

Motion of solitons in CGL-type equations

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Abstract:

The complex Ginzburg-Landau (CGL) equation is often used as a model for laser cavities, to describe the structuring of the optical field in both the transverse (spatial) and longitudinal (temporal) directions. In the temporal case, it writes as

$$u_z = \delta u + \left(\beta + i \frac{D}{2} \right) u_{tt} + (\varepsilon + i) u |u|^2 + (\mu + i\nu) u |u|^4, \quad (1)$$

and it is known that it admits traveling wave solutions of the form $u = u_0(t - Vz) \exp i[tV/D - zV^2/(2D)]$, with u_0 any solution independent of z , as soon as $\beta = 0$. We show that, for small values of β , the term which breaks the translation invariance acts as a viscous friction.

Experiments in mode-locked fiber lasers, however, have shown the existence of “soliton gas” states, containing a large number of solitons in permanent relative motion. Since β , which here accounts for the finiteness of the spectral bandwidth of the gain, is not zero, such a motion seems to be impossible in the frame of the CGL equation.

We consider numerically the effects of a sinusoidal source term added to Eq. (1), which describes continuous laser injection, and an integral term accounting for the gain dynamics, and show that both are able to induce relative soliton motion, and allow to retrieve the collective states of solitons found in experiments.

In the spatial configuration, β becomes zero, and solitons are free to move in the transverse plane. However, periodic potentials used to stabilize some patterns such as vortices can prevent soliton motion. We shown that solitons with an initial velocity can move, freely if their initial velocity is high enough, or jumping from site to site in the periodic potential if it is moderate. In the latter case, the solitons leaves copies of itself behind it.

References:

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