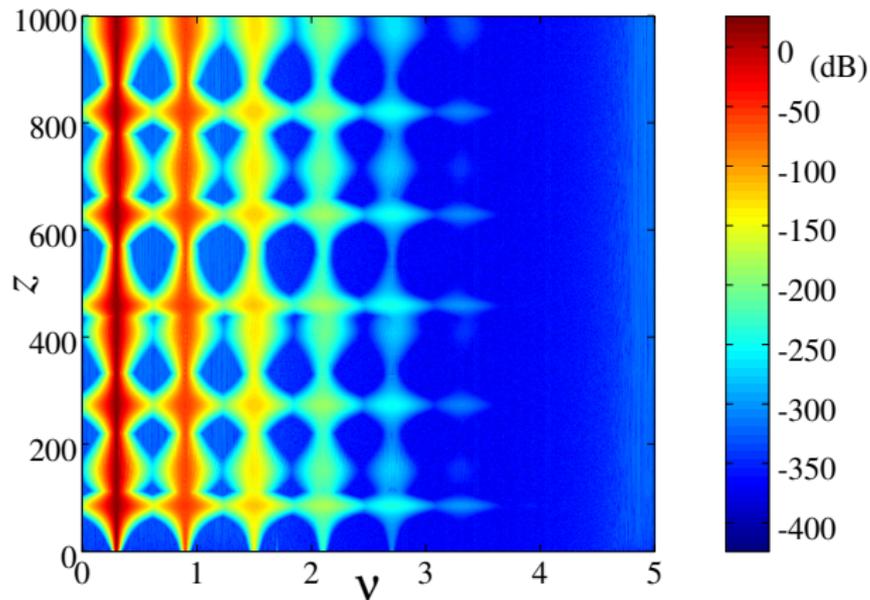


Initial pulse with  $FWHM = 100$ ,  $\nu = 0.3$ , and  $A = 0.17$ .

# Conclusion

- Generic non-slowly varying envelope approximation models: modified Korteweg-de Vries, sine-Gordon, mKdV-sG
- Adequately describe the supercontinuum generation over several octaves
- sG accounts for the Raman broadening
- Modulation instability
- High harmonics generation
- FCP soliton generation and interaction leads to ultrabroad spectrum.

Thank you for your attention.



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