



HAL
open science

Few-cycle optical soliton propagation in coupled waveguides

Hervé Leblond, Dumitru Mihalache, David Kremer, Said Terniche

► **To cite this version:**

Hervé Leblond, Dumitru Mihalache, David Kremer, Said Terniche. Few-cycle optical soliton propagation in coupled waveguides. Nonlinear Nanophotonics meets Nanomagnetism 2016, 2016, Le Mans, France. hal-02564158

HAL Id: hal-02564158

<https://univ-angers.hal.science/hal-02564158>

Submitted on 5 May 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Few-cycle optical soliton propagation in coupled waveguides

Hervé Leblond¹, Dumitru Mihalache², David Kremer³, Said Terniche^{1,4}

¹Laboratoire de Photonique d'Angers L ϕ A EA 4464, Université d'Angers.

²Horia Hulubei National Institute for Physics and Nuclear Engineering,
and Academy of Romanian Scientists, Bucharest.

³Laboratoire MOLTECH-Anjou, CNRS UMR 6200, Université d'Angers.

⁴Laboratoire Electronique Quantique, USTHB, Alger.

1 Models for few-cycle solitons

- The mKdV-sG equation
- Nonlinear widening of the linear guided modes

2 Waveguide coupling in the few-cycle regime

- Derivation of the coupling terms
- Examples of behavior of the linear non-dispersive coupling
- Few-cycle optical solitons in linearly coupled waveguides

3 Few cycle spatiotemporal solitons in waveguide arrays

- Formation of a solitons from a Gaussian pulse
- Two kind of solitons

1 Models for few-cycle solitons

- The mKdV-sG equation
- Nonlinear widening of the linear guided modes

2 Waveguide coupling in the few-cycle regime

- Derivation of the coupling terms
- Examples of behavior of the linear non-dispersive coupling
- Few-cycle optical solitons in linearly coupled waveguides

3 Few cycle spatiotemporal solitons in waveguide arrays

- Formation of a solitons from a Gaussian pulse
- Two kind of solitons

- The shortest laser pulses: a duration of a few optical cycles.

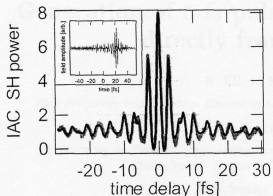


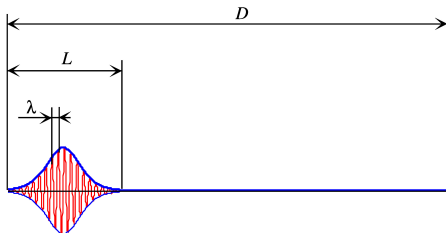
Fig. 4. Measured and reconstructed IAC of the pulse. A phase-retrieval algorithm reveals a pulse width of 5 fs. The reconstructed IAC fits perfectly and is not distinguishable. The reconstructed electric field is displayed in the inset. SH, second harmonic.

Autocorrelation trace, R. Ell et al., Optics Letters 26 (6), 373 (2001).

- Pulse duration down to a few fs. Ex. above: $5 \text{ fs} = 5 \times 10^{-15} \text{ s}$.
- How to model the propagation of such pulses?

Solitary wave vs envelope solitons

- Envelope soliton: the usual optical soliton in the ps range

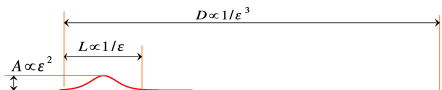


- Pulse duration $L \gg \lambda$ wavelength
- Typical model: NonLinear Schrödinger equation (NLS)

- It is a soliton if it propagates without deformation on $D \gg L$, due to nonlinearity.

In linear regime: spread out by dispersion.

- Solitary wave soliton: the hydrodynamical soliton



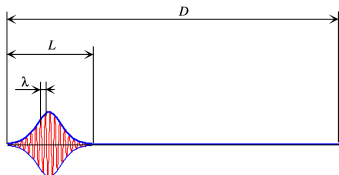
- A single oscillation
- Typical model: Korteweg-de Vries equation (KdV)

- Few-cycle optical solitons: $L \sim \lambda$

The slowly varying envelope approximation is not valid

Solitary wave vs envelope solitons

- Envelope soliton: the usual optical soliton in the ps range

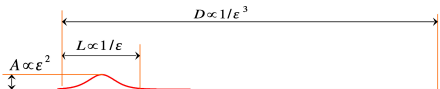


- Pulse duration $L \gg \lambda$ wavelength
- Typical model: NonLinear Schrödinger equation (NLS)

- It is a soliton if it propagates without deformation on $D \gg L$, due to nonlinearity.

In linear regime: spread out by dispersion.

- Solitary wave soliton: the hydrodynamical soliton



- A single oscillation
- Typical model: Korteweg-de Vries equation (KdV)

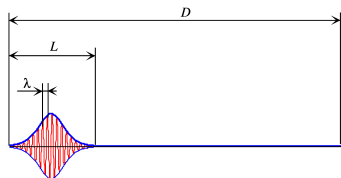
- Few-cycle optical solitons: $L \sim \lambda$

- The slowly varying envelope approximation is not valid
- Generalized NLS equation

We seek a different approach based on KdV type models

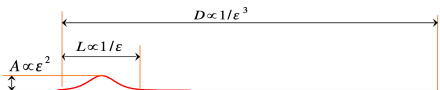
Solitary wave vs envelope solitons

- Envelope soliton: the usual optical soliton in the ps range



- Pulse duration $L \gg \lambda$ wavelength
- Typical model: NonLinear Schrödinger equation (NLS)

- Solitary wave soliton: the hydrodynamical soliton



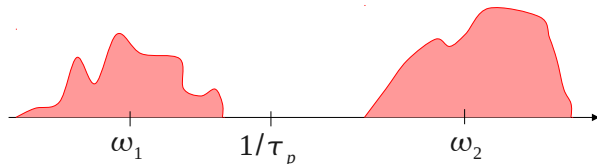
- A single oscillation
- Typical model: Korteweg-de Vries equation (KdV)

- Few-cycle optical solitons: $L \sim \lambda$

- The slowly varying envelope approximation is not valid
- Generalized NLS equation
- We seek a different approach based on KdV-type models

A transparent medium

- The general absorption spectrum of a transparent medium



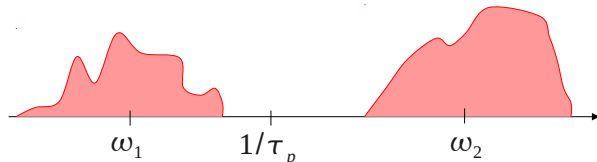
- A simple model: A two-component medium, each component is described by a two-level model
- We assume that the transparency range is very large:

$$\omega_1 \ll (1/\tau_p) \ll \omega_2$$

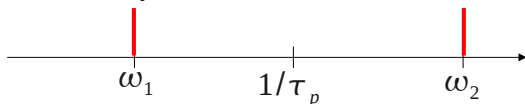
(τ_p : FCP duration).

A transparent medium

- The general absorption spectrum of a transparent medium



- A simple model: A two-component medium, each component is described by a two-level model



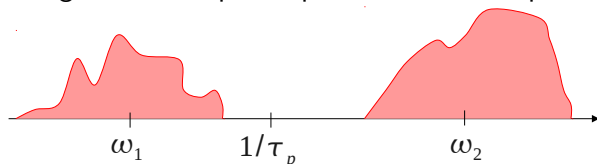
- We assume that the transparency range is very large:

$$\omega_1 \ll (1/\tau_p) \ll \omega_2$$

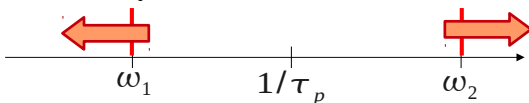
(τ_p : FCP duration).

A transparent medium

- The general absorption spectrum of a transparent medium



- A simple model: A two-component medium, each component is described by a two-level model



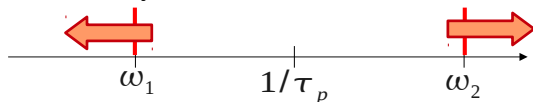
- We assume that the transparency range is very large:

$$\omega_1 \ll (1/\tau_p) \ll \omega_2$$

(τ_p : FCP duration).

A transparent medium

- A simple model: A two-component medium, each component is described by a two-level model



- We assume that the transparency range is very large:

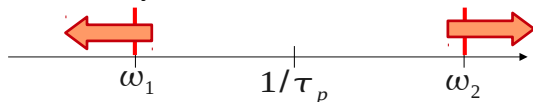
$$\omega_1 \ll (1/\tau_p) \ll \omega_2$$

(τ_p : FCP duration).

- In a first stage, the two components are treated separately
- UV transition only, with $(1/\tau_p) \ll \omega_2$
⇒ Long-wave approximation

A transparent medium

- A simple model: A two-component medium, each component is described by a two-level model



- We assume that the transparency range is very large:

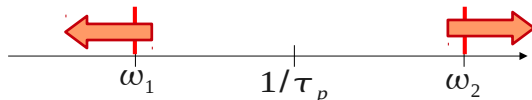
$$\omega_1 \ll (1/\tau_p) \ll \omega_2$$

(τ_p : FCP duration).

- In a first stage, the two components are treated separately
- UV transition only, with $(1/\tau_p) \ll \omega_2$
⇒ Long-wave approximation

A transparent medium

- A simple model: A two-component medium, each component is described by a two-level model

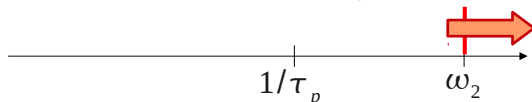


- We assume that the transparency range is very large:

$$\omega_1 \ll (1/\tau_p) \ll \omega_2$$

(τ_p : FCP duration).

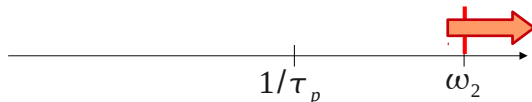
- In a first stage, the two components are treated separately
- UV transition only, with $(1/\tau_p) \ll \omega_2$



⇒ Long-wave approximation

A transparent medium

- In a first stage, the two components are treated separately
- UV transition only, with $(1/\tau_p) \ll \omega_2$



⇒ Long-wave approximation

modified Korteweg-de Vries (mKdV) equation

$$\frac{\partial E}{\partial \zeta} = \frac{1}{6} \frac{d^3 k}{d\omega^3} \Big|_{\omega=0} \frac{\partial^3 E}{\partial \tau^3} - \frac{6\pi}{nc} \chi^{(3)}(\omega; \omega, \omega, -\omega) \Big|_{\omega=0} \frac{\partial E^3}{\partial \tau}$$

H. Leblond and F. Sanchez, *Phys. Rev. A* **67**, 013804 (2003)

A transparent medium

- In a first stage, the two components are treated separately
- IR transition only, with $\omega_1 \ll (1/\tau_p)$
⇒ Short-wave approximation
- sine-Gordon (sG) equation: $\frac{\partial^2 \psi}{\partial z \partial t} = c_1 \sin \psi$

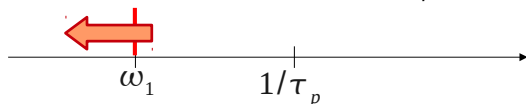
with $c_1 = \frac{w_\infty}{w_r}$: normalized initial population difference

and $\frac{\partial \psi}{\partial t} = \frac{E}{E_r}$: normalized electric field

H. Leblond and F. Sanchez, *Phys. Rev. A* **67**, 013804 (2003)

A transparent medium

- In a first stage, the two components are treated separately
- IR transition only, with $\omega_1 \ll (1/\tau_p)$



⇒ Short-wave approximation

- sine-Gordon (sG) equation: $\frac{\partial^2 \psi}{\partial z \partial t} = c_1 \sin \psi$

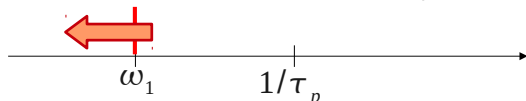
with $c_1 = \frac{w_\infty}{w_r}$: normalized initial population difference

and $\frac{\partial \psi}{\partial t} = \frac{E}{E_r}$: normalized electric field

H. Leblond and F. Sanchez, *Phys. Rev. A* **67**, 013804 (2003)

A transparent medium

- In a first stage, the two components are treated separately
- IR transition only, with $\omega_1 \ll (1/\tau_p)$



⇒ Short-wave approximation

- sine-Gordon (sG) equation: $\frac{\partial^2 \psi}{\partial z \partial t} = c_1 \sin \psi$

with $c_1 = \frac{w_\infty}{w_r}$: normalized initial population difference

and $\frac{\partial \psi}{\partial t} = \frac{E}{E_r}$: normalized electric field

H. Leblond and F. Sanchez, *Phys. Rev. A* **67**, 013804 (2003)

A transparent medium

- Then the two approximations are brought together to yield a general model:

The mKdV-sG equation

$$\frac{\partial^2 \psi}{\partial z \partial t} + c_1 \sin \psi + c_2 \frac{\partial}{\partial t} \left(\frac{\partial \psi}{\partial t} \right)^3 + c_3 \frac{\partial^4 \psi}{\partial t^4} = 0$$

- $\frac{\partial u}{\partial z} + c_1 \sin \int^t u + c_2 \frac{\partial u^3}{\partial t} + c_3 \frac{\partial^3 u}{\partial t^3} = 0$

with $u = \frac{\partial \psi}{\partial t} = \frac{E}{E_r}$: normalized electric field

- Integrable by inverse scattering transform in some cases:

,

A transparent medium

- Then the two approximations are brought together to yield a general model:

The mKdV-sG equation

$$\frac{\partial^2 \psi}{\partial z \partial t} + c_1 \sin \psi + c_2 \frac{\partial}{\partial t} \left(\frac{\partial \psi}{\partial t} \right)^3 + c_3 \frac{\partial^4 \psi}{\partial t^4} = 0$$

- Or

$$\frac{\partial u}{\partial z} + c_1 \sin \int^t u + c_2 \frac{\partial u^3}{\partial t} + c_3 \frac{\partial^3 u}{\partial t^3} = 0$$

with $u = \frac{\partial \psi}{\partial t} = \frac{E}{E_r}$: normalized electric field

- Integrable by inverse scattering transform in some cases:

A transparent medium

- Then the two approximations are brought together to yield a general model:

The mKdV-sG equation

- $$\frac{\partial u}{\partial z} + c_1 \sin \int^t u + c_2 \frac{\partial u^3}{\partial t} + c_3 \frac{\partial^3 u}{\partial t^3} = 0$$

with $u = \frac{\partial \psi}{\partial t} = \frac{E}{E_r}$: normalized electric field

- Integrable by inverse scattering transform in some cases:

,

A transparent medium

- Then the two approximations are brought together to yield a general model:

The mKdV-sG equation

- $$\frac{\partial u}{\partial z} + c_2 \frac{\partial u^3}{\partial t} + c_3 \frac{\partial^3 u}{\partial t^3} = 0$$

with $u = \frac{\partial \psi}{\partial t} = \frac{E}{E_r}$: normalized electric field

- Integrable by inverse scattering transform in some cases:
mKdV,

A transparent medium

- Then the two approximations are brought together to yield a general model:

The mKdV-sG equation

- $$\frac{\partial u}{\partial z} + c_1 \sin \int^t u = 0$$

with $u = \frac{\partial \psi}{\partial t} = \frac{E}{E_r}$: normalized electric field

- Integrable by inverse scattering transform in some cases:
mKdV, sG,

A transparent medium

- Then the two approximations are brought together to yield a general model:

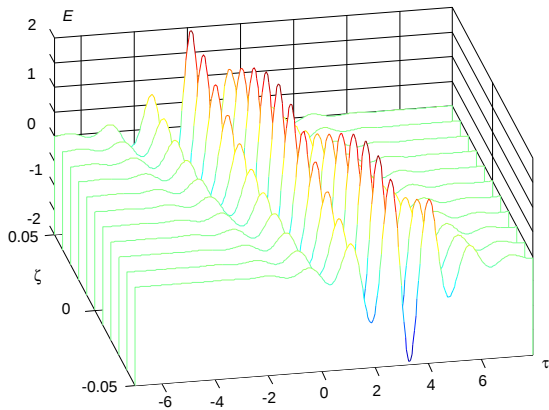
The mKdV-sG equation

- $$\frac{\partial u}{\partial z} + c_1 \sin \int^t u + c_2 \frac{\partial u^3}{\partial t} + 2c_2 \frac{\partial^3 u}{\partial t^3} = 0$$

with $u = \frac{\partial \psi}{\partial t} = \frac{E}{E_r}$: normalized electric field

- Integrable by inverse scattering transform in some cases:
mKdV, sG, and $c_3 = 2c_2$.

The analytical breather solution



- A few cycle soliton:
 - Not spread out by dispersion
 - Stable
 - However, oscillates (breather)

H. Leblond, S.V. Sazonov, I.V. Mel'nikov, D. Mihalache, and F. Sanchez,
Phys. Rev. A **74**, 063815 (2006)

1 Models for few-cycle solitons

- The mKdV-sG equation
- Nonlinear widening of the linear guided modes

2 Waveguide coupling in the few-cycle regime

- Derivation of the coupling terms
- Examples of behavior of the linear non-dispersive coupling
- Few-cycle optical solitons in linearly coupled waveguides

3 Few cycle spatiotemporal solitons in waveguide arrays

- Formation of a solitons from a Gaussian pulse
- Two kind of solitons

- The evolution of the electric field E :

In (1+1) dimensions:

→ The modified Korteweg-de Vries (mKdV) equation

$$\partial_z E + \beta \partial_\tau^3 E + \gamma \partial_\tau E^3 = 0$$

- Nonlinear coefficient $\gamma = \frac{1}{2nc} \chi^{(3)}$,

Dispersion parameter $\beta = \frac{(-n'')}{2c}$,

Waveguide description

- The evolution of the electric field E :

We generalize to (2+1) dimensions:

→ The cubic generalized Kadomtsev-Petviashvili (CGKP) equation

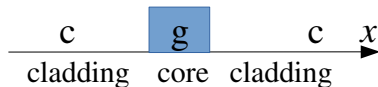
$$\partial_{\zeta} E + \beta \partial_{\tau}^3 E + \gamma \partial_{\tau} E^3 - \frac{V}{2} \int^{\tau} \partial_{\xi}^2 E d\tau' = 0$$

- Nonlinear coefficient $\gamma = \frac{1}{2nc} \chi^{(3)}$,
Dispersion parameter $\beta = \frac{(-n'')}{2c}$,
Linear group velocity: $V = \frac{c}{n}$.

Waveguide description

- The evolution of the electric field E :

A waveguide:



→ The cubic generalized Kadomtsev-Petviashvili (CGKP) equation

$$\partial_{\zeta} E + \beta_{\alpha} \partial_{\tau}^3 E + \gamma_{\alpha} \partial_{\tau} E^3 - \frac{V_{\alpha}}{2} \int^{\tau} \partial_{\xi}^2 E d\tau' = 0$$

with $\alpha = g$ in the core and $\alpha = c$ in the cladding.

- Nonlinear coefficient $\gamma_{\alpha} = \frac{1}{2n_{\alpha}c} \chi_{\alpha}^{(3)}$,

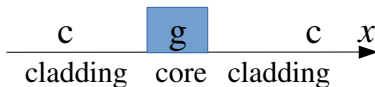
Dispersion parameter $\beta_{\alpha} = \frac{(-n''_{\alpha})}{2c}$,

Linear group velocity: $V_{\alpha} = \frac{c}{n_{\alpha}}$.

Waveguide description

- The evolution of the electric field E :

A waveguide:



→ The cubic generalized Kadomtsev-Petviashvili (CGKP) equation

$$\partial_{\zeta} E + \beta_{\alpha} \partial_{\tau}^3 E + \gamma_{\alpha} \partial_{\tau} E^3 + \frac{1}{V_{\alpha}} \partial_{\tau} E - \frac{V_{\alpha}}{2} \int^{\tau} \partial_{\xi}^2 E d\tau' = 0$$

with $\alpha = g$ in the core and $\alpha = c$ in the cladding.

Velocities: $V_g < V_c$

- Nonlinear coefficient $\gamma_{\alpha} = \frac{1}{2n_{\alpha}c} \chi_{\alpha}^{(3)}$,

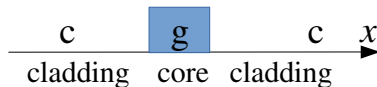
Dispersion parameter $\beta_{\alpha} = \frac{(-n''_{\alpha})}{2c}$,

Linear group velocity: $V_{\alpha} = \frac{c}{n_{\alpha}}$.

Waveguide description

- The evolution of the electric field E :

A waveguide:



→ The cubic generalized Kadomtsev-Petviashvili (CGKP) equation
In dimensionless form:

$$\partial_z u = A_\alpha \partial_t^3 u + B_\alpha \partial_t u^3 + v_\alpha \partial_t u + \frac{W_\alpha}{2} \int^t \partial_x^2 u dt'$$

with $\alpha = g$ in the core and $\alpha = c$ in the cladding.

Relative velocities: $v_g > v_c$

- Nonlinear coefficient $\gamma_\alpha = \frac{1}{2n_\alpha c} \chi_\alpha^{(3)}$,
- Dispersion parameter $\beta_\alpha = \frac{(-n''_\alpha)}{2c}$,
- Linear group velocity: $V_\alpha = \frac{c}{n_\alpha}$.

- The dimensionless CGKP equation

$$\partial_z u = A_\alpha \partial_t^3 u + B_\alpha \partial_t u^3 + v_\alpha \partial_t u + \frac{W_\alpha}{2} \int^t \partial_x^2 u dt'$$

- We linearize and seek for solutions of the form $u = f(x) \exp[i(\omega t - k_z z)]$.
- Waveguide dispersion relation: $\tan(k_x a) = \frac{\kappa}{k_x}$
- Mode profiles of the form $\begin{cases} f = R \cos(k_x x) & \text{in the core} \\ f = C e^{-\kappa|x|} & \text{in the cladding} \end{cases}$

- The dimensionless CGKP equation

$$\partial_z u = A_\alpha \partial_t^3 u + v_\alpha \partial_t u + \frac{W_\alpha}{2} \int^t \partial_x^2 u dt'$$

- We linearize and seek for solutions of the form $u = f(x) \exp[i(\omega t - k_z z)]$.

- Material dispersion relation: $k_{x,\alpha}^2 = \frac{2\omega}{V_\alpha} (A_\alpha \omega^3 - k_z - \omega v_\alpha)$

- Waveguide dispersion relation: $\tan(k_x a) = \frac{\kappa}{k_x}$

- Mode profiles of the form $\begin{cases} f = R \cos(k_x x) & \text{in the core} \\ f = C e^{-\kappa|x|} & \text{in the cladding} \end{cases}$

- The dimensionless CGKP equation

$$\partial_z u = A_\alpha \partial_t^3 u + v_\alpha \partial_t u + \frac{W_\alpha}{2} \int^t \partial_x^2 u dt'$$

- We linearize and seek for solutions of the form $u = f(x) \exp[i(\omega t - k_z z)]$.
- Waveguide dispersion relation: $\tan(k_x a) = \frac{\kappa}{k_x}$
- Mode profiles of the form $\begin{cases} f = R \cos(k_x x) & \text{in the core} \\ f = C e^{-\kappa|x|} & \text{in the cladding} \end{cases}$
- Guiding condition: $A_2 \omega^3 - \omega v_2 < k_z < A_1 \omega^3$

As in the usual symmetric dielectric planar waveguide.

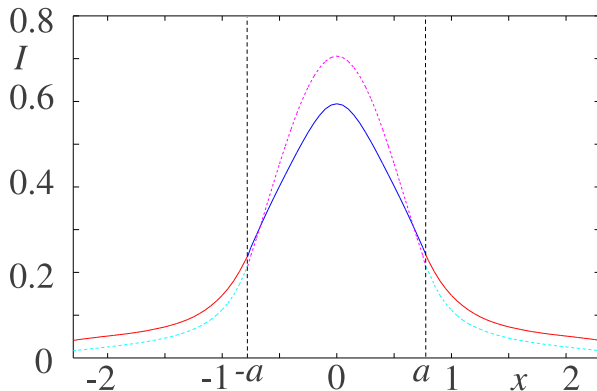
Nonlinear propagation in linear guide

- We solve the CGKP equation starting from

$$u(x, t, z = 0) = A \cos(\omega t) f(x) e^{-t^2/w^2},$$

- $f(x) = \begin{cases} \cos(k_x x), & \text{for } |x| \leq a, \\ C e^{-\kappa|x|}, & \text{for } |x| > a, \end{cases}$ is a linear mode profile
- Normalized coefficients $A_1 = A_2 = B_1 = B_2 = W_1 = W_2 = 1$,
→ we assume that
 - Temporal compression occurs
 - Spatial defocusing occurs, (else it collapses!)
 - Dispersion and nonlinearity are identical in core and cladding.

- Guided wave profiles



(Normalized so that the total power is 1. $v_2 = 3$, $w = 2$.)

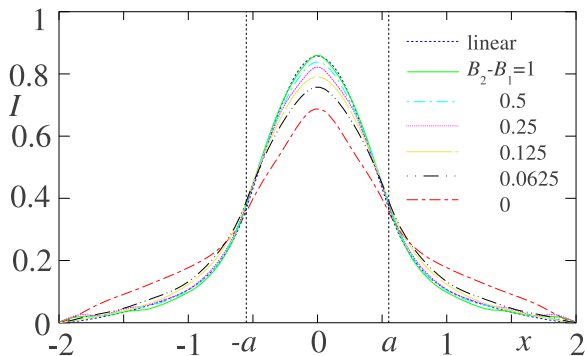
- The pulse is less confined in nonlinear (blue, and red) than in linear (pink and cyan) regime.

Nonlinear may reduce confinement. Indeed:

- Waveguiding is due to total internal reflection.
→ when the wave propagates too fast in the cladding, it cannot match the field oscillations in the core.
- In the few-cycle regime, not only linear, but also nonlinear velocity (large and positive here).
- The nonlinear velocity is larger in the core and creates a nonlinear variation of the index.
- It may compensate the linear part, and reduce the confinement.
- Important in very narrow guides, where linear confinement itself is low.

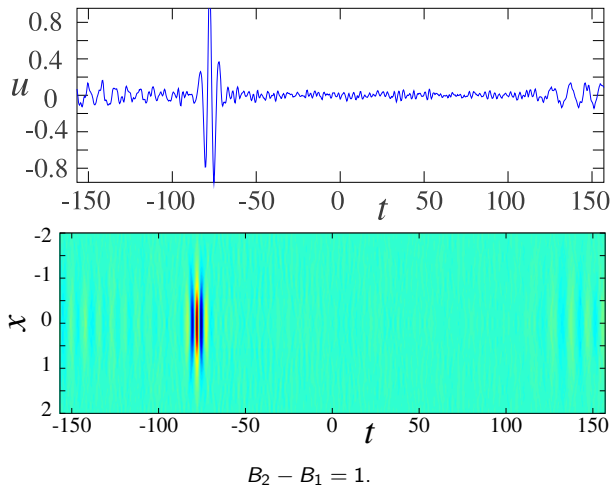
Nonlinear waveguide

- Wave guided and confined by using nonlinear velocity: a higher nonlinear coefficient in the cladding than that in the core.



Guided profiles of the nonlinear waveguide. Normalized so that the total power is 1.

Two-cycle soliton of the nonlinear waveguide



H. Leblond and D. Mihalache, *Phys. Rev. A* **88**, 023840 (2013)

1 Models for few-cycle solitons

- The mKdV-sG equation
- Nonlinear widening of the linear guided modes

2 Waveguide coupling in the few-cycle regime

- Derivation of the coupling terms
- Examples of behavior of the linear non-dispersive coupling
- Few-cycle optical solitons in linearly coupled waveguides

3 Few cycle spatiotemporal solitons in waveguide arrays

- Formation of a solitons from a Gaussian pulse
- Two kind of solitons

1 Models for few-cycle solitons

- The mKdV-sG equation
- Nonlinear widening of the linear guided modes

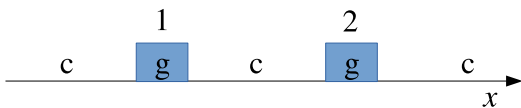
2 Waveguide coupling in the few-cycle regime

- Derivation of the coupling terms
- Examples of behavior of the linear non-dispersive coupling
- Few-cycle optical solitons in linearly coupled waveguides

3 Few cycle spatiotemporal solitons in waveguide arrays

- Formation of a solitons from a Gaussian pulse
- Two kind of solitons

- 2D waveguiding structure: two cores 1 and 2 and dielectric cladding



- The generalized Kadomtsev-Petviashvili (GKP) equation (dimensionless)

$$\partial_z u = A_\alpha \partial_t^3 u + B_\alpha \partial_t u^3 + V_\alpha \partial_t u + \frac{w_\alpha}{2} \int^t \partial_x^2 u dt,$$

$\alpha = g$ in the cores 1 and 2, $\alpha = c$ in the cladding.

- Individual waveguides, in the linearized model ($B_\alpha = 0$):
The field is $u = Rf_j(x)e^{i(\omega t - \beta z)} = Rf_j(x)e^{i\varphi}$,
 f_j , ($j = 1, 2$): guided mode profile, R : amplitude.
- We seek for a solution as

$$u = R(z)f_1(x)e^{i\varphi} + S(z)f_2(x)e^{i\varphi},$$

i.e., two interacting modes.

- We report it into the GKP equation and get
 - in the cladding $\partial_z Rf_1 + \partial_z Sf_2 = 0$
 - in guide 1 $\partial_z Rf_1 + \partial_z Sf_2 = \frac{i\omega_g}{2\omega} (K_c - K_g) Sf_2$
 and so on.
- Multiplying this by f_1 and integrating over all x removes the x -dependency of f_1 , f_2 and we get

$$\partial_z R = \partial_z S = \frac{i\omega_g}{2\omega} \frac{(K_c - K_g) l_2}{1 + l_1} (R + S),$$

- Involve overlap integrals $l_1 = \int_{-\infty}^{\infty} f_1 f_2 dx$, $l_2 = \int_{g_1} f_1 f_2 dx = \int_{g_2} f_1 f_2 dx$
 (“ $\int_{g_j} \cdot dx$ ” is the integral over the core $j = 1$ or 2 .)

- Individual waveguides, in the linearized model ($B_\alpha = 0$):
The field is $u = Rf_j(x)e^{i(\omega t - \beta z)} = Rf_j(x)e^{i\varphi}$,
 f_j , ($j = 1, 2$): guided mode profile, R : amplitude.
- We seek for a solution as

$$u = R(z)f_1(x)e^{i\varphi} + S(z)f_2(x)e^{i\varphi},$$

i.e., two interacting modes.

- We report it into the GKP equation and get
 - in the cladding $\partial_z Rf_1 + \partial_z Sf_2 = 0$
 - in guide 1 $\partial_z Rf_1 + \partial_z Sf_2 = \frac{i\omega_g}{2\omega} (K_c - K_g) Sf_2$
 and so on.
- Multiplying this by f_1 and integrating over all x removes the x -dependency of f_1 , f_2 and we get

$$\partial_z R = \partial_z S = \frac{i\omega_g}{2\omega} \frac{(K_c - K_g) l_2}{1 + l_1} (R + S),$$

- Involve overlap integrals $l_1 = \int_{-\infty}^{\infty} f_1 f_2 dx$, $l_2 = \int_{g_1} f_1 f_2 dx = \int_{g_2} f_1 f_2 dx$
 (“ $\int_{g_j} \cdot dx$ ” is the integral over the core $j = 1$ or 2 .)

- Individual waveguides, in the linearized model ($B_\alpha = 0$):
The field is $u = Rf_j(x)e^{i(\omega t - \beta z)} = Rf_j(x)e^{i\varphi}$,
 f_j , ($j = 1, 2$): guided mode profile, R : amplitude.
- We seek for a solution as

$$u = R(z)f_1(x)e^{i\varphi} + S(z)f_2(x)e^{i\varphi},$$

i.e., two interacting modes.

- We report it into the GKP equation and get
 - in the cladding $\partial_z Rf_1 + \partial_z Sf_2 = 0$
 - in guide 1 $\partial_z Rf_1 + \partial_z Sf_2 = \frac{i\omega_g}{2\omega} (K_c - K_g) Sf_2$
 and so on.

- Multiplying this by f_1 and integrating over all x removes the x -dependency of f_1 , f_2 and we get

$$\partial_z R = \partial_z S = \frac{i\omega_g}{2\omega} \frac{(K_c - K_g) l_2}{1 + l_1} (R + S),$$

- Involve overlap integrals $l_1 = \int_{-\infty}^{\infty} f_1 f_2 dx$, $l_2 = \int_{g_1} f_1 f_2 dx = \int_{g_2} f_1 f_2 dx$
 (“ $\int_{g_j} \cdot dx$ ” is the integral over the core $j = 1$ or 2 .)

- Individual waveguides, in the linearized model ($B_\alpha = 0$):
The field is $u = Rf_j(x)e^{i(\omega t - \beta z)} = Rf_j(x)e^{i\varphi}$,
 f_j , ($j = 1, 2$): guided mode profile, R : amplitude.
- We seek for a solution as

$$u = R(z)f_1(x)e^{i\varphi} + S(z)f_2(x)e^{i\varphi},$$

i.e., two interacting modes.

- We report it into the GKP equation and get
 - in the cladding $\partial_z Rf_1 + \partial_z Sf_2 = 0$
 - in guide 1 $\partial_z Rf_1 + \partial_z Sf_2 = \frac{i\omega_g}{2\omega} (K_c - K_g) Sf_2$
 and so on.
- Multiplying this by f_1 and integrating over all x removes the x -dependency of f_1 , f_2 and we get

$$\partial_z R = \partial_z S = \frac{i\omega_g}{2\omega} \frac{(K_c - K_g) I_2}{1 + I_1} (R + S),$$

- Involve overlap integrals $I_1 = \int_{-\infty}^{\infty} f_1 f_2 dx$, $I_2 = \int_{g_1} f_1 f_2 dx = \int_{g_2} f_1 f_2 dx$
 (“ $\int_{g_j} \cdot dx$ ” is the integral over the core $j = 1$ or 2.)

- Individual waveguides, in the linearized model ($B_\alpha = 0$):
The field is $u = Rf_j(x)e^{i(\omega t - \beta z)} = Rf_j(x)e^{i\varphi}$,
 f_j , ($j = 1, 2$): guided mode profile, R : amplitude.
- We seek for a solution as

$$u = R(z)f_1(x)e^{i\varphi} + S(z)f_2(x)e^{i\varphi},$$

i.e., two interacting modes.

- We report it into the GKP equation and get
 - in the cladding $\partial_z Rf_1 + \partial_z Sf_2 = 0$
 - in guide 1 $\partial_z Rf_1 + \partial_z Sf_2 = \frac{i\omega_g}{2\omega} (K_c - K_g) Sf_2$
 and so on.
- Multiplying this by f_1 and integrating over all x removes the x -dependency of f_1 , f_2 and we get

$$\partial_z R = \partial_z S = \frac{i\omega_g}{2\omega} \frac{(K_c - K_g) l_2}{1 + l_1} (R + S),$$

- Involve overlap integrals $l_1 = \int_{-\infty}^{\infty} f_1 f_2 dx$, $l_2 = \int_{g_1} f_1 f_2 dx = \int_{g_2} f_1 f_2 dx$
 (“ $\int_{g_j} \cdot dx$ ” is the integral over the core $j = 1$ or 2.)

- We seek for a solution as

$$u = R(z)f_1(x)e^{i\varphi} + S(z)f_2(x)e^{i\varphi},$$

i.e., two interacting modes.

- Multiplying this by f_1 and integrating over all x removes the x -dependency of f_1 , f_2 and we get

$$\partial_z R = \partial_z S = \frac{i\omega_g}{2\omega} \frac{(K_c - K_g) I_2}{1 + I_1} (R + S),$$

- The few-cycle pulse is expanded as a Fourier integral of such modes, $u_1 = \int R e^{i\varphi} d\omega$, $u_2 = \int S e^{i\varphi} d\omega$.
- We report $\partial_z R$ and $\partial_z S$ into $\partial_z u_1$, and get the linear coupling terms.
- Finally, we get the system of two coupled modified Korteweg-de Vries (mKdV) equations

$$\partial_z u_1 = A \partial_t^3 u_1 + B \partial_t u_1^3 + V \partial_t u_1 + C \partial_t u_2 + D \partial_t^3 u_2,$$

$$\partial_z u_2 = A \partial_t^3 u_2 + B \partial_t u_2^3 + V \partial_t u_2 + C \partial_t u_1 + D \partial_t^3 u_1,$$

- We seek for a solution as

$$u = R(z)f_1(x)e^{i\varphi} + S(z)f_2(x)e^{i\varphi},$$

i.e., two interacting modes.

- Multiplying this by f_1 and integrating over all x removes the x -dependency of f_1 , f_2 and we get

$$\partial_z R = \partial_z S = \frac{i\omega_g}{2\omega} \frac{(K_c - K_g) I_2}{1 + I_1} (R + S),$$

- The few-cycle pulse is expanded as a Fourier integral of such modes, $u_1 = \int R e^{i\varphi} d\omega$, $u_2 = \int S e^{i\varphi} d\omega$.
- We report $\partial_z R$ and $\partial_z S$ into $\partial_z u_1$, and get the linear coupling terms.
- Finally, we get the system of two coupled modified Korteweg-de Vries (mKdV) equations

$$\partial_z u_1 = A\partial_t^3 u_1 + B\partial_t u_1^3 + V\partial_t u_1 + C\partial_t u_2 + D\partial_t^3 u_2,$$

$$\partial_z u_2 = A\partial_t^3 u_2 + B\partial_t u_2^3 + V\partial_t u_2 + C\partial_t u_1 + D\partial_t^3 u_1,$$

- We seek for a solution as

$$u = R(z)f_1(x)e^{i\varphi} + S(z)f_2(x)e^{i\varphi},$$

i.e., two interacting modes.

- Multiplying this by f_1 and integrating over all x removes the x -dependency of f_1 , f_2 and we get

$$\partial_z R = \partial_z S = \frac{i\omega_g}{2\omega} \frac{(K_c - K_g) I_2}{1 + I_1} (R + S),$$

- The few-cycle pulse is expanded as a Fourier integral of such modes, $u_1 = \int R e^{i\varphi} d\omega$, $u_2 = \int S e^{i\varphi} d\omega$.
- We report $\partial_z R$ and $\partial_z S$ into $\partial_z u_1$, and get the linear coupling terms.
- Finally, we get the system of two coupled modified Korteweg-de Vries (mKdV) equations

$$\partial_z u_1 = A\partial_t^3 u_1 + B\partial_t u_1^3 + V\partial_t u_1 + C\partial_t u_2 + D\partial_t^3 u_2,$$

$$\partial_z u_2 = A\partial_t^3 u_2 + B\partial_t u_2^3 + V\partial_t u_2 + C\partial_t u_1 + D\partial_t^3 u_1,$$

- We seek for a solution as

$$u = R(z)f_1(x)e^{i\varphi} + S(z)f_2(x)e^{i\varphi},$$

i.e., two interacting modes.

- Multiplying this by f_1 and integrating over all x removes the x -dependency of f_1 , f_2 and we get

$$\partial_z R = \partial_z S = \frac{i\omega_g}{2\omega} \frac{(K_c - K_g) I_2}{1 + I_1} (R + S),$$

- The few-cycle pulse is expanded as a Fourier integral of such modes, $u_1 = \int R e^{i\varphi} d\omega$, $u_2 = \int S e^{i\varphi} d\omega$.
- We report $\partial_z R$ and $\partial_z S$ into $\partial_z u_1$, and get the linear coupling terms.
- Finally, we get the system of two coupled modified Korteweg-de Vries (mKdV) equations

$$\partial_z u_1 = A\partial_t^3 u_1 + B\partial_t u_1^3 + V\partial_t u_1 + C\partial_t u_2 + D\partial_t^3 u_2,$$

$$\partial_z u_2 = A\partial_t^3 u_2 + B\partial_t u_2^3 + V\partial_t u_2 + C\partial_t u_1 + D\partial_t^3 u_1,$$

- An analogous procedure, treating the nonlinear term as a perturbation, yields a evolution equation

$$\partial_z u_1 = \mathcal{L}(u_1) + \partial_t [I_3 u_1^3 + I_4 (3u_1^2 u_2 + u_2^3)].$$

- The integrals involved are $I_3 = \int_{-\infty}^{\infty} B_\alpha f_1^4 dx$, $I_4 = \int_{-\infty}^{\infty} B_\alpha f_1^3 f_2 dx$.
- The complete final system is

$$\begin{aligned}\partial_z u_1 &= A\partial_t^3 u_1 + B\partial_t u_1^3 + V\partial_t u_1 \\ &\quad + C\partial_t u_2 + D\partial_t^3 u_2 + E\partial_t (3u_1^2 u_2 + u_2^3) \\ \partial_z u_2 &= A\partial_t^3 u_2 + B\partial_t u_2^3 + V\partial_t u_2 \\ &\quad + C\partial_t u_1 + D\partial_t^3 u_1 + E\partial_t (3u_1 u_2 + u_1^3)\end{aligned}$$

H. Leblond, and S. Terniche, *Phys. Rev. A* **93**, 043839 (2016)

- The complete final system is

$$\begin{aligned}\partial_z u_1 &= A\partial_t^3 u_1 + B\partial_t u_1^3 + V\partial_t u_1 \\ &\quad + C\partial_t u_2 + D\partial_t^3 u_2 + E\partial_t (3u_1^2 u_2 + u_2^3)\end{aligned}$$

$$\begin{aligned}\partial_z u_2 &= A\partial_t^3 u_2 + B\partial_t u_2^3 + V\partial_t u_2 \\ &\quad + C\partial_t u_1 + D\partial_t^3 u_1 + E\partial_t (3u_1 u_2 + u_1^3)\end{aligned}$$

- We evidence
 - a standard linear coupling term,
 - a linear coupling term based on dispersion,
 - a nonlinear coupling term

H. Leblond, and S. Terniche, *Phys. Rev. A* **93**, 043839 (2016)

- The complete final system is

$$\begin{aligned}\partial_z u_1 = & A\partial_t^3 u_1 + B\partial_t u_1^3 + V\partial_t u_1 \\ & + C\partial_t u_2 + D\partial_t^3 u_2 + E\partial_t (3u_1^2 u_2 + u_2^3)\end{aligned}$$

$$\begin{aligned}\partial_z u_2 = & A\partial_t^3 u_2 + B\partial_t u_2^3 + V\partial_t u_2 \\ & + C\partial_t u_1 + D\partial_t^3 u_1 + E\partial_t (3u_1 u_2 + u_1^3)\end{aligned}$$

- We evidence
 - a standard **linear coupling** term,
 - a linear **coupling** term **based on dispersion**,
 - a **nonlinear coupling** term

H. Leblond, and S. Terniche, *Phys. Rev. A* **93**, 043839 (2016)

- The complete final system is

$$\begin{aligned}\partial_z u_1 &= A\partial_t^3 u_1 + B\partial_t u_1^3 + V\partial_t u_1 \\ &\quad + C\partial_t u_2 + D\partial_t^3 u_2 + E\partial_t (3u_1^2 u_2 + u_2^3) \\ \partial_z u_2 &= A\partial_t^3 u_2 + B\partial_t u_2^3 + V\partial_t u_2 \\ &\quad + C\partial_t u_1 + D\partial_t^3 u_1 + E\partial_t (3u_1 u_2 + u_1^3)\end{aligned}$$

- We evidence
 - a standard **linear coupling** term,
 - a linear **coupling** term **based on dispersion**,
 - a **nonlinear coupling** term

H. Leblond, and S. Terniche, *Phys. Rev. A* **93**, 043839 (2016)

- The complete final system is

$$\begin{aligned}\partial_z u_1 &= A\partial_t^3 u_1 + B\partial_t u_1^3 + V\partial_t u_1 \\ &\quad + C\partial_t u_2 + D\partial_t^3 u_2 + E\partial_t (3u_1^2 u_2 + u_2^3) \\ \partial_z u_2 &= A\partial_t^3 u_2 + B\partial_t u_2^3 + V\partial_t u_2 \\ &\quad + C\partial_t u_1 + D\partial_t^3 u_1 + E\partial_t (3u_1 u_2 + u_1^3)\end{aligned}$$

- We evidence
 - a standard **linear coupling** term,
 - a linear **coupling** term **based on dispersion**,
 - a **nonlinear coupling** term

H. Leblond, and S. Terniche, *Phys. Rev. A* **93**, 043839 (2016)

1 Models for few-cycle solitons

- The mKdV-sG equation
- Nonlinear widening of the linear guided modes

2 Waveguide coupling in the few-cycle regime

- Derivation of the coupling terms
- **Examples of behavior of the linear non-dispersive coupling**
- Few-cycle optical solitons in linearly coupled waveguides

3 Few cycle spatiotemporal solitons in waveguide arrays

- Formation of a solitons from a Gaussian pulse
- Two kind of solitons

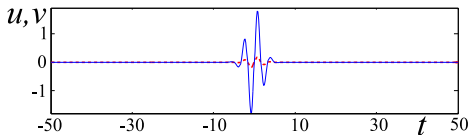
- We assume a purely linear and non-dispersive coupling

$$\begin{aligned}\partial_z u &= -\partial_t(u^3) - \partial_t^3 u - C\partial_t v, \\ \partial_z v &= -\partial_t(v^3) - \partial_t^3 v - C\partial_t u,\end{aligned}$$

- The initial data is

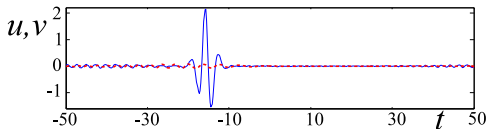
$$\begin{aligned}u &= A_u \sin(\omega_u t + \varphi_u) e^{-(t-t_u)^2/\tau_u^2}, \\ v &= A_v \sin(\omega_v t + \varphi_v) e^{-(t-t_v)^2/\tau_v^2}.\end{aligned}$$

- A **soliton** is launched in channel u and a **smaller input** in channel v .
Initial pulses:

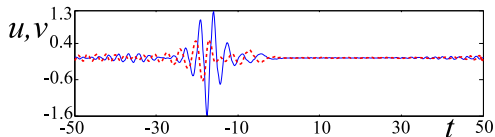


u forms a **soliton**, v is **spread out**.

- Output, uncoupled:



- Output, coupled:



A **vector soliton** forms.

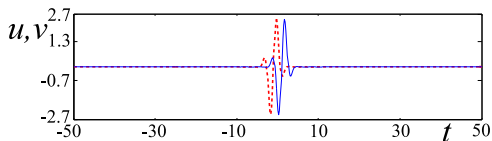
(Blue line: u , red line: v . Output at $z = 2$, coupling $C = -1$ or 0 . $A_u = 2$, $A_v = 0.2$, $\lambda_u = \lambda_v = 1$, $FWHM_u = FWHM_v = 3$, $\varphi_u = \varphi_v = 0$, $t_u = t_v = 0$.)

- Some energy is transferred from one channel to the other.

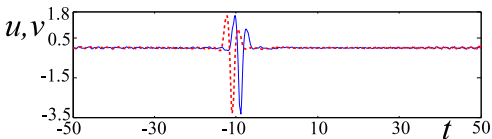
- **Mutual trapping** of solitons:

It can occur if their **center don't coincide** initially

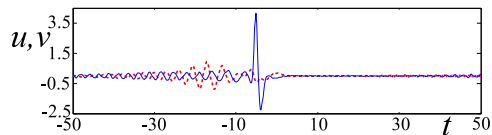
Initial pulses:



- Output, uncoupled:



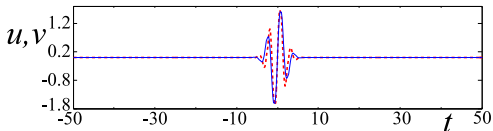
- Output, coupled:



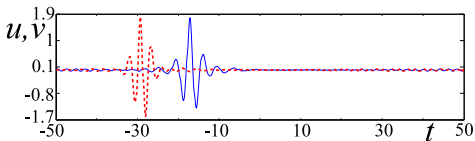
Blue line: u , red line: v . Output at $z = 2$, coupling $C = -1$ or 0 . $A_u = A_v = 3$, $\lambda_u = \lambda_v = 1$, $FWHM_u = FWHM_v = 2$, $\varphi_u = \varphi_v = 0$, $t_u = 1$, $t_v = -1$).

- Two solitons with different frequencies can lock together.

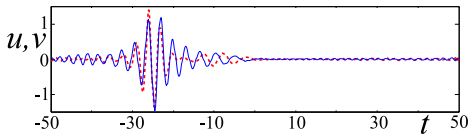
Initial pulses:



- Output, uncoupled:



- Output, coupled:



Blue line: u , red line: v . Output at $z = 2$, coupling $C = -1$ or 0 . $A_u = A_v = 1.8$, $\lambda_u = 1$, $\lambda_v = 0.8$, $FWHM_u = FWHM_v = 3$, $\varphi_u = \varphi_v = 0$, $t_u = t_v = 0$.

1 Models for few-cycle solitons

- The mKdV-sG equation
- Nonlinear widening of the linear guided modes

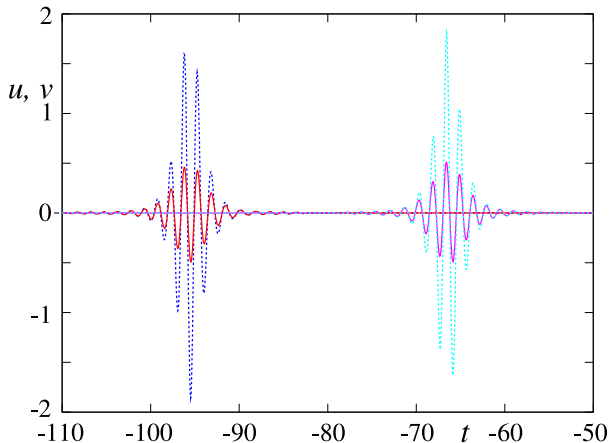
2 Waveguide coupling in the few-cycle regime

- Derivation of the coupling terms
- Examples of behavior of the linear non-dispersive coupling
- Few-cycle optical solitons in linearly coupled waveguides

3 Few cycle spatiotemporal solitons in waveguide arrays

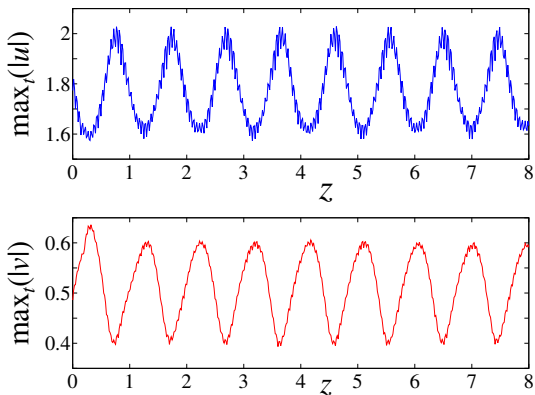
- Formation of a solitons from a Gaussian pulse
- Two kind of solitons

- We look for stationary states (vector solitons) in this model
- The "stationary" states oscillate with t and z : .
few-cycle solitons are **breathers**.
- A **typical example** of few-cycle vector soliton



(Dotted lines: u , solid lines: v . Left: at $z = 0$, right: at $z = 60$. $\langle A_u \rangle = 1.837$).

- Evolution of soliton's maximum amplitude during propagation.



Soliton with $\langle A_u \rangle = 1.789$.

- Two types of oscillations:
 - **Fast:** phase - group velocity mismatch
 - **Slower:** periodic energy exchange, as in linear regime

- Consider now the coupled equations in the **linearized** case.
- The **monochromatic** solutions are

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix} e^{-i(\omega t + b\omega^3 z)},$$

- With, due to **coupling**,

$$A = u_0 \cos c\omega z + i v_0 \sin c\omega z,$$

$$B = v_0 \cos c\omega z + i u_0 \sin c\omega z.$$

- The maximum amplitude and the power density of the wave oscillate with spatial frequency $c\omega/\pi = \sigma_0 = 1.326$.

- Consider now the coupled equations in the **linearized** case.
- The **monochromatic** solutions are

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix} e^{-i(\omega t + b\omega^3 z)},$$

- With, due to **coupling**,

$$A = u_0 \cos c\omega z + i v_0 \sin c\omega z,$$

$$B = v_0 \cos c\omega z + i u_0 \sin c\omega z.$$

- The maximum amplitude and the power density of the wave oscillate with spatial frequency $c\omega/\pi = \sigma_0 = 1.326$.

- Consider now the coupled equations in the **linearized** case.
- The **monochromatic** solutions are

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix} e^{-i(\omega t + b\omega^3 z)},$$

- With, due to **coupling**,

$$A = u_0 \cos c\omega z + iv_0 \sin c\omega z,$$

$$B = v_0 \cos c\omega z + iu_0 \sin c\omega z.$$

- The maximum amplitude and the power density of the wave oscillate with spatial frequency $c\omega/\pi = \sigma_0 = 1.326$.

- Consider now the coupled equations in the **linearized** case.
- The **monochromatic** solutions are

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix} e^{-i(\omega t + b\omega^3 z)},$$

- With, due to **coupling**,

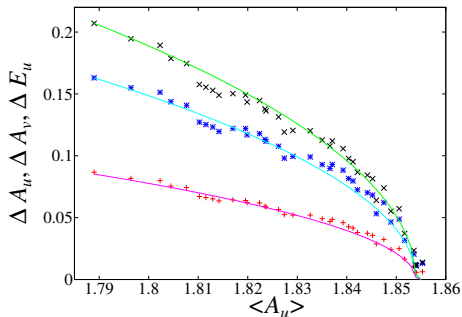
$$A = u_0 \cos c\omega z + iv_0 \sin c\omega z,$$

$$B = v_0 \cos c\omega z + iu_0 \sin c\omega z.$$

- The maximum amplitude and the power density of the wave oscillate with spatial frequency $c\omega/\pi = \sigma_0 = 1.326$.

Oscillations of the few-cycle vector solitons

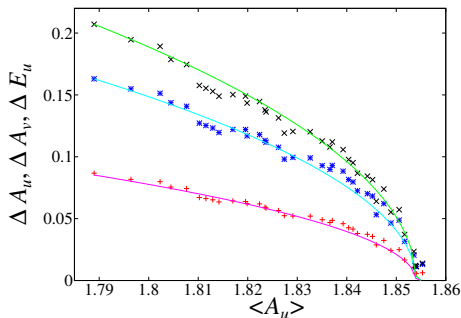
- The energies $E_u = \int u^2 dt$ and $E_v = \int v^2 dt$ oscillate almost harmonically, as $E_u = \langle E_u \rangle + \Delta E_u \sin(2\pi\sigma_a z + \phi_{E,u})$,
- The same for $A_u = \max_t(|u|)$ and $A_v = \max_t(|v|)$
- Spatial frequency $\sigma_a \in [1.06, 1.17]$, increasing with $\langle A_u \rangle$.
(linear: $\sigma_0 = 1.326$).
- Amplitudes of oscillations vs amplitude of field u



black saltires: ΔE_u ; blue stars: ΔA_u ; red crosses: ΔA_v

Oscillations of the few-cycle vector solitons

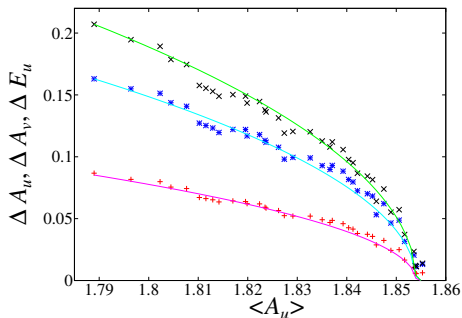
- The energies $E_u = \int u^2 dt$ and $E_v = \int v^2 dt$ oscillate almost harmonically, as $E_u = \langle E_u \rangle + \Delta E_u \sin(2\pi\sigma_a z + \phi_{E,u})$,
- The same for $A_u = \max_t(|u|)$ and $A_v = \max_t(|v|)$
- Spatial frequency $\sigma_a \in [1.06, 1.17]$, increasing with $\langle A_u \rangle$.
(linear: $\sigma_0 = 1.326$).
- Amplitudes of oscillations vs amplitude of field u



black saltires: ΔE_u ; blue stars: ΔA_u ; red crosses: ΔA_v

Oscillations of the few-cycle vector solitons

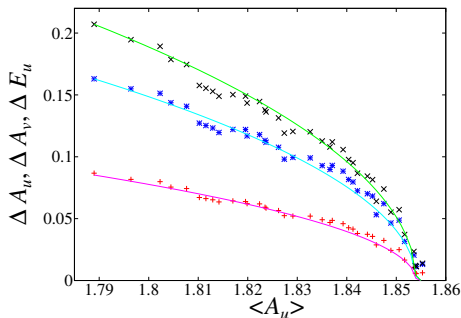
- The energies $E_u = \int u^2 dt$ and $E_v = \int v^2 dt$ oscillate almost harmonically, as $E_u = \langle E_u \rangle + \Delta E_u \sin(2\pi\sigma_a z + \phi_{E,u})$,
- The same for $A_u = \max_t(|u|)$ and $A_v = \max_t(|v|)$
- Spatial frequency $\sigma_a \in [1.06, 1.17]$, increasing with $\langle A_u \rangle$.
(linear: $\sigma_0 = 1.326$).
- Amplitudes of oscillations vs amplitude of field u



black saltires: ΔE_u ; blue stars: ΔA_u ; red crosses: ΔA_v

Oscillations of the few-cycle vector solitons

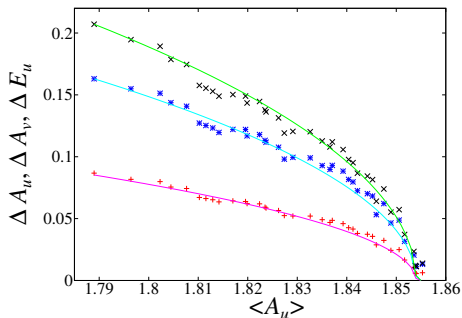
- The energies $E_u = \int u^2 dt$ and $E_v = \int v^2 dt$ oscillate almost harmonically, as $E_u = \langle E_u \rangle + \Delta E_u \sin(2\pi\sigma_a z + \phi_{E,u})$,
- The same for $A_u = \max_t(|u|)$ and $A_v = \max_t(|v|)$
- Spatial frequency $\sigma_a \in [1.06, 1.17]$, increasing with $\langle A_u \rangle$.
(linear: $\sigma_0 = 1.326$).
- Amplitudes of oscillations vs amplitude of field u



black saltires: ΔE_u ; blue stars: ΔA_u ; red crosses: ΔA_v .

Oscillations of the few-cycle vector solitons

- The energies $E_u = \int u^2 dt$ and $E_v = \int v^2 dt$ oscillate almost harmonically, as $E_u = \langle E_u \rangle + \Delta E_u \sin(2\pi\sigma_a z + \phi_{E,u})$,
- Amplitudes of oscillations vs amplitude of field u

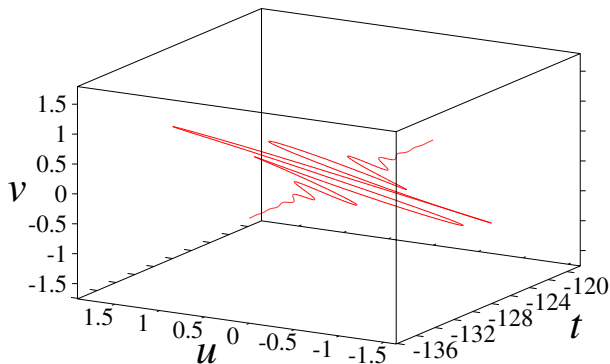


black saltires: ΔE_u ; blue stars: ΔA_u ; red crosses: ΔA_v .

- Well fitted with $\Delta E_u \simeq R\sqrt{A_0 - \langle A_u \rangle}$, etc., with $A_0 = 1.854$.

Evolution of the ratio v/u

- Almost constant vs t

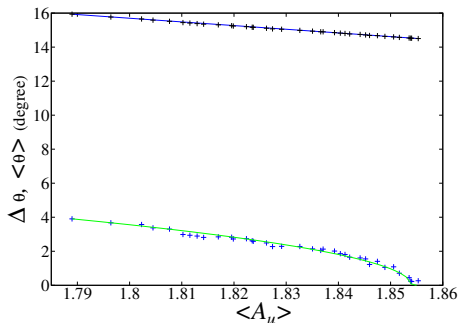


Soliton with $\langle A_u \rangle = 1.855$.

Evolution of the ratio v/u

Or $\theta = \arctan \frac{v}{u}$.

- Oscillates almost harmonically with z .
- Amplitudes of oscillations vs field u amplitude:



Black line: mean value $\langle \theta \rangle$; green line: $\Delta \theta$.

Crosses: raw numerical data; solid lines: linear or parabolic fits.

S. Terniche, H. Leblond, D. Mihalache, and A. Kellou, submitted to *Phys.Rev. A*

1 Models for few-cycle solitons

- The mKdV-sG equation
- Nonlinear widening of the linear guided modes

2 Waveguide coupling in the few-cycle regime

- Derivation of the coupling terms
- Examples of behavior of the linear non-dispersive coupling
- Few-cycle optical solitons in linearly coupled waveguides

3 Few cycle spatiotemporal solitons in waveguide arrays

- Formation of a solitons from a Gaussian pulse
- Two kind of solitons

1 Models for few-cycle solitons

- The mKdV-sG equation
- Nonlinear widening of the linear guided modes

2 Waveguide coupling in the few-cycle regime

- Derivation of the coupling terms
- Examples of behavior of the linear non-dispersive coupling
- Few-cycle optical solitons in linearly coupled waveguides

3 Few cycle spatiotemporal solitons in waveguide arrays

- Formation of a solitons from a Gaussian pulse
- Two kind of solitons

- A set of coupled waveguides within the same model, as:

$$\partial_z u_n = -a \partial_t (u_n^3) - b \partial_t^3 u_n - c \partial_t (u_{n-1} + u_{n+1}),$$

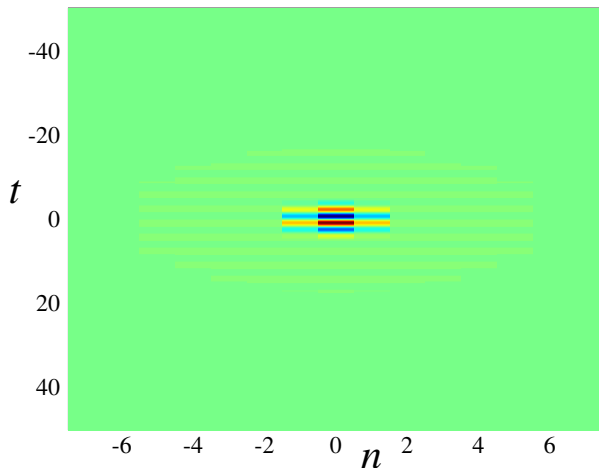
- Initial data

$$u_n(z = 0, t) = A_0 \sin(\omega t + \varphi) \exp\left(-\frac{n^2}{x^2} - \frac{t^2}{\tau^2}\right);$$

- We fix $\varphi_0 = 0$, $x = 1$, $\lambda = 1$, and we vary A_0 and τ .

Formation of a solitons from a Gaussian pulse

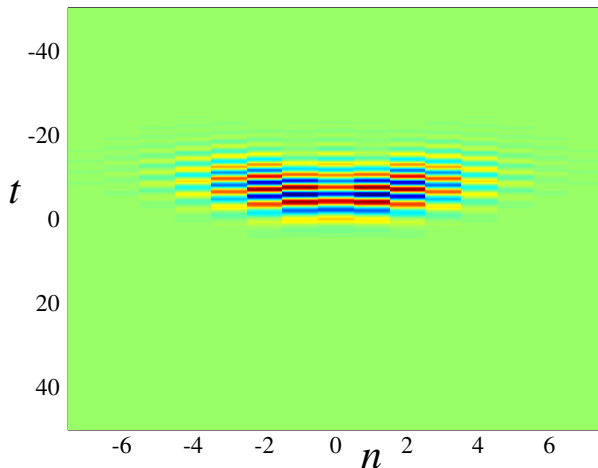
- Input



$z=0, \quad fwhm = 3.5.$

Formation of a solitons from a Gaussian pulse

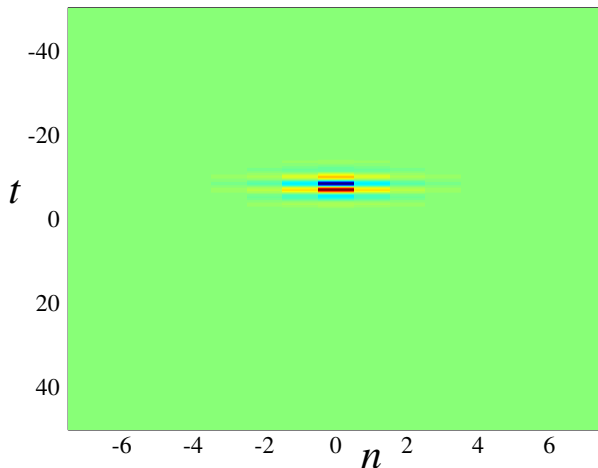
- Low amplitude output: diffraction and dispersion



$z = 0.72, A_0 = 0.2, fwhm = 3.5.$

Formation of a solitons from a Gaussian pulse

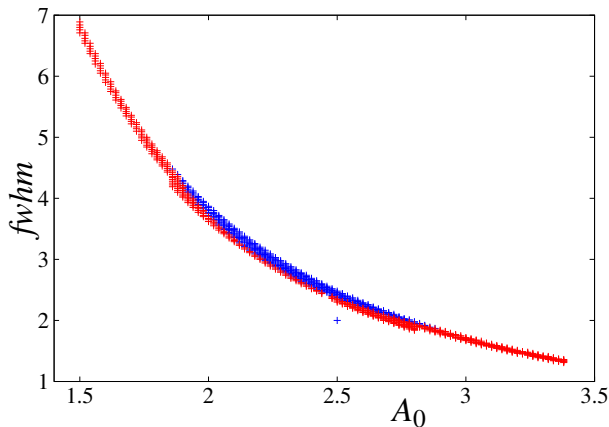
- High amplitude output: space-time localization



$z = 288, A_0 = 2.06, fwhm = 3.5.$

An energy threshold for soliton formation?

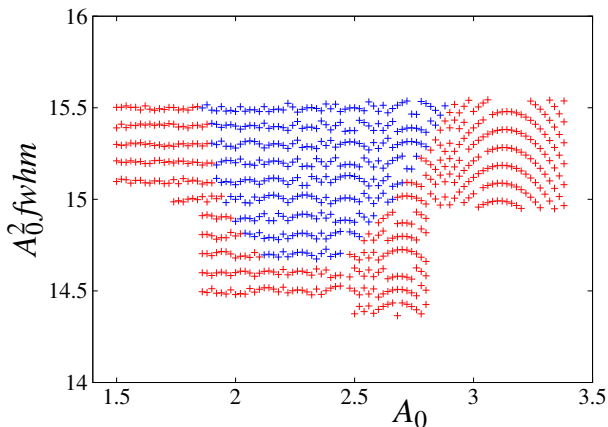
- Domain for soliton formation



Blue: soliton; red: dispersion-diffraction.

An energy threshold for soliton formation?

- Domain for soliton formation



Blue: soliton; red: dispersion-diffraction.

1 Models for few-cycle solitons

- The mKdV-sG equation
- Nonlinear widening of the linear guided modes

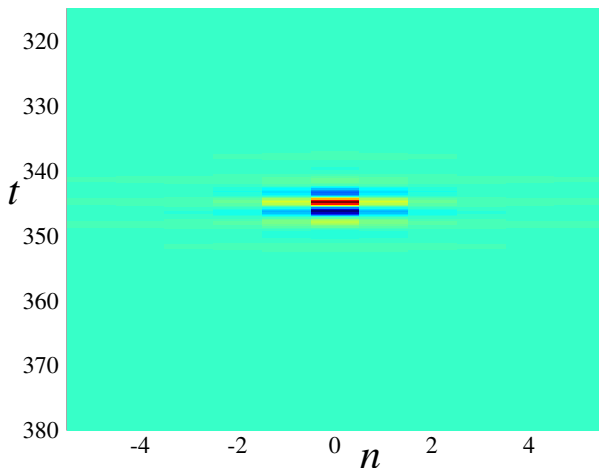
2 Waveguide coupling in the few-cycle regime

- Derivation of the coupling terms
- Examples of behavior of the linear non-dispersive coupling
- Few-cycle optical solitons in linearly coupled waveguides

3 Few cycle spatiotemporal solitons in waveguide arrays

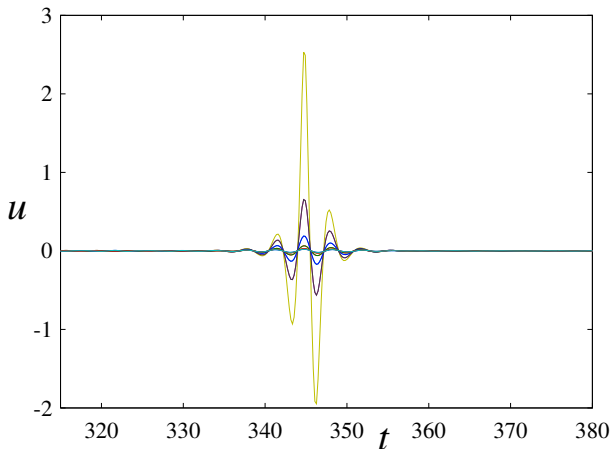
- Formation of a solitons from a Gaussian pulse
- Two kind of solitons

- Two kind of solitons: breathing and fundamental.
- Breathing soliton: { localized in space and time
oscillating wave packet



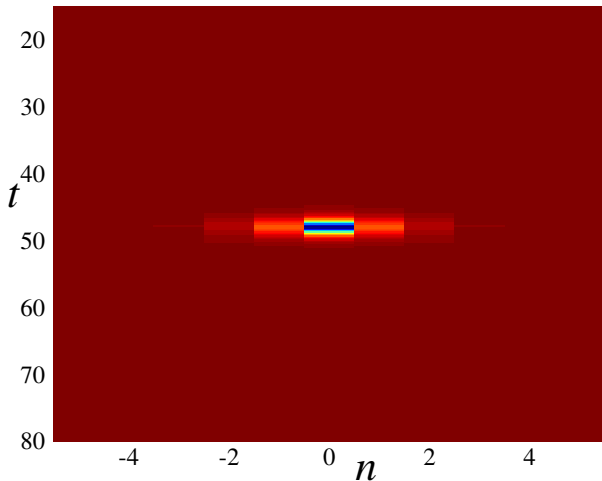
$$\max |u| = 3.1801$$

- Two kind of solitons: breathing and fundamental.
- Breathing soliton: { localized in space and time
oscillating wave packet



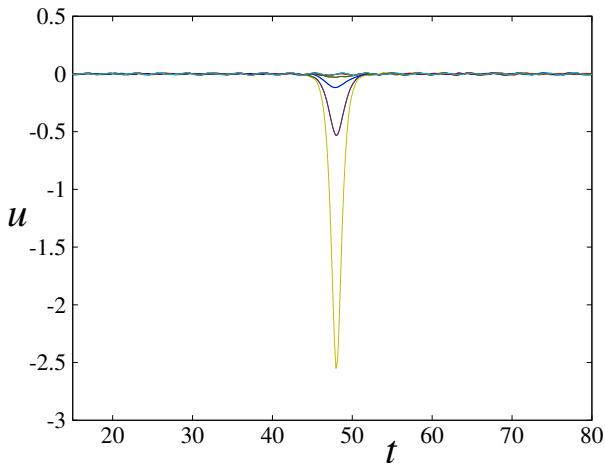
$\max |u| = 3.1801$

- Fundamental soliton: $\left\{ \begin{array}{l} \text{localized in space and time} \\ \text{single humped} \end{array} \right.$



$\max |u| = 2.5667$

- Fundamental soliton: $\left\{ \begin{array}{l} \text{localized in space and time} \\ \text{single humped} \end{array} \right.$



$$\max |u| = 2.5667$$

- Thank you for your attention.

1 Models for few-cycle solitons

- The mKdV-sG equation
- Nonlinear widening of the linear guided modes

2 Waveguide coupling in the few-cycle regime

- Derivation of the coupling terms
- Examples of behavior of the linear non-dispersive coupling
- Few-cycle optical solitons in linearly coupled waveguides

3 Few cycle spatiotemporal solitons in waveguide arrays

- Formation of a solitons from a Gaussian pulse
- Two kind of solitons