



Solitons à quelques cycles dans la dynamique de génération de supercontinuum

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Non-SVEA models for supercontinuum generation

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- 1 Models for few-cycle solitons
 - The mKdV-sG equation
- 2 A few generalizations
 - General Hamiltonian
 - Circular polarization - long wave
 - Circular polarization - short wave
- 3 Supercontinuum generation
 - Evidence for SC generation
 - Stokes side
 - Self-phase modulation
 - High harmonics generation
- 4 Few cycle solitons in supercontinuum generation
 - The sG model
 - The mKdV model

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- The shortest laser pulses: a duration of a few optical cycles.

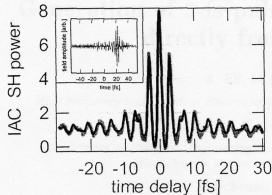


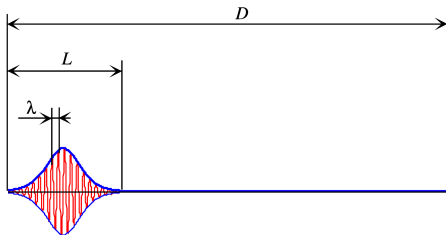
Fig. 4. Measured and reconstructed IAC of the pulse. A phase-retrieval algorithm reveals a pulse width of 5 fs. The reconstructed IAC fits perfectly and is not distinguishable. The reconstructed electric field is displayed in the inset. SH, second harmonic.

Autocorrelation trace, R. Ell et al., Optics Letters 26 (6), 373 (2001).

- Pulse duration down to a few fs. Ex. above: $5 \text{ fs} = 5 \times 10^{-15} \text{ s}$.
- How to model the propagation of such pulses?

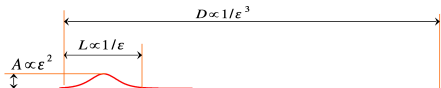
Solitary wave vs envelope solitons

- Envelope soliton: the usual optical soliton in the ps range



- Pulse duration $L \gg \lambda$ wavelength
- Typical model: NonLinear Schrödinger equation (NLS)

- Solitary wave soliton: the hydrodynamical soliton



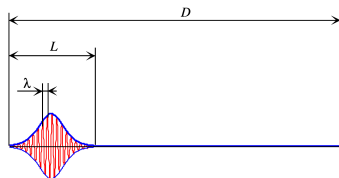
- A single oscillation
- Typical model: Korteweg-de Vries equation (KdV)

- Few-cycle optical solitons: $L \sim \lambda$

- The slowly varying envelope approximation is not valid
- Generalized NLS equation
- We seek a different approach based on KdV-type models

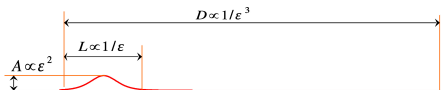
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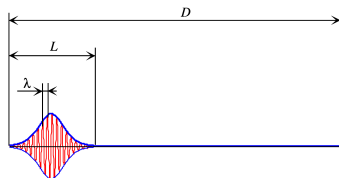
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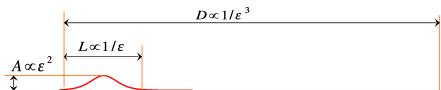
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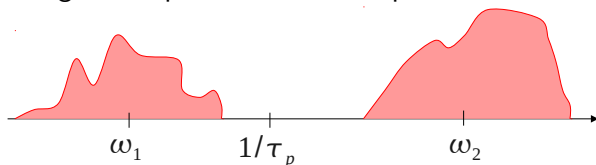
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A transparent medium

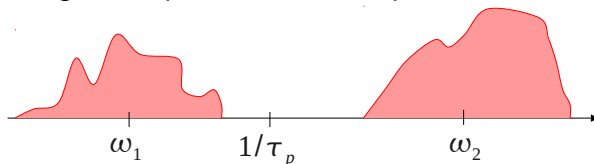
- The general spectrum of a transparent medium



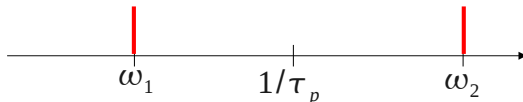
- A simple model: A two-component medium, each component is described by a two-level model
- We assume that the FCP duration τ_p is such that $\omega_1 \ll (1/\tau_p) \ll \omega_2$

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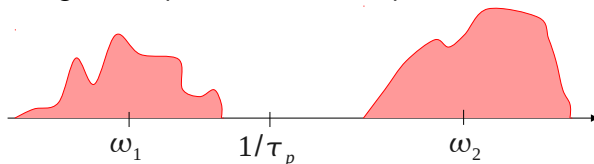
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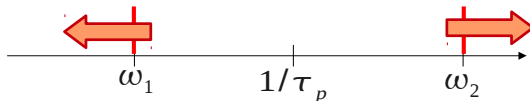
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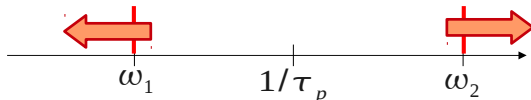
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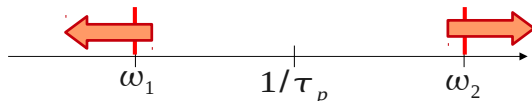
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 $\Rightarrow$ Long-wave approximation

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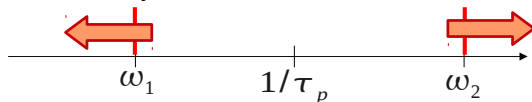
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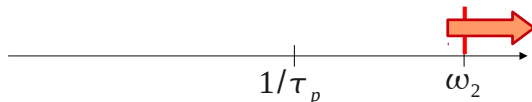
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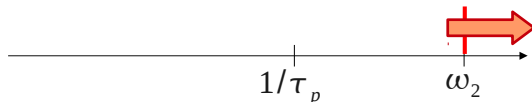
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⇒ Long-wave approximation

modified Korteweg-de Vries (mKdV) equation

$$\frac{\partial E}{\partial \zeta} = \frac{1}{6} \frac{d^3 k}{d\omega^3} \bigg|_{\omega=0} \frac{\partial^3 E}{\partial \tau^3} - \frac{6\pi}{nc} \chi^{(3)}(\omega; \omega, \omega, -\omega) \bigg|_{\omega=0} \frac{\partial E^3}{\partial \tau}$$

H. Leblond and F. Sanchez, *Phys. Rev. A* **67**, 013804 (2003)

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⇒ Short-wave approximation
- sine-Gordon (sG) equation: $\frac{\partial^2 \psi}{\partial z \partial t} = c_1 \sin \psi$

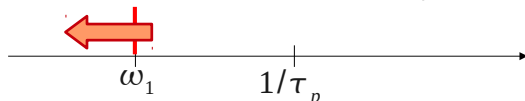
with $c_1 = \frac{w_\infty}{w_r}$: normalized initial population difference

and $\frac{\partial \psi}{\partial t} = \frac{E}{E_r}$: normalized electric field

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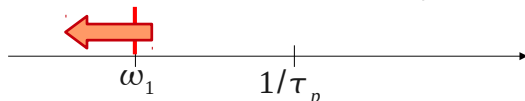
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- Then the two approximations are brought together to yield a general model:

The mKdV-sG equation

$$\frac{\partial^2 \psi}{\partial z \partial t} + c_1 \sin \psi + c_2 \frac{\partial}{\partial t} \left(\frac{\partial \psi}{\partial t} \right)^3 + c_3 \frac{\partial^4 \psi}{\partial t^4} = 0$$

- $\frac{\partial u}{\partial z} + c_1 \sin \int^t u + c_2 \frac{\partial u^3}{\partial t} + c_3 \frac{\partial^3 u}{\partial t^3} = 0$

with $u = \frac{\partial \psi}{\partial t} = \frac{E}{E_r}$: normalized electric field

- Integrable by inverse scattering transform in some cases:

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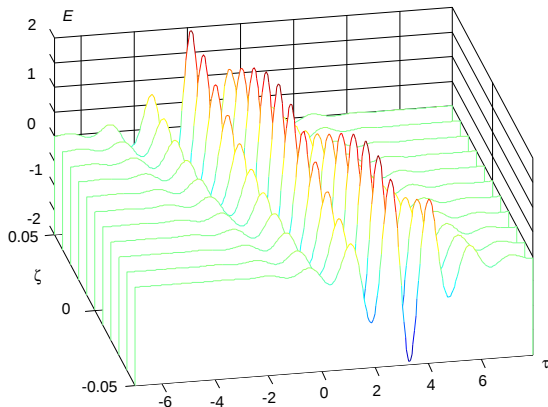
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- $$\frac{\partial u}{\partial z} + c_1 \sin \int^t u + c_2 \frac{\partial u^3}{\partial t} + 2c_2 \frac{\partial^3 u}{\partial t^3} = 0$$

with $u = \frac{\partial \psi}{\partial t} = \frac{E}{E_r}$: normalized electric field

- Integrable by inverse scattering transform in some cases:
mKdV, sG, and $c_3 = 2c_2$.

The analytical breather solution



- A few cycle soliton:

- Not spread out by dispersion
- Stable
- However, oscillates (breather)

H. Leblond, S.V. Sazonov, I.V. Mel'nikov, D. Mihalache, and F. Sanchez,
Phys. Rev. A **74**, 063815 (2006)

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More than two An arbitrary number of atomic levels

- UV transitions:
mKdV model generalizes without modification to a general Hamiltonian,
with an arbitrary number of atomic levels.

H. Triki, H. Leblond, D. Mihalache, *Opt. Comm.* **285**, 3179-3186 (2012)

- IR transitions: Not so simple
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- Hence population inversion w is explicitly involved.
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$$\begin{aligned}\frac{\partial E}{\partial z} &= \frac{-N}{\epsilon_0 c} \Omega Q \\ \hbar \frac{\partial w}{\partial t} &= -EQ \\ \hbar \frac{\partial Q}{\partial t} &= |\mu|^2 E w\end{aligned}$$

identical to Self Induced Transparency equations,
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More than two An arbitrary number of atomic levels

- IR transitions: Not so simple
- The sG model for 2 transitions (4 level), generalises to:

$$\begin{aligned}\frac{\partial E}{\partial z} &= \frac{-N\Omega}{\varepsilon_0 c} (\Omega_1 Q_1 + \Omega_2 Q_2) \\ \hbar \frac{\partial w_j}{\partial t} &= -EQ_j, \quad j = 1, 2 \\ \hbar \frac{\partial Q_j}{\partial t} &= |\mu_j|^2 E w_j, \quad j = 1, 2\end{aligned}$$

- Hence population inversion w is explicitly involved.
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- The above system reduces to

$$\frac{\partial u}{\partial z} + c_1 \sin \int^\tau u d\tau' + qc_1 \sin \nu \int^\tau u d\tau' = 0$$

→ a generalized double sine-Gordon equation

- Both admit breather solutions:

H. Triki, H. Leblond, D. Mihalache, *Phys. Rev. A* **86**, 063825 (2012)

- The above system reduces to

$$\frac{\partial u}{\partial z} + c_1 \sin \int^\tau u d\tau' + qc_1 \sin 2 \int^\tau u d\tau' = 0$$

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- Under certain conditions: the double sine Gordon equation ($\nu = 2$).
- Both admit breather solutions:

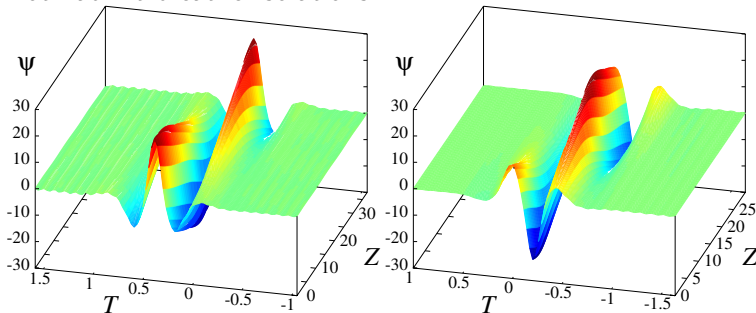
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Left: $\nu = 2$, $q = 0.2$; right: $\nu = \sqrt{3}$, $q = 0.4$.

H. Triki, H. Leblond, D. Mihalache, *Phys. Rev. A* **86**, 063825 (2012)

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Complex mKdV

- Consider UV transitions only, and vectorial electric field (U, V)
- Two coupled mKdV equations (normalized form):

$$\begin{aligned}U_Z &= U_{TTT} + [(U^2 + V^2) U]_T \\V_Z &= V_{TTT} + [(U^2 + V^2) V]_T\end{aligned}$$

- Setting $f = U + iV$, reduce to

$$f_Z = f_{TTT} + (|f|^2 f)_T, \quad (1)$$

→ The complex modified Korteweg-de Vries (cmKdV) equation.

- Avoid confusion with

$$f_Z = f_{TTT} + (|f|^2) f_T. \quad (2)$$

Eq. (2) is completely integrable while Eq. (1) is not.

- Eq. (1): cmKdV I; Eq. (2) cmKdV II.

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with f_0 solution of mKdV (real) and ϕ constant.

$$U = f_0(Z, T) \cos \phi, \quad V = f_0(Z, T) \sin \phi$$

- More interesting would be a circularly polarized soliton:

$$f = u(T - wZ)e^{i(\omega T - kZ)}.$$

$$\text{If } u = ae^{i\phi}, \quad U = a(T - wZ) \cos(\omega T - kZ + \phi(T - wZ))$$

$$V = a(T - wZ) \sin(\omega T - kZ + \phi(T - wZ))$$

- However no *exact*, even numerical, steady state solution of this type, but very robust *approximate* ones.

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$$f_Z = f_{TTT} + \left(|f|^2 f \right)_T,$$

- Linearly polarized solitons: $f = f_0(Z, T)e^{i\phi}$,
with f_0 solution of mKdV (real) and ϕ constant.

$$U = f_0(Z, T) \cos \phi, \quad V = f_0(Z, T) \sin \phi$$

- More interesting would be a circularly polarized soliton:

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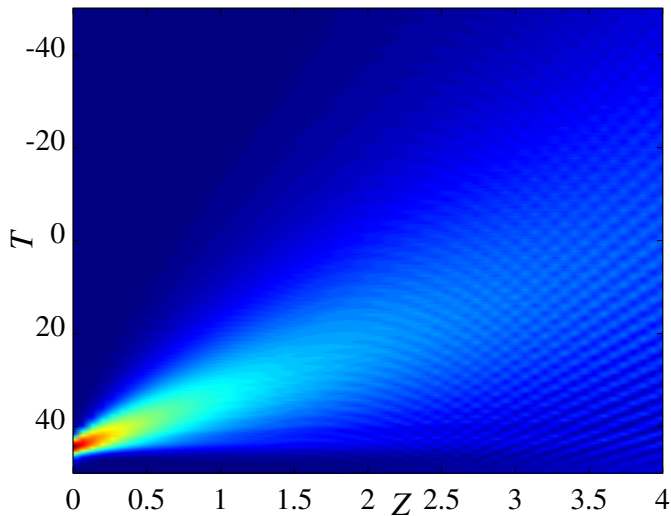
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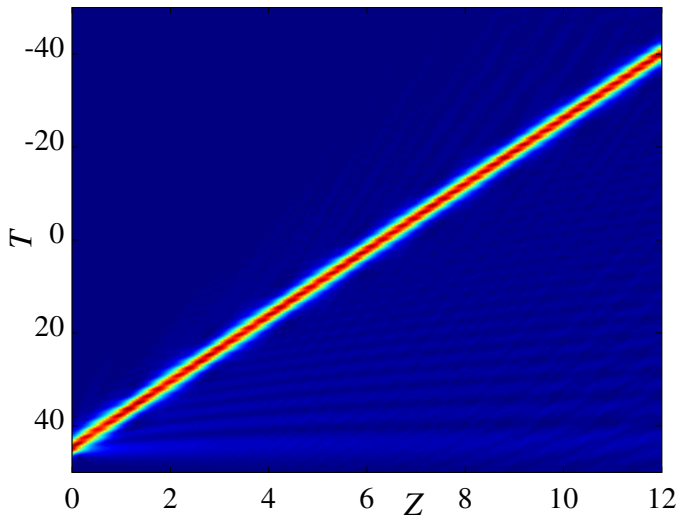
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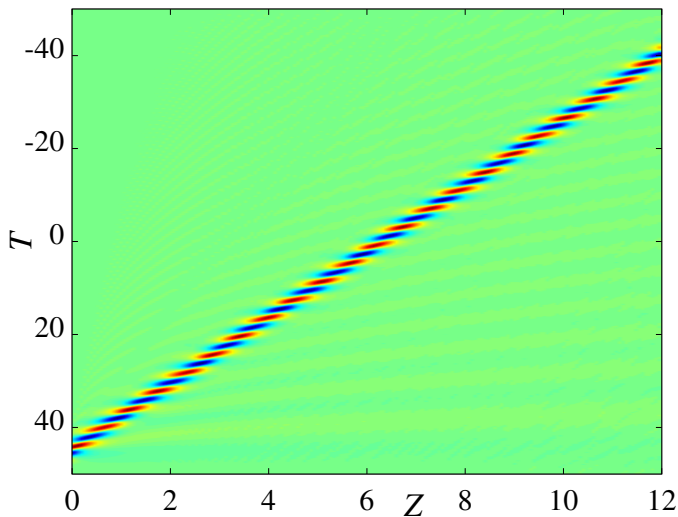
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- Linear propagation

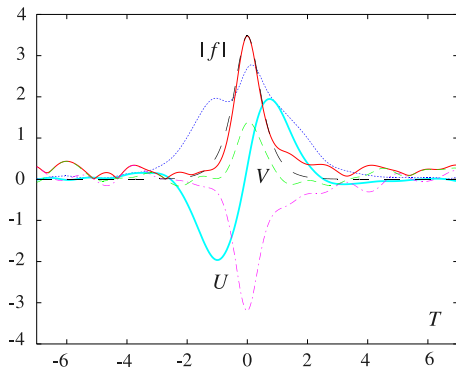


- The circularly polarized FCP: amplitude $|u|$



- The circularly polarized FCP: $E_x = \text{Re}(U)$

- The circularly polarized FCP is unstable in subcycle range
(for $\omega\tau_p < 1.4$)



- Initial ($Z = 100$): breather with $\omega = 1$, $\tau_p = 1$, $|f|$; $U = \text{Re}(f)$.
- Final ($Z = 10000$): $|f|$; U ; $V = \text{Im}(f)$.
- Black: fit of final $|f|$ by a fundamental soliton of real mKdV.

H. Leblond, H. Triki, F. Sanchez, D. Mihalache, *Phys. Rev. A* **83**, 063802 (2011)

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- One IR transition, for a vectorial field (U, V) .
- Excited state for x-polarization differs from excited state for y-polarization
 → two population differences w_1 and w_2 .
- No reduction to sG, but a complicated system

$$\begin{aligned}
 U_Z &= -P_T \\
 P_T &= -M - iUK \\
 M_T &= wU - sU^* \\
 K_T &= \text{Im}(U^*P) \\
 s_T &= UM \\
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 \end{aligned}$$

$U = u + iv$: electric field; $P = p + iq$: polarization density

$w = (w_1 + w_2)/2$, $s = (w_2 - w_1)/2 - iS$: population differences

K real, M complex

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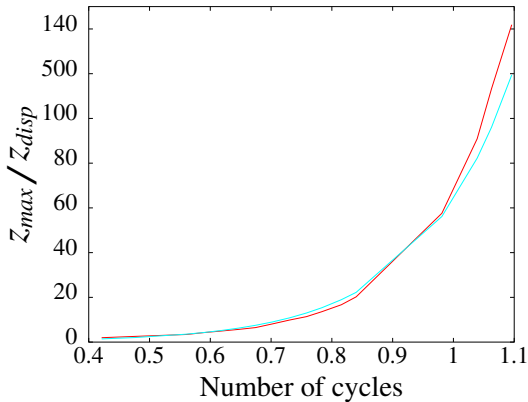
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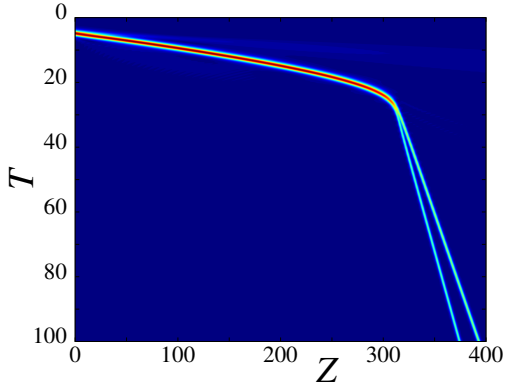
- If we set $U = fe^{i\phi}$, with ϕ constant,
i.e. we look for a linearly polarized pulse:
The system reduces to sine-Gordon.
- We look for a circularly polarized FCP.
- No exact solution
- An approximate solution valid in the SVEA:

$$U = be^{i\left[\left(k - \frac{b^2 k}{\omega^2}\right)Z - \omega T\right]} \operatorname{sech}\left[b\left(T + \frac{k}{\omega}Z\right)\right].$$

- Stable in the SVEA limit
- But unstable in the sub-cycle range



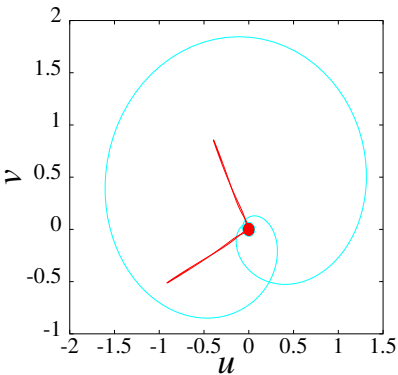
- The lifetime of the circularly polarized FCP,
(Red: numerical data. Light blue: exponential fit)



- The circularly polarized FCP decays into orthogonally polarized single-humped pulses.
($\omega = 5$, $b = 2$)

Polarization vector ($\text{Re}(U), \text{Im}(U)$) Time profiles of $\text{Re}(U)$ and $\text{Im}(U)$

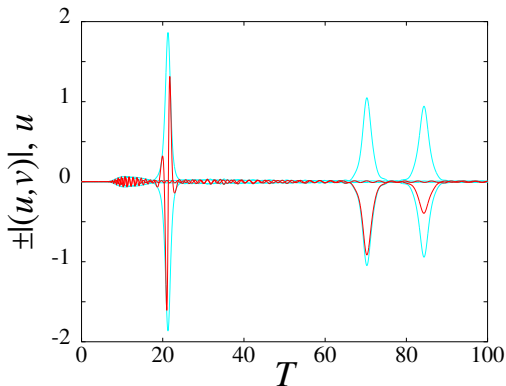
H. Leblond, H. Triki, D. Mihalache *Phys. Rev. A* **84**, 023833 (2011)



Polarization vector
($\text{Re}(U)$, $\text{Im}(U)$)

Initial, 1 circularly pol. pulse,
Final, 2 linearly pol. pulses.

H. Leblond, H. Triki, D. Mihalache *Phys. Rev. A* **84**, 023833 (2011)



Time profiles
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- An intense laser pulse launched in a fiber (photonic crystal fiber especially) turns into white coherent light
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- The mKdV equation:

$$\frac{\partial u}{\partial z} + c_2 \frac{\partial u^3}{\partial t} + c_3 \frac{\partial^3 u}{\partial t^3} = 0$$

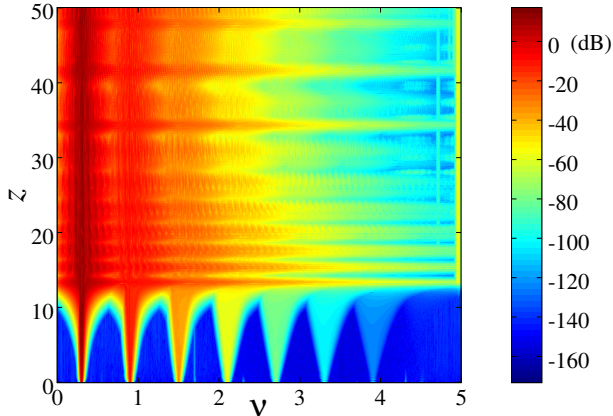
- Input is a Gaussian pulse:

$$u(0,t) = A \sin(\omega t) e^{-t^2/\tau^2}.$$

- Normalized (dimensionless), $c_2 = c_3 = 1$

- Optical period $\frac{2\pi}{\omega}$ and pulse duration τ :
same as a 100 fs long pulse, $\lambda = 1\mu\text{m}$.

- Numerical resolution.....



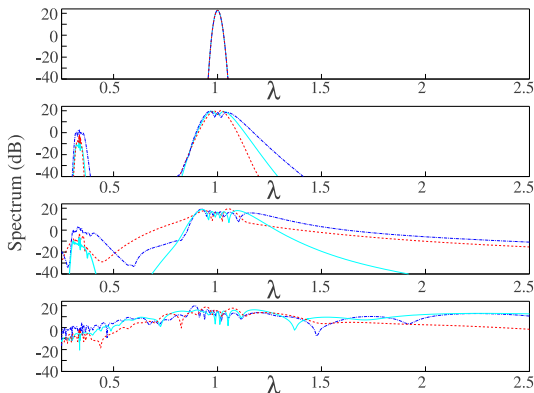
mKdV, input with $FWHM = 100$, $\nu = 0.3$, $A = 0.7$.

- A very broad spectrum is reached quickly.

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- Question: generation of frequencies lower than ω ?
- We solve numerically the mKdV, sG and mKdV-sG models starting with a 100 fs pulse, $\lambda = 1\mu\text{m}$
→ Compare the evolution of the spectrum

- Comparison between **mKdV**, **sG**, and **mKdV-sG** models



For $z = 0, 6, 12,$ and 16 .

- sG** extends first towards **Stokes** side,
- mKdV** extends first towards **anti-Stokes**.
- mKdV-sG** model lies in between.

- Recall that mKdV accounts for UV transitions, and sG for IR transitions.
- Usual Raman broadening yields extension of the spectrum towards low frequencies at the beginning of the process and is due to IR transitions.
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- A quasi monochromatic wave

$$u = U(z,t)e^{i(kz-\omega t)} + cc + u_1(z,t), \quad (3)$$

U : fundamental wave amplitude; u_1 : small correction.

- Neglect dispersion, and disregard third harmonic generation:

$$U = \frac{A}{2} \exp \left[i \left(\frac{c_1}{2\omega^3} + 3\omega c_2 \right) \frac{A^2}{4} z \right]. \quad (4)$$

- Self phase modulation.
 → broadening of the spectrum
 with typical oscillations of the spectral density.

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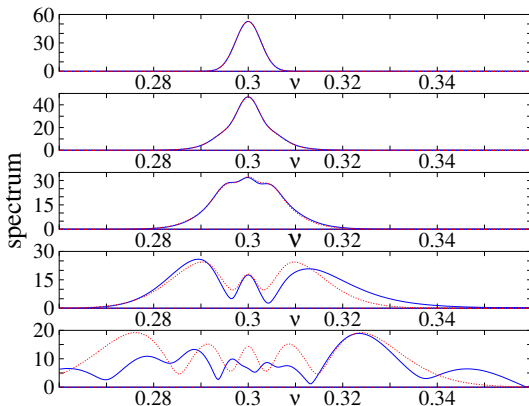
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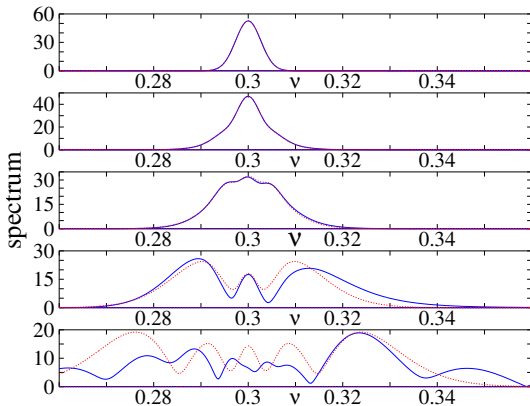
- Numerical **mKdV** vs analytical **self-phase modulation**, Eq. (4)



$FWHM = 100$, $\nu = 0.3$, $A = 0.7$, for $z = 0, 2, 4, 10, 20$.

- Then **actual broadening** becomes asymmetric, while **analytic formula** remains symmetric.
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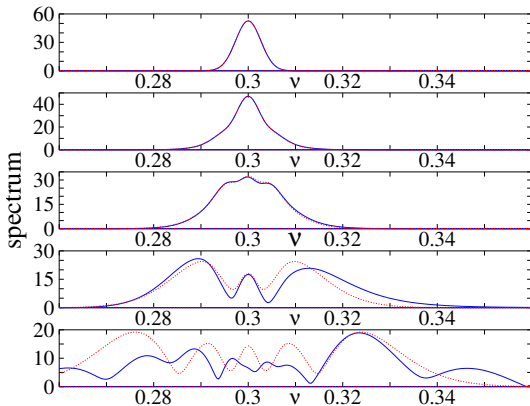
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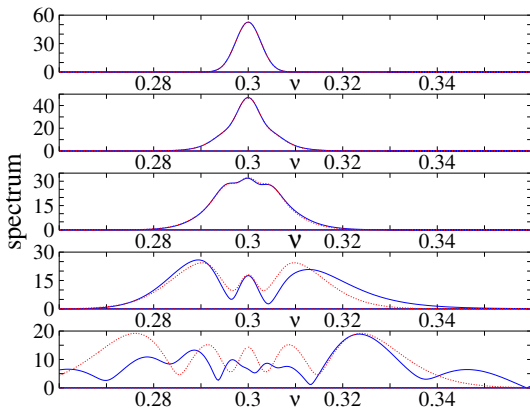
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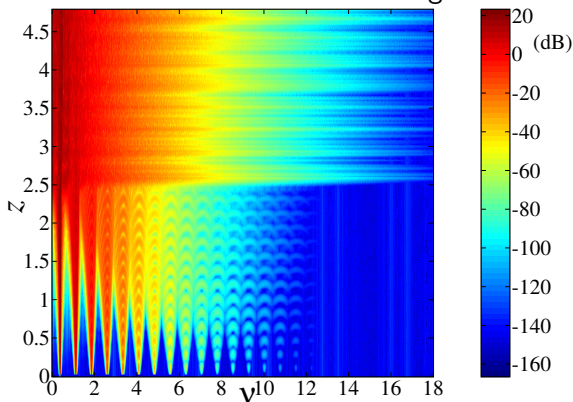
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- A lot of high harmonics are created and involved in the process
- The SVEA does not account for high harmonic generation



- Up to 15 harmonics can be seen
 $FWHM = 80$, $\nu = 0.375$, and $A = 5$, sG model.

Spectral width of harmonics

- Assume a Gaussian profile $u = Ae^{\frac{-t^2}{\tau^2}} e^{-i\omega_0 t}$.

- Its Fourier transform is $\hat{u} = \frac{2\sqrt{\pi}}{A\tau} e^{\frac{-(\omega-\omega_0)^2 \tau^2}{4}}$.

- The n th harmonic is then $u^n = Ae^{\frac{-nt^2}{\tau^2}} e^{-in\omega_0 t}$,
and its width is $\frac{\tau}{\sqrt{n}}$.

- Consequently its Fourier transform is

$$\widehat{(u^n)} = \frac{2\sqrt{n\pi}}{A\tau} e^{\frac{-(\omega-n\omega_0)^2 \tau^2}{4n}}.$$

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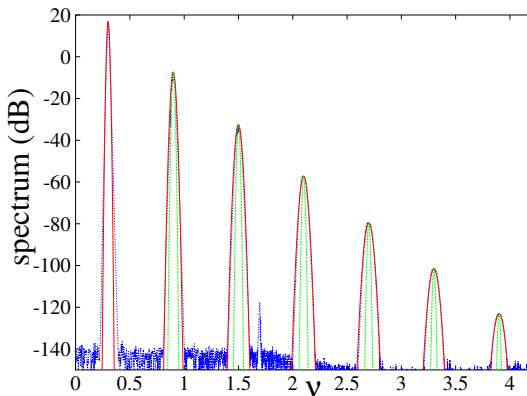
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- Spectral widths of the harmonics.
mKdV compared to Gaussian with width increasing as $\sqrt{n}$



$FWHM = 100$, $\nu = 0.3$, and $A = 0.7$, for $z = 2$.

- Spectral broadening due to parametric interaction between the sidebands of the harmonics and the ones of the fundamental.
- ω_0 : fundamental frequency. Assume a sideband $\omega_0 + \delta\omega$,
- the third harmonics contains the sideband $3(\omega_0 + \delta\omega)$.
- It may interact with the fundamental $\pm\omega_0$:
 $\rightarrow 3(\omega_0 + \delta\omega) - \omega_0 - \omega_0 = \omega_0 + 3\delta\omega$
- Hence $\omega_0 + \delta\omega$ creates $\omega_0 + 3\delta\omega$:
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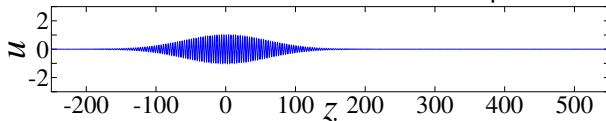
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- Input is a Gaussian pulse:

$$u(0,t) = A \sin(\omega t) e^{-t^2/\tau^2}.$$

- A few FCP solitons form and tend to separate



- The sG equation,

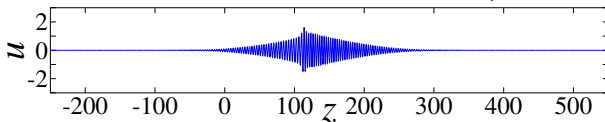
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$$u(0,t) = A \sin(\omega t) e^{-t^2/\tau^2}.$$

- A few FCP solitons form and tend to separate



- The sG equation,

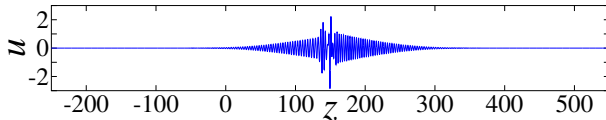
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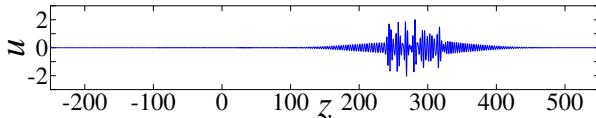
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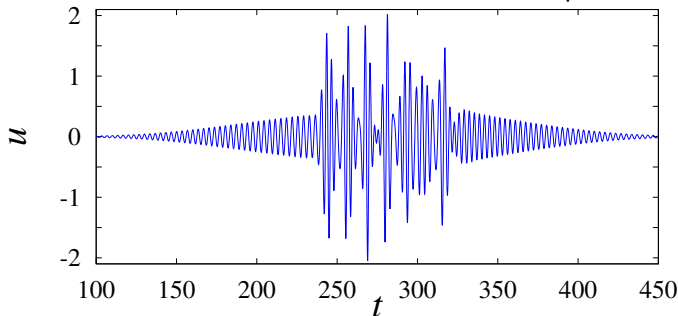
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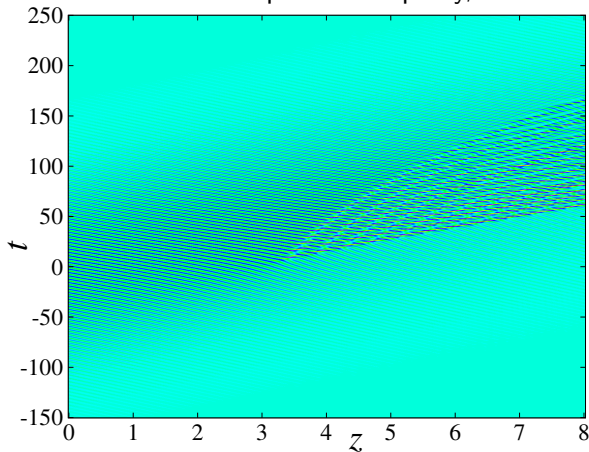
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- FCP solitons don't separate completely, but interact
- $A = 1$ a few FCP solitons form and tend to separate

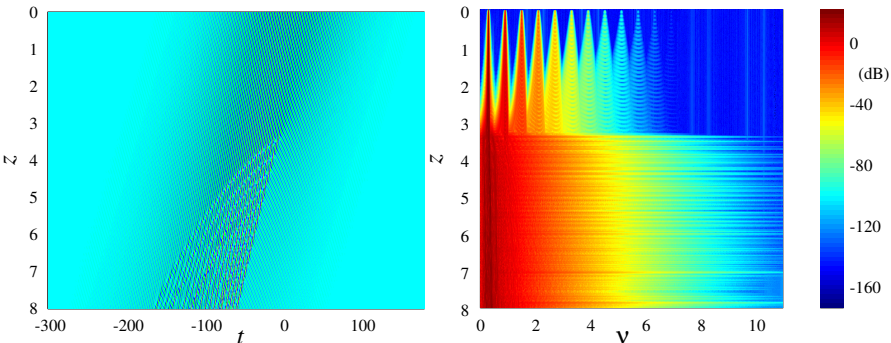


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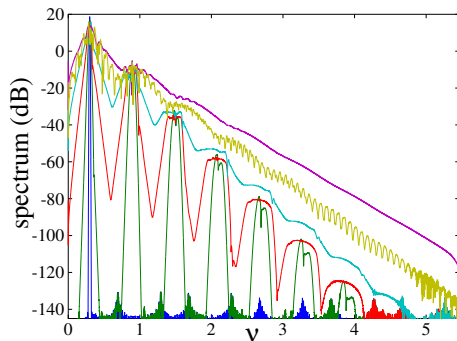
Initial pulse with $FWHM = 100$, $\nu = 0.3$, and $A = 2.5$

- FCP solitons form, don't separate, but interact $\rightarrow$ ultrabroad supercontinuum



Initial pulse with $FWHM = 100$, $\nu = 0.3$, and $A = 2.5$, according to the sG model.

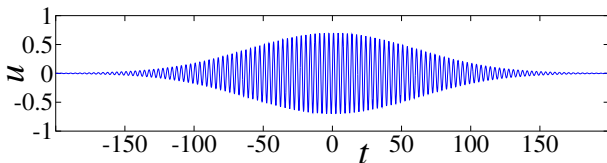
Evolution of the spectrum according to sG



- Input as above, $z = 0$
- Harmonic generation, $z = 4$
- Broadening of harmonics, $z = 8$
- Arising of the first soliton, $z = 9.2$
- The second soliton is just formed, $z = 11.16$
- Final ultrabroad spectrum, $z = 20$

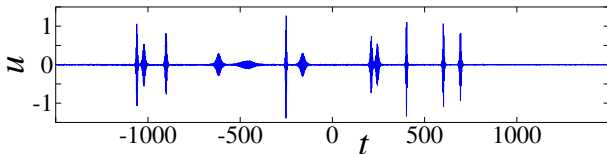
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- At high amplitude level ($A \sim 2$) an initial 100 fs pulse splits into a set of FCP solitons, which are mKdV breathers



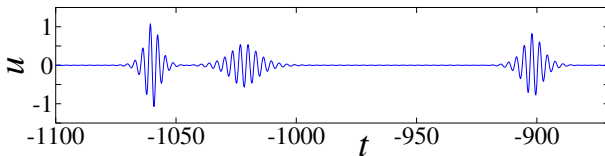
- The FCPs form in a first stage until $z \simeq 12$ then slowly go away one from the other.
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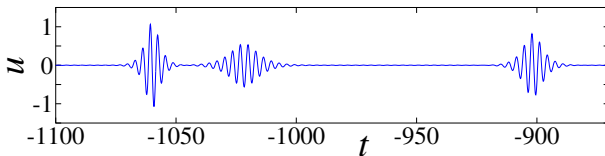
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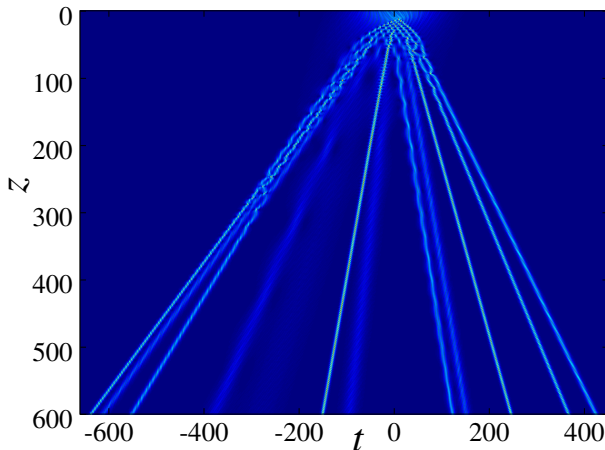


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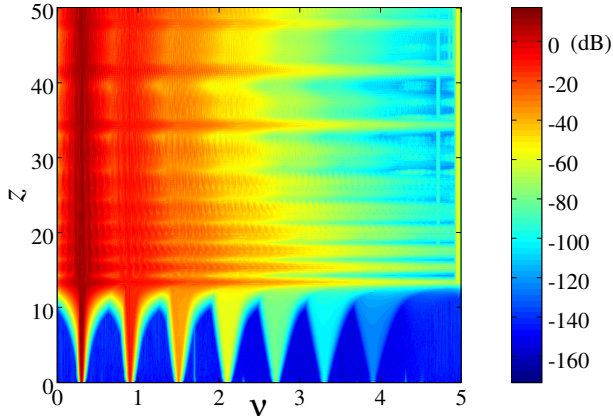


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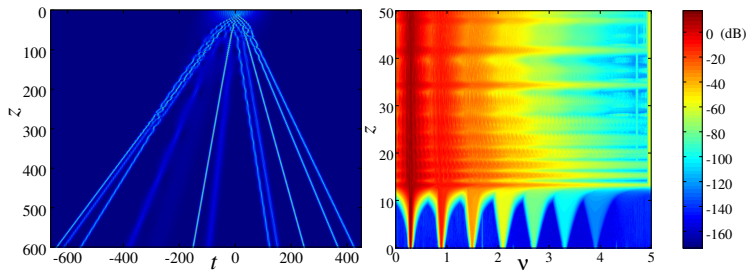
Initial pulse with $FWHM = 100$, $\nu = 0.3$, and $A = 0.7$, according to mKdV.

- The input pulse splits into a set of FCP solitons



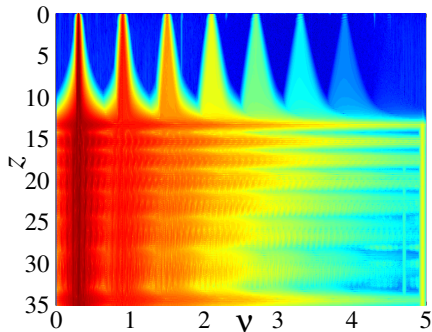
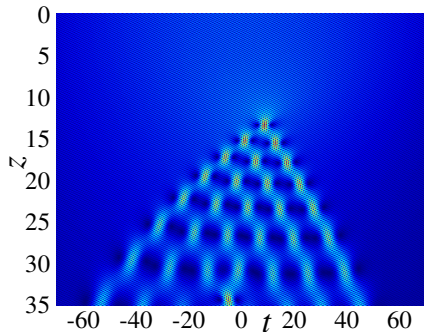
mKdV, input with $FWHM = 100$, $\nu = 0.3$, $A = 0.7$.

- A very broad spectrum is reached quickly.



mKdV, input with $FWHM = 100$, $\nu = 0.3$, $A = 0.7$.

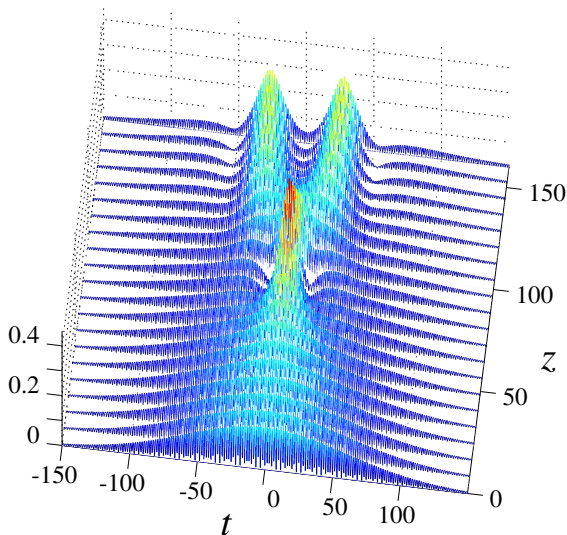
- Compare temporal profile and spectrum
- The spectrum extends when the solitons separate



mKdV, input with $FWHM = 100$, $\nu = 0.3$, $A = 0.7$.

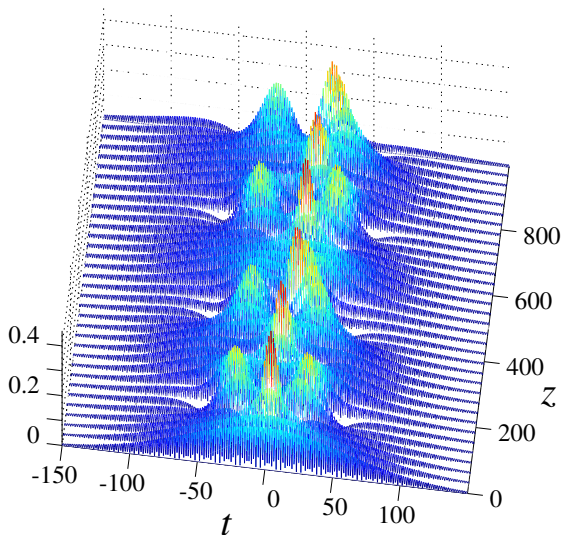
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- At a lower amplitude ($A \sim 0.1, 0.2$), it may happen, that two solitons only form



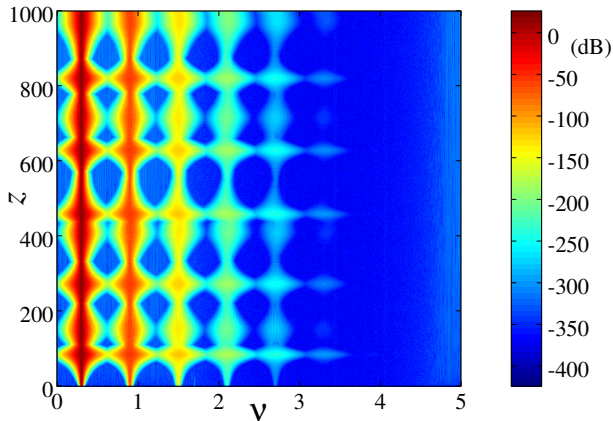
Initial pulse with $FWHM = 100$, $\nu = 0.3$, and $A = 0.17$.

- At a lower amplitude ($A \sim 0.1, 0.2$), they interact with each other and form a kind of superbreather



Initial pulse with $FWHM = 100$, $\nu = 0.3$, and $A = 0.17$.

- Then the spectral width oscillates



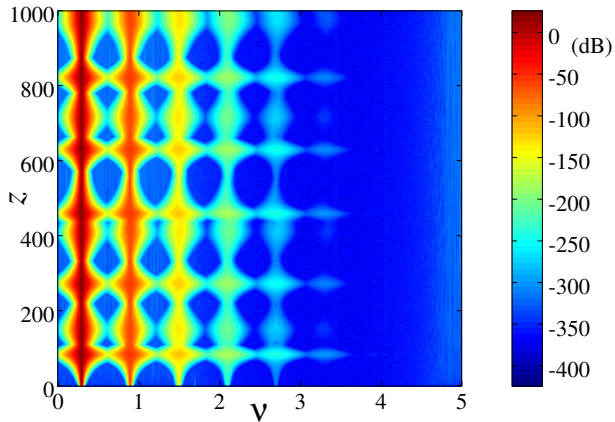
Initial pulse with $FWHM = 100$, $\nu = 0.3$, and $A = 0.17$.

Conclusion

- Generic non-slowly varying envelope approximation models: modified Korteweg-de Vries, sine-Gordon, mKdV-sG
- Adequately describe the supercontinuum generation over several octaves
- sG accounts for the Raman broadening
- Modulation instability
- High harmonics generation
- FCP soliton generation and interaction leads to ultrabroad spectrum.

H. Leblond, Ph. Grelu, D. Mihalache, *Phys. Rev. A* **90**, 053816 (2014)

Thank you for your attention.



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