Motion of solitons in CGL-type equations

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Galilean invariance of CGL

- The CGL equation is

$$\frac{\partial E}{\partial z} = \delta E + \left(\beta + i\frac{D}{2}\right)\frac{\partial^2 E}{\partial t^2} + (\varepsilon + i)\, E\, |E|^2 + (\mu + i\nu)\, E\, |E|^4$$

- If $\beta = 0$, and $E_0(z,t)$ solution to CGL

$$E = E_0\,(z,t - wz)\exp i\left[Dwt - \left(Dw^2/2\right)z\right]$$

is a solution moving at inverse speed w.

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E : electric field amplitude;
δ: net linear gain;
β: spectral gain bandwidth;
$D = \pm 1$: dispersion;
ε: cubic nonlinear gain;
μ: quintic nonlinear gain;
ν: 4th-order nonlinear index;
z: number of round-trips;

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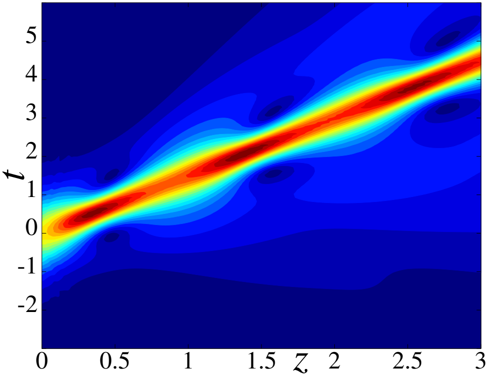
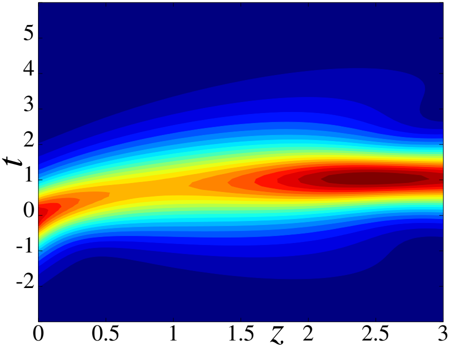


- $\beta\partial^2 E/\partial t^2$ breaks Galilean invariance and prevents any motion of the solitons.
- If $\beta \neq 0$: the moving soliton does not exist.



At $z = 0$, $E = E_0 \exp i\Delta\omega t$, $\beta = 0.55$ instead of 0

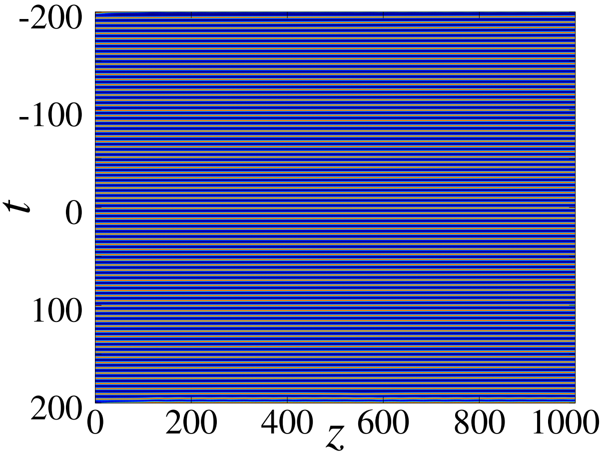
- Strong breaking due to the limited gain bandwidth.

- Experiments in mode-locked fiber lasers:
  $\Longrightarrow$ existence of "soliton gas"
- A large number of solitons in motion.
- Spectral bandwidth of gain is finite: $\beta \neq 0$
- Is the motion it due to the cw component?
- Try to inject cw.

$$\begin{aligned}\frac{\partial E}{\partial z} &= \delta E + \left(\beta + i\frac{D}{2}\right)\frac{\partial^2 E}{\partial t^2} + (\varepsilon + i)\, E\, |E|^2 \\ &\quad + (\mu + i\nu)\, E\, |E|^4 + A\exp\left(-i\Delta\omega_0 t\right)\end{aligned}$$

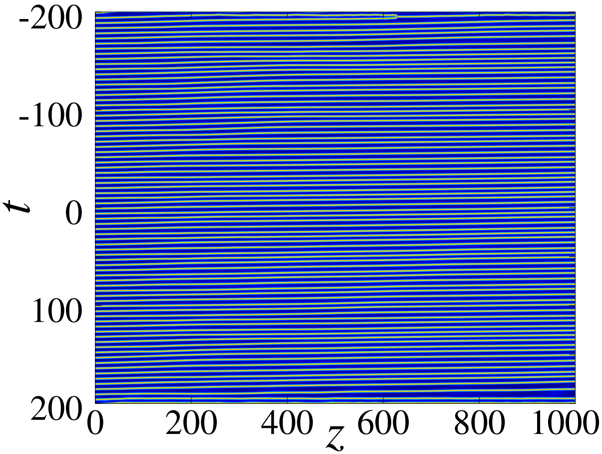
A: amplitude of injected cw; $\Delta\omega_0$: frequency shift.

Changing “states of matter”: soliton cristal



- Soliton crystal.
  $\Delta\nu_0 = 1.2$.

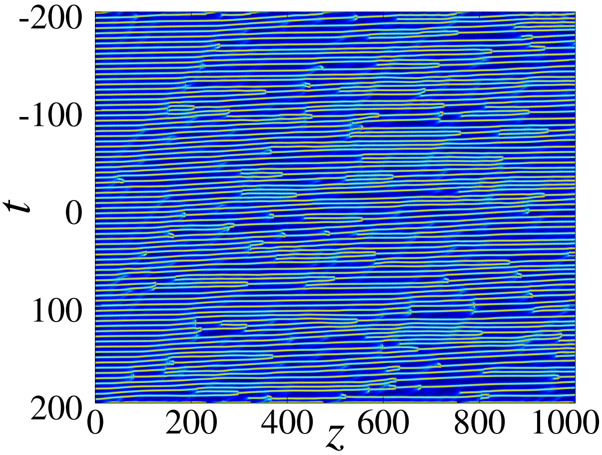
Changing “states of matter”: soliton liquid



- Soliton liquid.

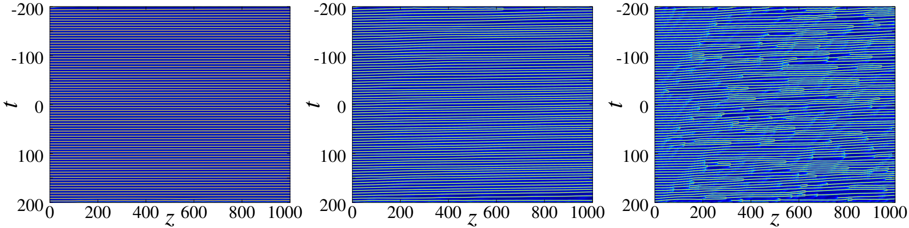
  $\Delta\nu_0 = 0.9$.

Changing “states of matter”: soliton gas



- Soliton gas.
  $\Delta\nu_0 = 0.8$.

Changing "states of matter" of solitons

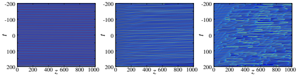


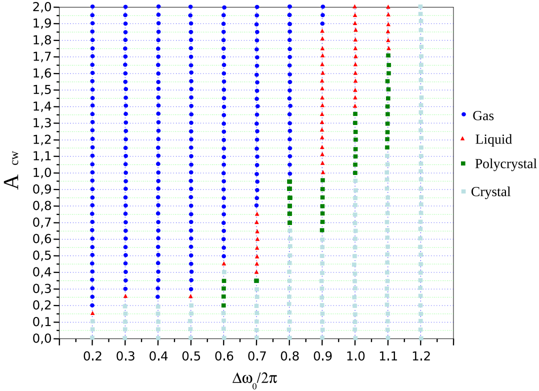
Soliton crystal, $\Delta\nu_0 = 1.2$,

Soliton liquid, $\Delta\nu_0 = 0.9$,

Soliton gas. $\Delta\nu_0 = 0.8$.

Changing "states of matter" of solitons





- The different regimes vs detuning $\Delta\omega_0$ and amplitude $A = A_{cw}$ of injected cw

- For a single soliton, input of the form $E = E_0 \exp i\Delta\omega_1 t$, varying A, $\Delta\omega_0$ and $\Delta\omega_1 \Longrightarrow$ Pulse motion.

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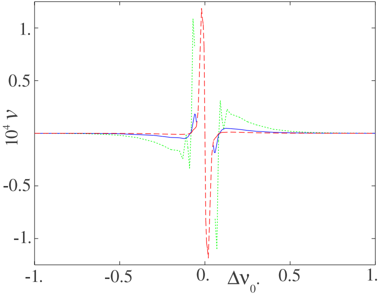
- The velocity depends on A and $\Delta\omega_0$ (injected cw), but not on $\Delta\omega_1$ (initial speed).

  i.e. Speed is entirely determined by injected cw.



Soliton velocity vs $\Delta\nu_0 = \Delta\omega_0/2\pi$ for $A = 0.004$ (green dotted), 0.002 (red dashed), 0.001 (blue solid line).

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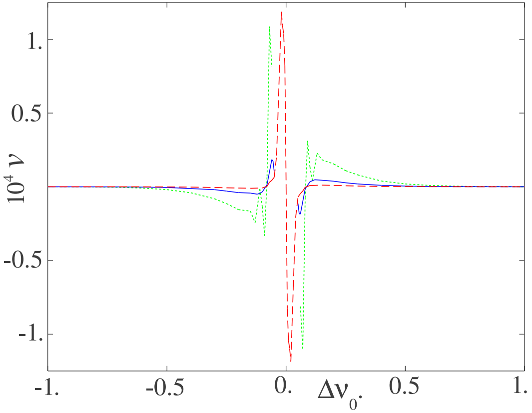

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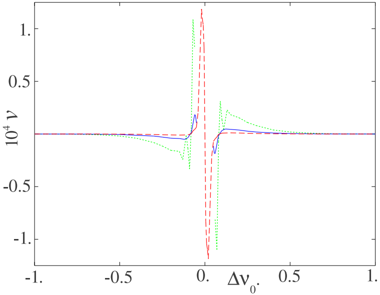

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- Motion is restored but Galilean invariance is not.

"Brownian motion" induced by injected cw

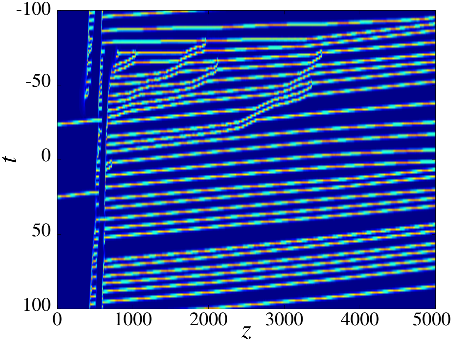
- Soliton velocity is fixed by the cw.
- At higher amplitudes of injected cw:
  More complex nonlinear interaction between cw and solitons.
  $\Longrightarrow$ The amplitude of cw (radiation) varies with t.
- A tiny variation of the cw component in either amplitude or frequency changes radically the soliton velocity
  $\Longrightarrow$ apparently random variations of the the soliton speed.
- $\Longrightarrow$ Erratic motion of solitons,
  and soliton gas.

- An integral term accounting for the fast gain dynamics

$$\frac{\partial E}{\partial z} = \delta E + \left(\beta + i\frac{D}{2}\right)\frac{\partial^2 E}{\partial t^2} + (\varepsilon + i)\, E|E|^2 + (\mu + i\nu)\, E|E|^4 - \Gamma E \int_{-\infty}^{t} \left(|E|^2 - < |E|^2 >\right) dt'$$

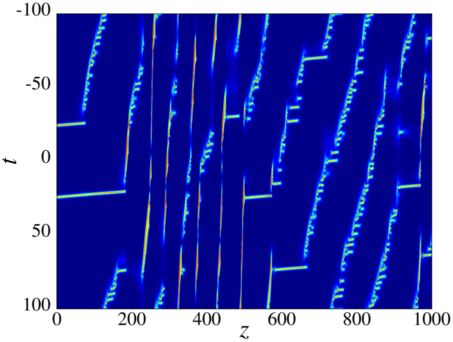
- Represents the decrease of the population inversion
  (and hence of gain)
  when stimulated emission occurs.

- A single pulse (or two-pulse) input is unstable:
  New pulses form in front of the input (towards $t < 0$), and quickly disappear.
- Unstable pulse emission repeats all along the cavity,
  $\Longrightarrow$ multi-pulse pattern
- Solitons move slowly



$\Gamma = 0.01$

- For larger Γ, the instability increases: pulses form and vanish faster.
- Then, the pulse train does not stabilize any more: pulses are created and vanish permanently



$\Gamma = 0.03$

- Generation and vanishing process $\Longrightarrow$ effective soliton motion

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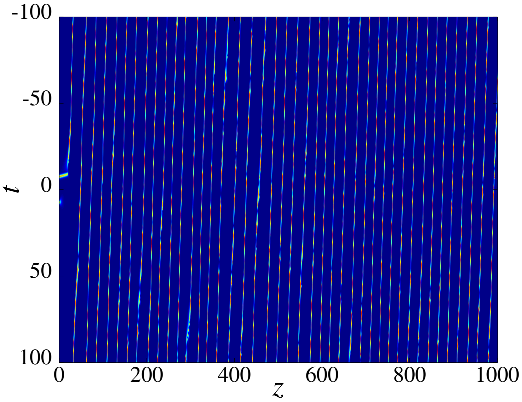
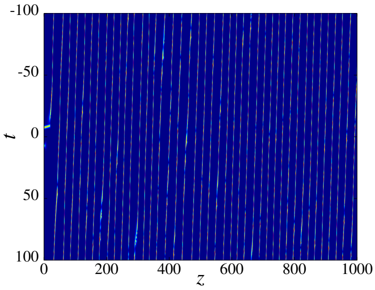
- The inverse velocity w of this motion is very large



$\Gamma = 0.1.$

- Generation and vanishing process $\Longrightarrow$ effective soliton motion
- The inverse velocity w of this motion is very large



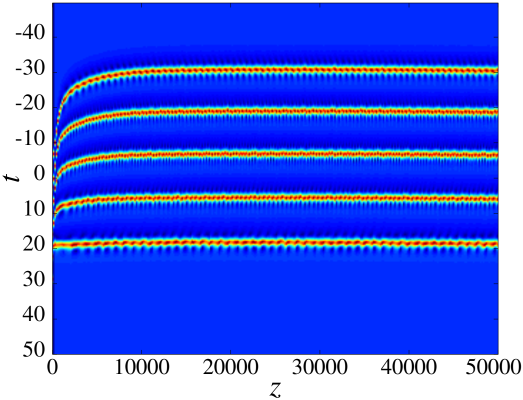
$\Gamma = 0.1.$

- For high values of Γ, the moving soliton is unstable and vanishes.
- Gain dynamics can induce soliton motion.

- Model including external injection, fast gain dynamics, and gain saturation.

$$
\begin{aligned}
\frac{\partial E}{\partial z} = & \left(\frac{g_0}{1+<|E|^2>/I_s}-r\right)E+\left(\beta+i\frac{D}{2}\right)\frac{\partial^2 E}{\partial t^2} \\
& +(\varepsilon+i)\,E|E|^2+(\mu+i\nu)\,E|E|^4 \\
& -\Gamma E\int_{-\infty}^{t}\left(|E|^2-<|E|^2>\right)dt'+A\exp\left(-i\Delta\omega_0 t\right)
\end{aligned}
$$

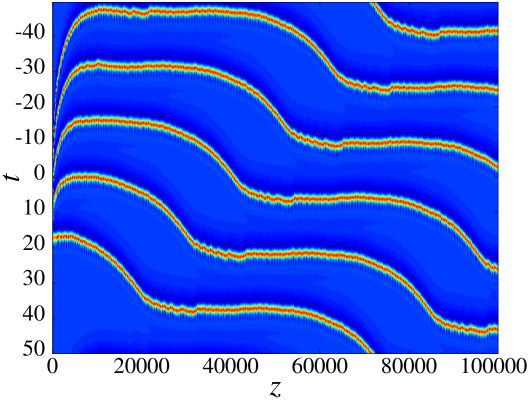
- Gain saturation limits the number of solitons
  $\Longrightarrow$ A liquid: condensed phase, which does not fill the box



$\Delta\nu_0 = 0.1$, $A = 0.115$, with velocity compensation, $w = -0.05877$.

- Not a crystal: no phase-locking

- We can still have a soliton gas:

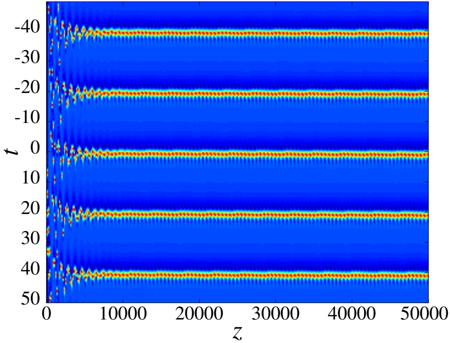


$\Delta\nu_0 = 0.1, \ A = 0.120, \ w = -0.07040.$

Harmonic mode-locking

- Equidistant solitons filling all the box, stable state.

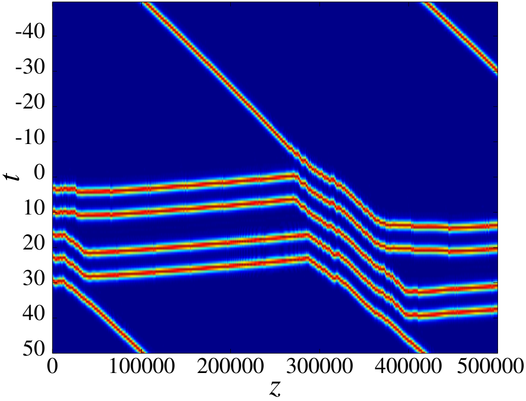
When $0.125 \leq A \leq 0.133$.



$\Delta\nu_0 = 0.1,\ A = 0.130,\ w = -0.02755.$

- Consecutive pulses are phase-locked: a crystal, but the crystal length exactly matches the box length.

- In a three bunch pattern,
  elastic interaction according to the Newton's cradle scenario



$A = 0.130$, $\Delta\nu_0 = 0.5$ and $w = -0.0024$.

- The CGL equation $(u = E)$

$$u_z = \delta u + \left(\beta + i\frac{D}{2}\right) u_{tt} + (\varepsilon + i)\, u\, |u|^2 + (\mu + i\nu)\, u\, |u|^4 .$$

- Moving solution: $u = u_0(t - T, z)e^{i(\omega t - kz)}$
with $u_0(t,z)$ solution to CGL with $\beta = 0$,
$T = Vz, \quad \omega = \frac{V}{D} \quad$ and $\quad k = \frac{V^2}{2D}$.
- Perturbative approach: Consider some small non zero β.
u a soliton solution,

$$M = \int_{-\infty}^{+\infty} |u|^2 dt : \text{its mass,}$$

$$T = \int_{-\infty}^{+\infty} t|u|^2 dt / M : \text{position of its center of mass.}$$

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- The velocity of the pulse is then

$$\frac{dT}{dz} = \frac{1}{M} \int t \left(u_z u^* + cc \right) dt$$

(cc: complex conjugate, $u_z = \partial u / \partial z$).

- Using the CGL equation:

$$\frac{dT}{dz} = \frac{1}{M} \left(I_1 + \frac{iD}{2} I_2 + \beta I_3 \right) dt,$$

where

$$I_1 = 2 \int t \left(\delta |u|^2 + \varepsilon |u|^4 + \mu |u|^6 \right) dt,$$

$$I_2 = \int t \left(u_{tt} u^* - cc \right) dt,$$

$$I_3 = \int t \left(u_{tt} u^* + cc \right) dt.$$

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- Assumption: u_0 is a symmetrical pulse, centered at $t = T$, consequently the function $u_0(t')$, with $t' = t - T$, is even.
- Then

$$I_1 = 2\int t\left(\delta|u|^2 + \varepsilon|u|^4 + \mu|u|^6\right)dt,$$

becomes

$$I_1 = 2T\int \left(\delta|u_0|^2 + \varepsilon|u_0|^4 + \mu|u_0|^6\right)dt'.$$

and so on.
- Then we can compute the acceleration d^2T/dz^2:

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- Using integration by parts and parity we compute a set of integrals
- Finally; we obtain the expression of the force $F = Md^2T/dz^2$:

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$$M\frac{dV}{dz} = F = -4\beta \int \left|u_{0t'}\right|^2 dt' \, V,$$

- Hence, the velocity evolves as $V(z) = V(0)e^{-\lambda z}$ with the decay rate

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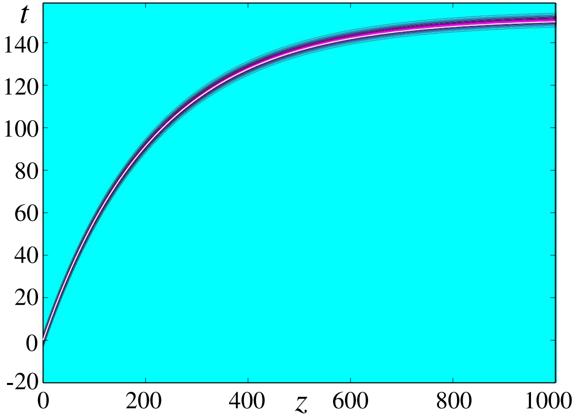
1. Motion induced by a continuous wave
   - Crystal, liquid and gas of solitons
   - Pulse motion due to gain dynamics
   - Injected continuous wave

2. The finite bandwidth of gain as a viscous friction
   - Analytical expression of the viscous friction
   - **Numerical validation of the approximation**

3. Transverse mobility in the presence of periodic potential
   - Fondamental soliton
   - Dipoles and vortices

- An example of calculation

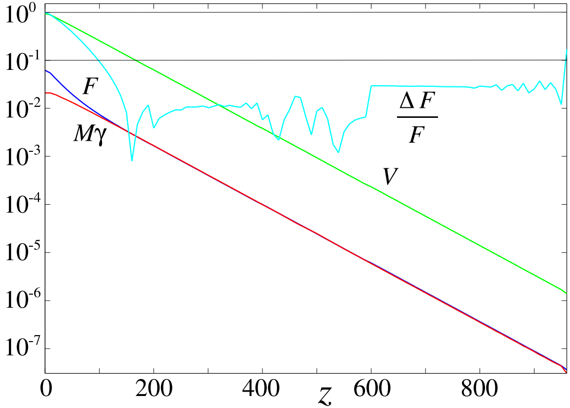


Initial velocity $V_0 = 0.7$ and gain bandwidth coefficient $\beta = 0.004$.

White line: approximate analytical solution

- Good agreement with the numerical solution.

- We plot the characteristics of the pulse motion vs z



Logarithmic scale.
V:velocity; γ: acceleration and M: mass; F: force from above theory;
$\Delta F/F$: relative difference between F and $M\gamma$.

Parameters: $\omega = 1$, $\beta = 0.0124$.

- Effect of finite bandwidth of gain on CGL soliton
  with anomalous dispersion
  is equivalent to a viscous friction force,
  if it is not too large.
- To construct simplified models
  to describe CGL soliton interactions
  as forces between effective particles :
- The lack of Galilean invariance of CGL
  was a major difficulty,
  since the concept of force is based on it.
- With our result, we can approach soliton interaction
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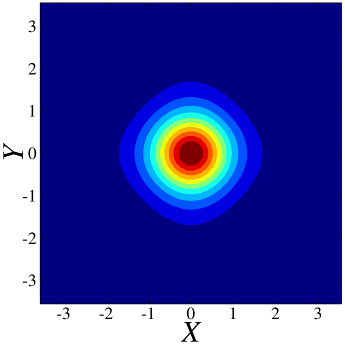
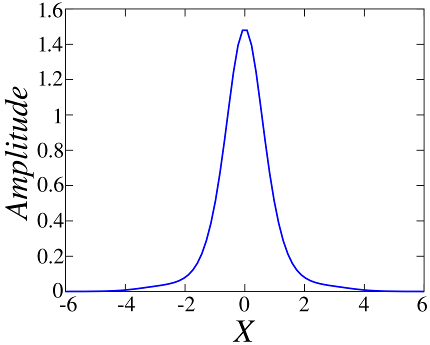
- (2+1)-D spatial Ginzburg-Landau equation:

$$\frac{\partial u}{\partial Z} = \left[-\delta + iV(X,Y) + \frac{i}{2}\nabla_\perp^2 + (i+\epsilon)|u|^2 - (i\nu+\mu)|u|^4 \right] u,$$

- $\nabla_\perp^2 = \partial^2/\partial X^2 + \partial^2/\partial Y^2$: the paraxial diffraction
- A periodic potential: $V(X,Y) = -V_0\left[\cos(2X) + \cos(2Y)\right]$ breaks Galilean invariance
- $\delta = 0.4$,$\varepsilon = 1.85$,$\mu = 1$,$\nu = 0.1$, $V_0 = 1$,
  for which the quiescent fundamental soliton is stable.





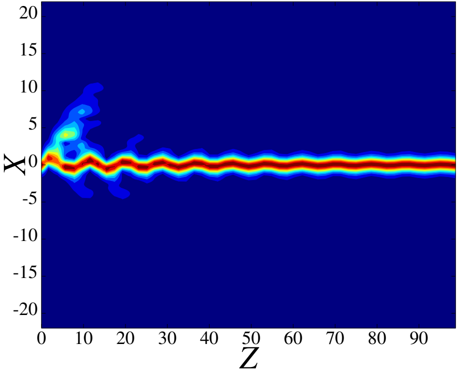
$|u(X,Y)|$; $|u(X)|$ at $Y = 0$.

The stable fundamental soliton

- Input $u = u_0 \exp\left(i\mathbf{k}_0\mathbf{R}\right)$,
  with $\mathbf{R} = (X, Y)$, and $\mathbf{k}_0 = (k_0 \cos\theta, k_0 \sin\theta)$ $(0 \leq \theta \leq \pi/4)$
- In an amplifier, the factor $\left(i\mathbf{k}_0\mathbf{R}\right)$
  represents a deviation of the wave vector $\mathbf{k}$ from the Z-axis.
- Indeed, CGL is derived within the SVEA:
  Either $E = \mathcal{U}(X, Y, Z - vT)e^{i(k_x X + k_Y Y + k_Z Z - \omega T)} + \mathrm{c.c.}$,
  or $E = u(X, Y, Z - vT)e^{i(k_Z Z - \omega T)} + \mathrm{c.c.}$,
  with $u(X, Y, Z - vT) = \mathcal{U}(X, Y, Z - vT)e^{i(k_X X + k_Y Y)}$.
  Equivalent if k_X, k_Y are small enough.

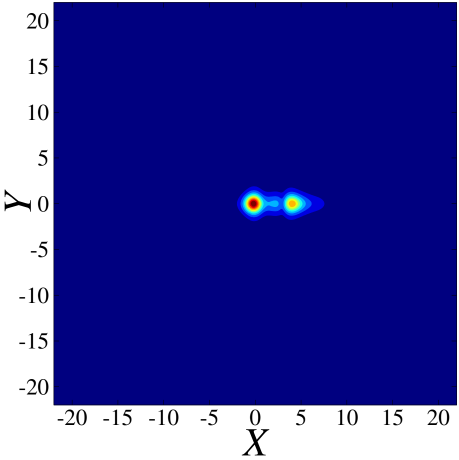
Fundamental soliton with initial kick

- If k_0 is small, the pulse oscillates in the potential site



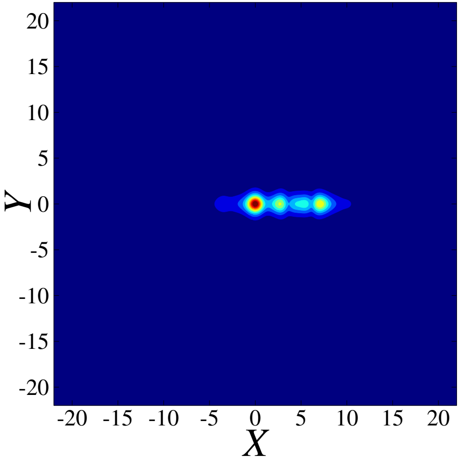
$|u(X,Z)|$ in the cross section $Y=0$, for $k_0=1.61$, $\theta=0$.

- For larger k_0, the pulse starts to move



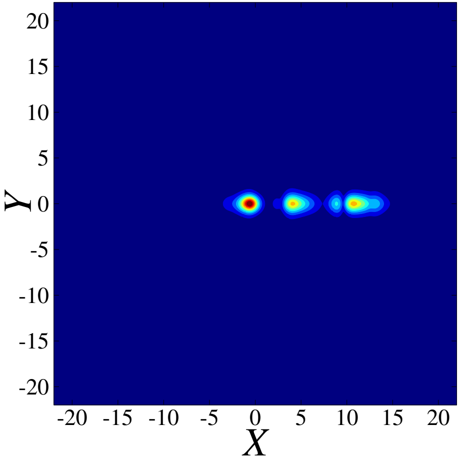
$Z = 5.29$

- For larger k_0, the pulse starts to move



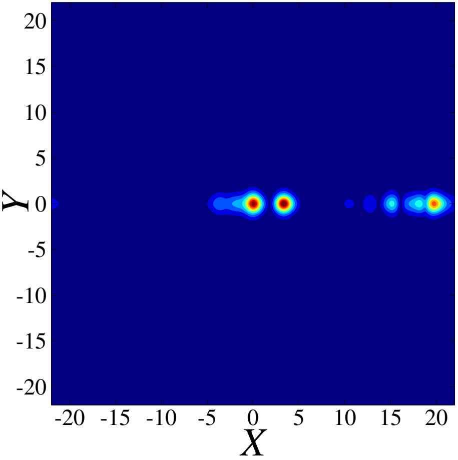
$Z = 9.39$

- For larger k_0, the pulse starts to move



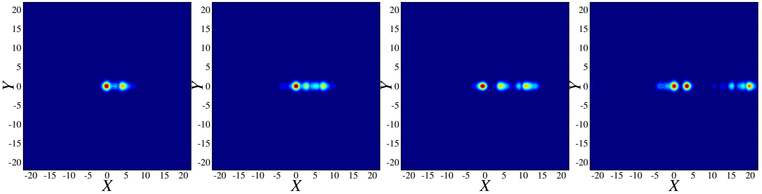
$Z = 17.01$

- For larger k_0, the pulse starts to move



$Z = 28.00$

- For larger k_0, the pulse starts to move

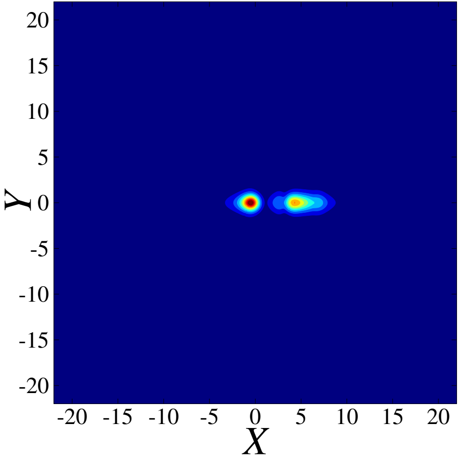


$Z = 5.29$ $Z = 9.39$ $Z = 17.01$ $Z = 28.00$

$|u(X,Z)|$ at $Y = 0$, for $k_0 = 1.6878$, $\theta = 0$.

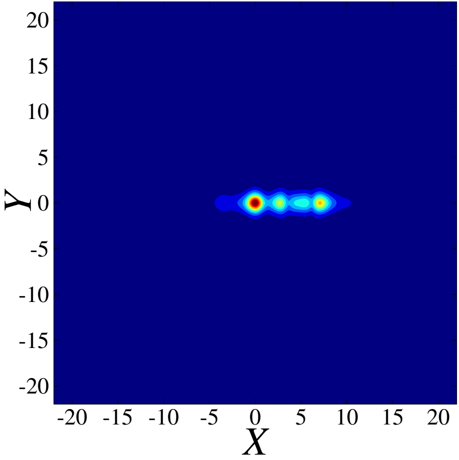
- The pulse leaves a copy of it behind it

- Another example, increasing k_0:



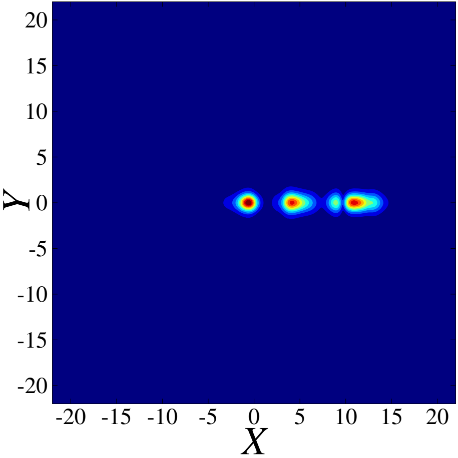
$Z = 5.26$

- Another example, increasing k_0:



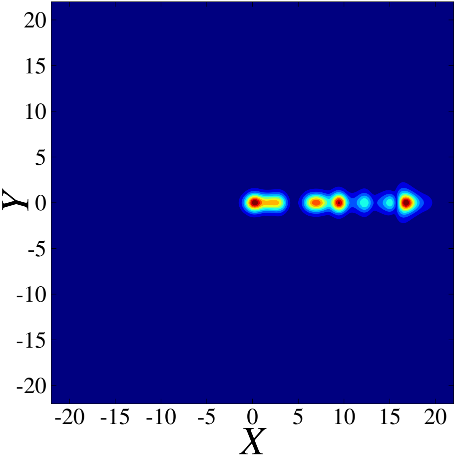
$Z = 9.35$

- Another example, increasing k_0:



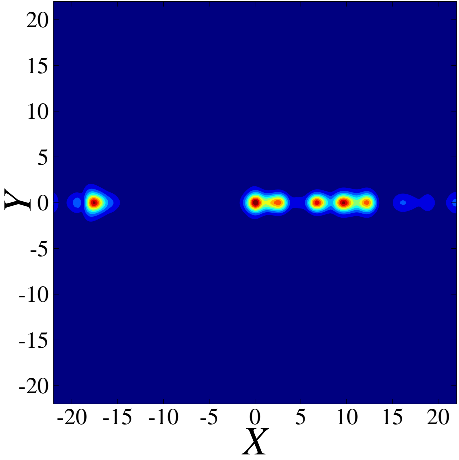
$Z = 16.95$

- Another example, increasing k_0:



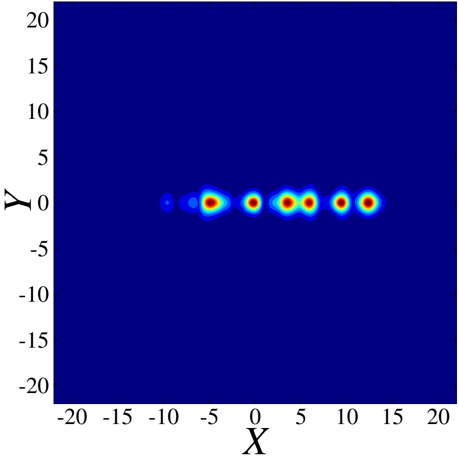
$Z = 24.10$

- Another example, increasing k_0:



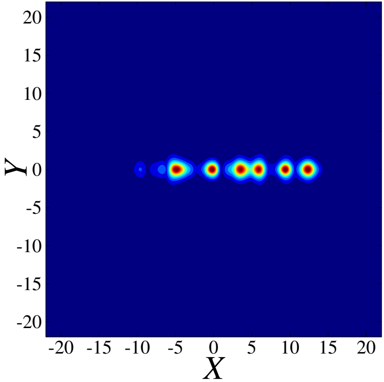
$Z = 35.33$

- Another example, increasing k_0:



$Z = 48.24.77$

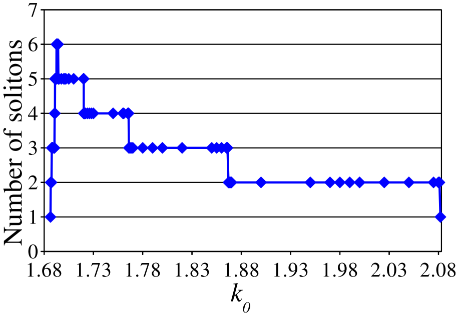
- Another example, increasing k_0:



$Z = 48.24.77$, for $k_0 = 1.694$

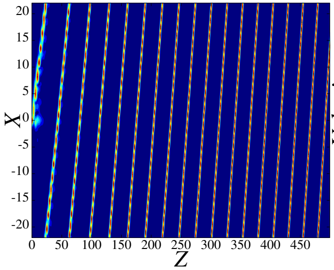
- An arrayed set of 5 fix + 1 moving solitons.

- The total number of emitted solitons first grows fast with k_0.

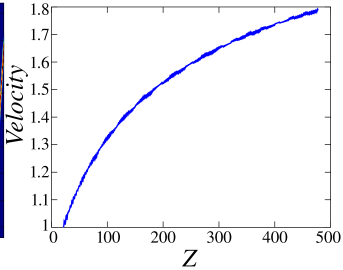


- It reaches a maximum of 5 (6 with the initial one)
- Then slowly goes down to 0 (the initial one only)

- For the highest k_0, the soliton moves freely



$|u(X,Z)|$ at $Y = 0$,

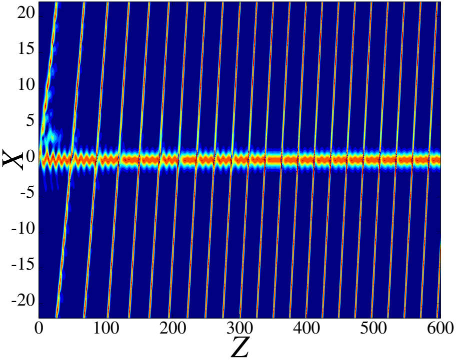


transverse speed, $k_0 = 2.1$, $\theta = 0$.

- The soliton velocity increases, approaching a certain limit value

Periodic elastic collisions

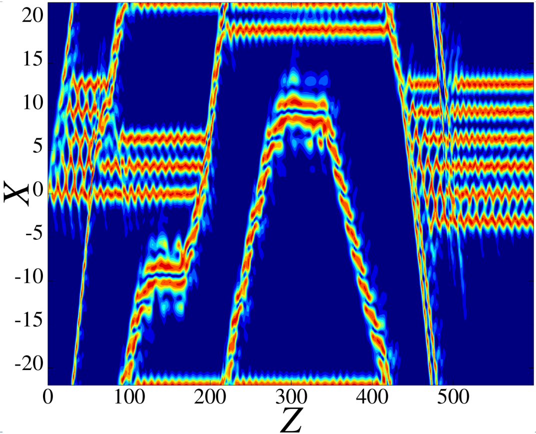
- A moving soliton with one fix soliton



$k_0 = 1.867,\ \theta = 0.$

- An example of the Newton's-cradle scenario

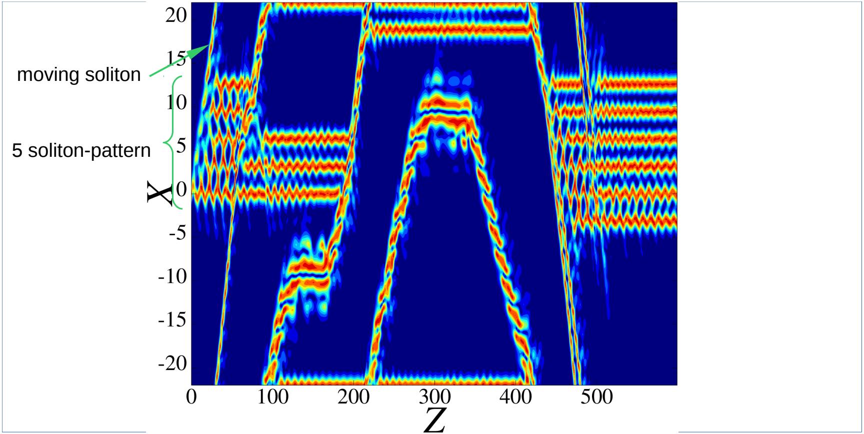
- 1 moving and 5 fix solitons



$|u(X,Z)|$ at $Y=0$; $k_0 = 1.693$, $\theta = 0$.

- An quite complex interaction

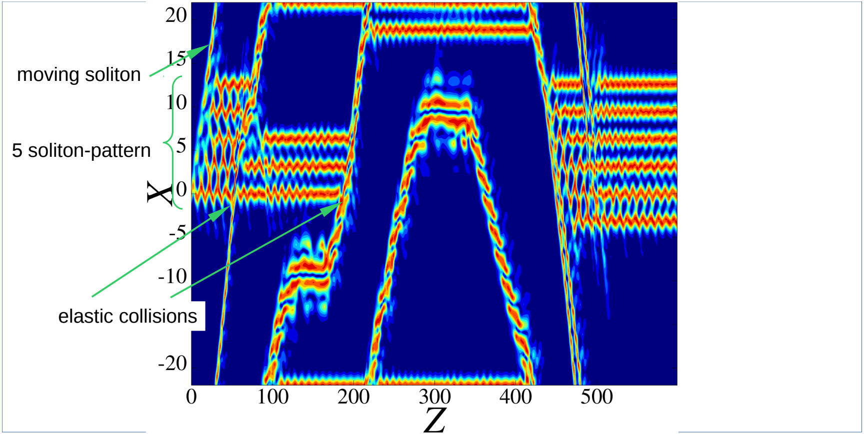
- 1 moving and 5 fix solitons



$|u(X,Z)|$ at $Y = 0$; $k_0 = 1.693$, $\theta = 0$.

- An quite complex interaction

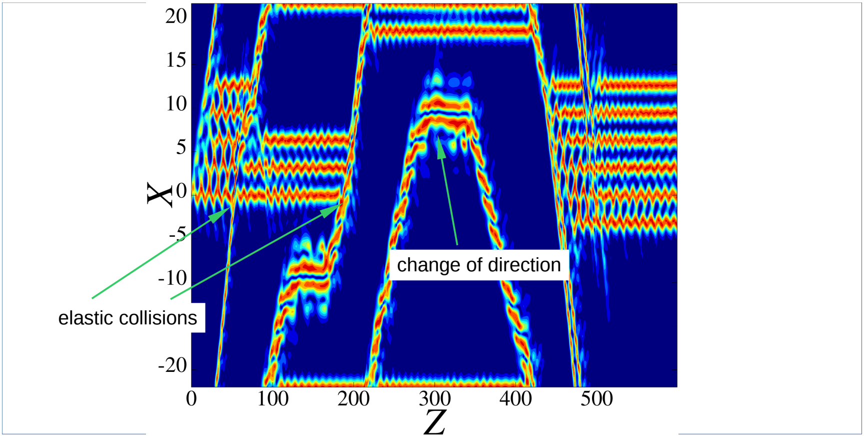
- 1 moving and 5 fix solitons



$|u(X,Z)|$ at $Y = 0$; $k_0 = 1.693$, $\theta = 0$.

- An quite complex interaction

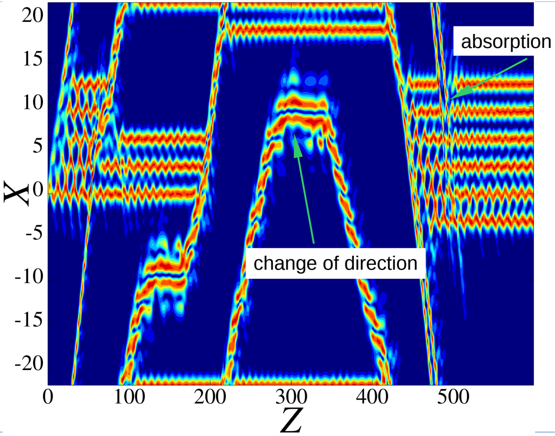
- 1 moving and 5 fix solitons



$|u\,(X,Z)|$ at $Y = 0$; $k_0 = 1.693$, $\theta = 0$.

- An quite complex interaction

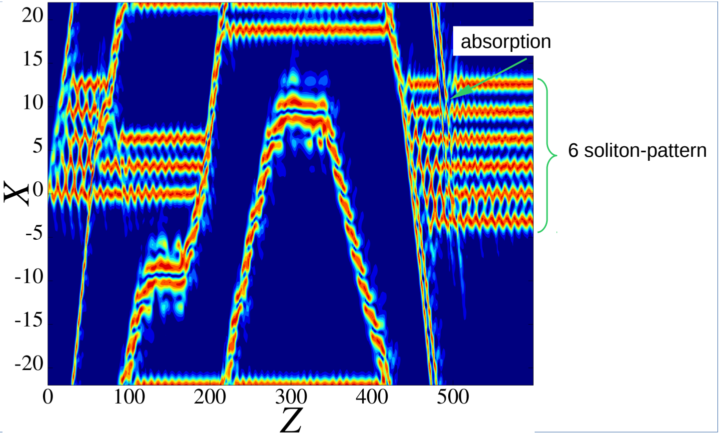
- 1 moving and 5 fix solitons



$|u(X,Z)|$ at $Y=0$; $k_0 = 1.693$, $\theta = 0$.

- An quite complex interaction

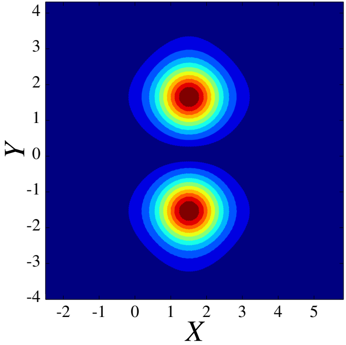
- 1 moving and 5 fix solitons



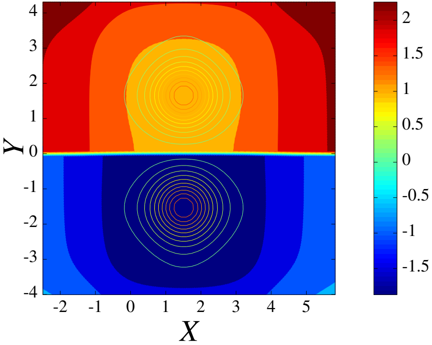
$|u(X,Z)|$ at $Y = 0$; $k_0 = 1.693$, $\theta = 0$.

- An quite complex interaction

The dipole

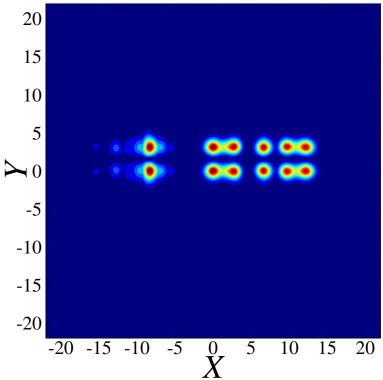


Amplitude



phase (in units of π)

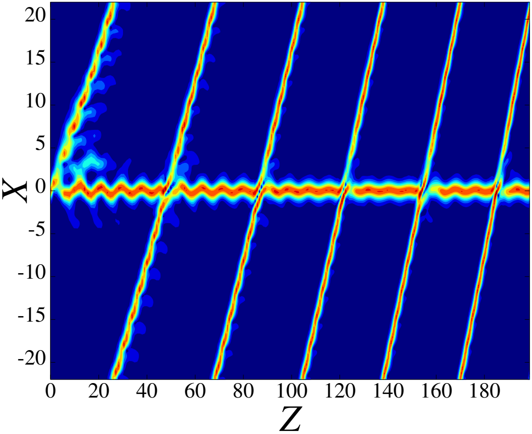
- Moving the dipole



$|u(X,Y)|$, at $Z = 22.410$, for $k_0 = 1.665$, $\theta = 0$.

- 5 fix and 1 moving dipoles

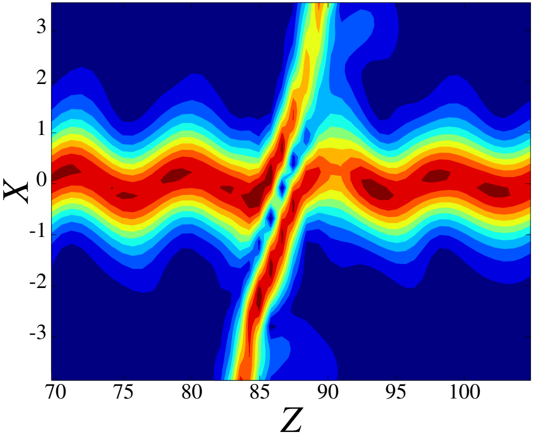
- Interaction of 1 moving dipole with 1 fix dipole



$|u(X,Y,Z)|$ at $Y=0$, for $k_0=1.865$.

- Repeated elastic collisions
- An example of the Newton's cradle scenario

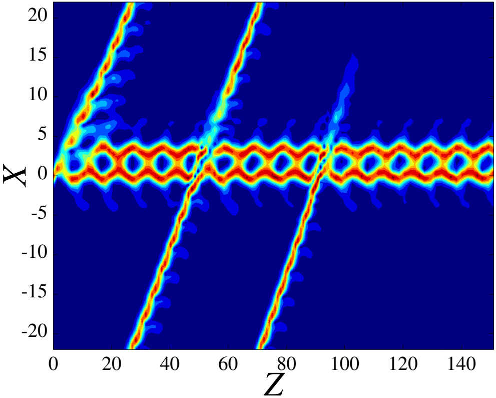
- Interaction of 1 moving dipole with 1 fix dipole



$|u(X,Y,Z)|$ at $Y = 0$, for $k_0 = 1.865$.

- Repeated elastic collisions
- An example of the Newton's cradle scenario

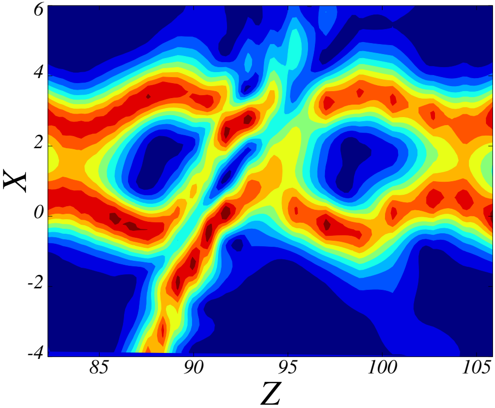
- The Newton's cradle with absorption scenario



$|u(X,Y,Z)|$ at $Y=0$, for $k_0 = 1.816$.

- After several quasi-elastic elastic collisions, the moving dipole is evntually absorbed by the quiescent
- Interaction of 1 moving dipole with 2 fix dipoles

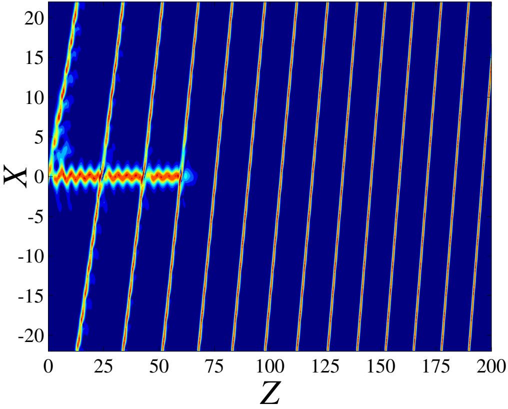
- The Newton's cradle with absorption scenario



$|u(X,Y,Z)|$ at $Y=0$, for $k_0 = 1.816$.

- After several quasi-elastic elastic collisions, the moving dipole is evntually absorbed by the quiescent
- Interaction of 1 moving dipole with 2 fix dipoles

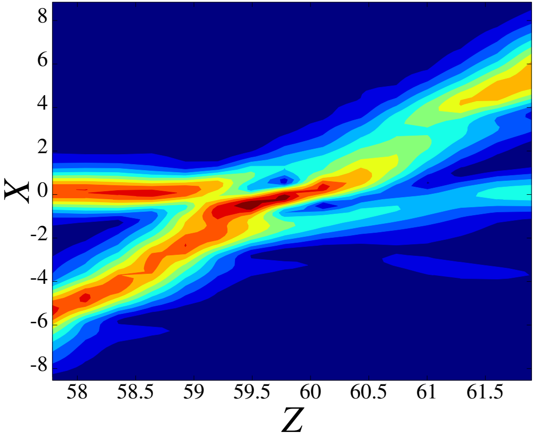
- Transient Newton's cradle with clearing the obstacle



$|u(X,Y,Z)|$ at $Y=0$, for $k_0 = 1.884$.

- After several quasi-elastic elastic collisions, the moving dipole absorbs the stationary chain

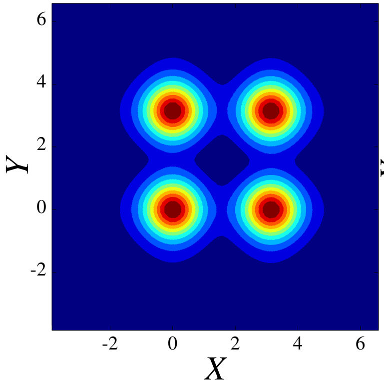
- Transient Newton's cradle with clearing the obstacle



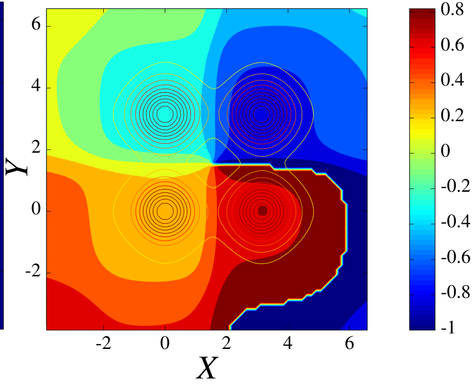
$|u\,(X,Y,Z)|$ at $Y=0$, for $k_0 = 1.884$.

- After several quasi-elastic elastic collisions, the moving dipole absorbs the stationary chain

Square-shaped (offsite-centered) vortex.



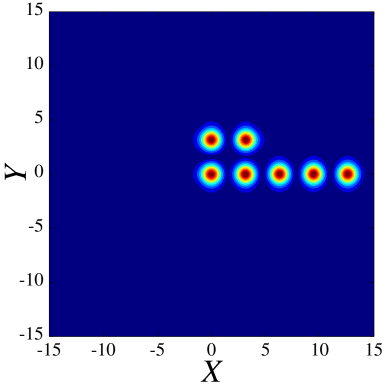
Amplitude



phase.

- It is unstable

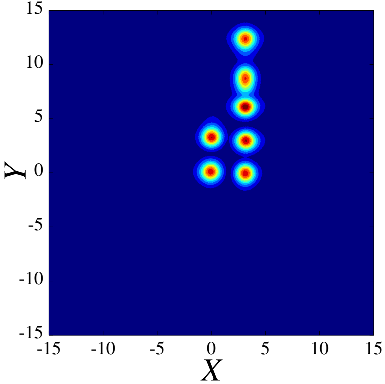
- Moving the vortex



Amplitude at $Z \simeq 300$, for $k_0 = 1.5$, and $\theta = \pi/8,\ 5\pi/8,\ 9\pi/8,\ 13\pi/8$.

- A set of fundamental solitons is formed
- For a clockwise rotating vortex, solitons form on the other line

- Moving the vortex



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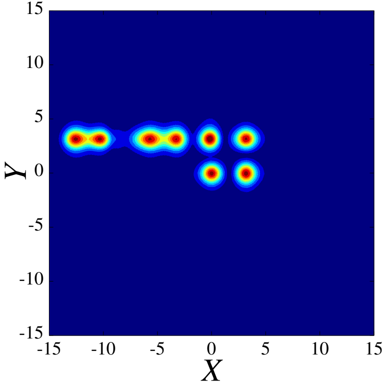
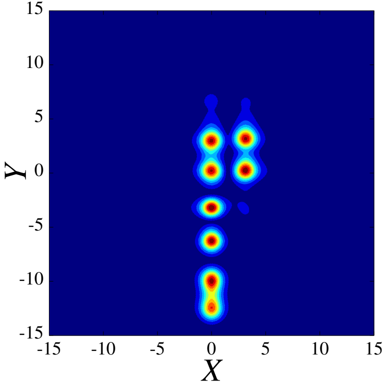
- Moving the vortex



Amplitude at $Z \simeq 300$, for $k_0 = 1.5$, and $\theta = \pi/8,\ 5\pi/8,\ 9\pi/8,\ 13\pi/8$.

- A set of fundamental solitons is formed
- For a clockwise rotating vortex, solitons form on the other line

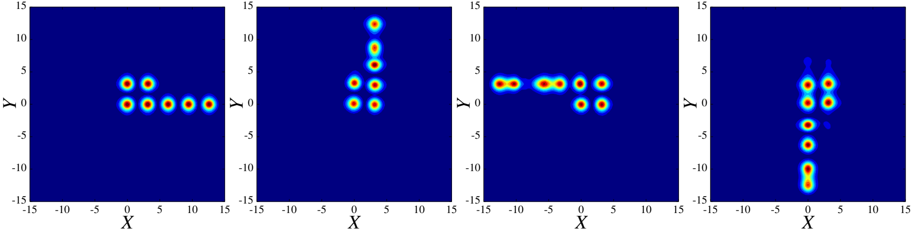
- Moving the vortex



Amplitude at $Z \simeq 300$, for $k_0 = 1.5$, and $\theta = \pi/8,\ 5\pi/8,\ 9\pi/8,\ 13\pi/8$.

- A set of fundamental solitons is formed
- For a clockwise rotating vortex, solitons form on the other line

- Moving the vortex



- A set of fundamental solitons is formed
- For a clockwise rotating vortex, solitons form on the other line
- The position of the soliton set depends of the direction of the kick with respect to vortex orientation

1. Motion induced by a continuous wave
   - Crystal, liquid and gas of solitons
   - Pulse motion due to gain dynamics
   - Injected continuous wave
2. The finite bandwidth of gain as a viscous friction
   - Analytical expression of the viscous friction
   - Numerical validation of the approximation
3. Transverse mobility in the presence of periodic potential
   - Fondamental soliton
   - Dipoles and vortices