Few-cycle optical soliton propagation in coupled waveguides

Hervé Leblond, Dumitru Mihalache, David Kremer, Said Terniche

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Hervé Leblond[1], Dumitru Mihalache[2], David Kremer[3], Said Terniche[1,4]

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[2]Horia Hulubei National Institute for Physics and Nuclear Engineering, and Academy of Romanian Scientists, Bucharest.

[3]Laboratoire MOLTECH-Anjou, CNRS UMR 6200, Université d'Angers.

[4]Laboratoire Electronique Quantique, USTHB, Alger.

1. Models for few-cycle solitons
   - The mKdV-sG equation
   - Nonlinear widening of the linear guided modes

2. Waveguide coupling in the few-cycle regime
   - Derivation of the coupling terms
   - Examples of behavior of the linear non-dispersive coupling
   - Few-cycle optical solitons in linearly coupled waveguides

3. Few cycle spatiotemporal solitons in waveguide arrays
   - Formation of a solitons from a Gaussian pulse
   - Two kind of solitons

Introduction

- The shortest laser pulses: a duration of a few optical cycles.

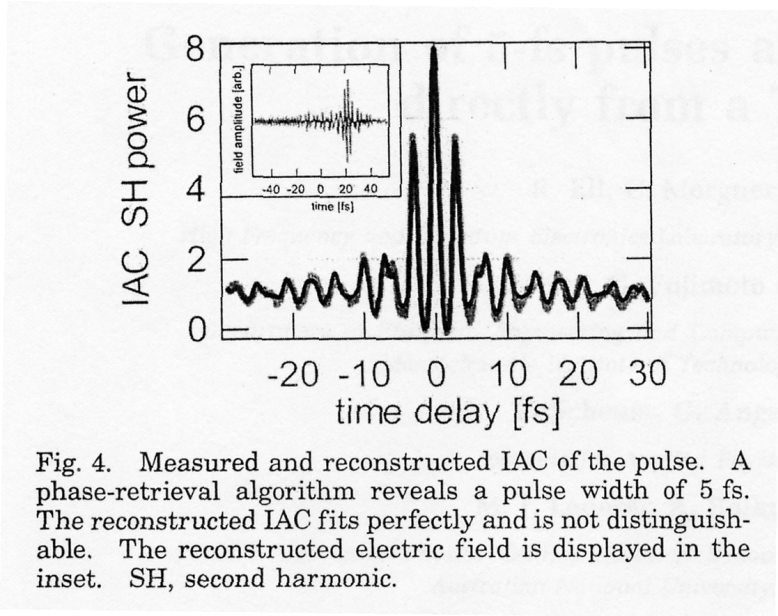


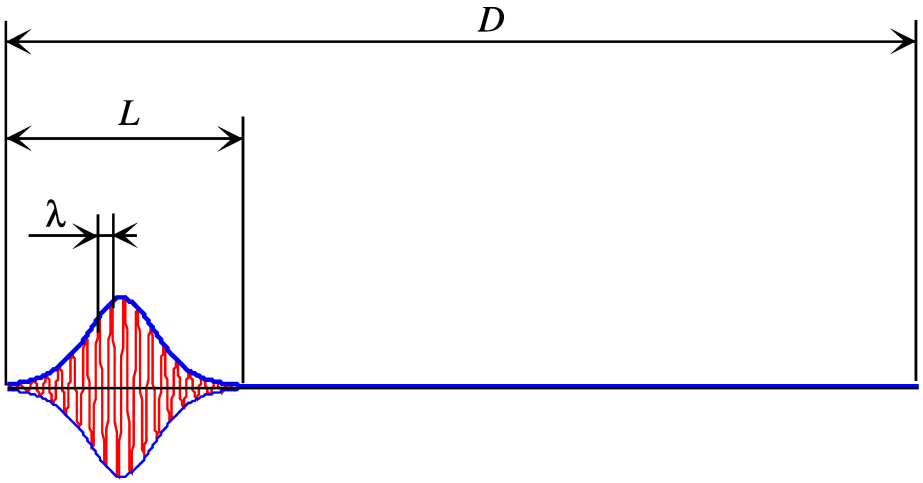
Fig. 4. Measured and reconstructed IAC of the pulse. A phase-retrieval algorithm reveals a pulse width of 5 fs. The reconstructed IAC fits perfectly and is not distinguishable. The reconstructed electric field is displayed in the inset. SH, second harmonic.

Autocorrelation trace, R. Ell et al., Optics Letters 26 (6), 373 (2001).

- Pulse duration down to a few fs. Ex. above: $5\,\mathrm{fs} = 5 \times 10^{-15}\mathrm{s}$.
- How to model the propagation of such pulses?

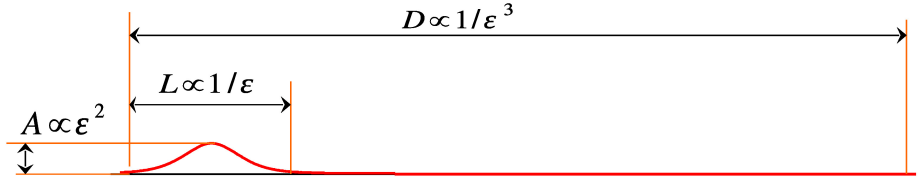
Solitary wave vs envelope solitons

- Envelope soliton: the usual optical soliton in the ps range



  - Pulse duration $L \gg \lambda$ wavelength
  - Typical model: NonLinear Schrödinger equation (NLS)

- It is a soliton if it propagates without deformation on $D \gg L$, due to nonlinearity.
  In linear regime: spread out by dispersion.
- Solitary wave soliton: the hydrodynamical soliton



  - A single oscillation
  - Typical model: Korteweg-de Vries equation (KdV)

- Few-cycle optical solitons: $L \sim \lambda$
  - The slowly varying envelope approximation is not valid

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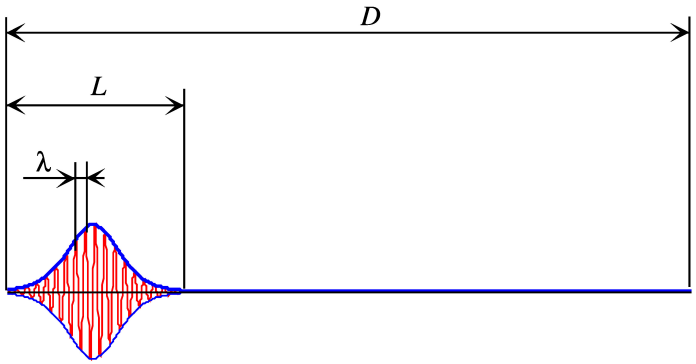

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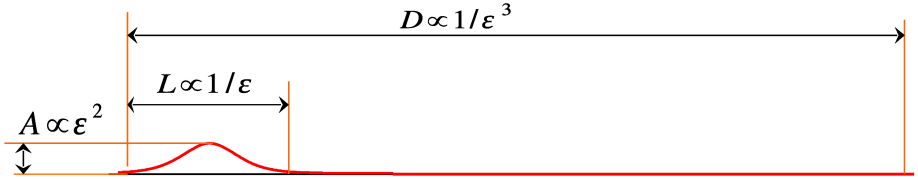

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  - We seek a different approach based on KdV-type models

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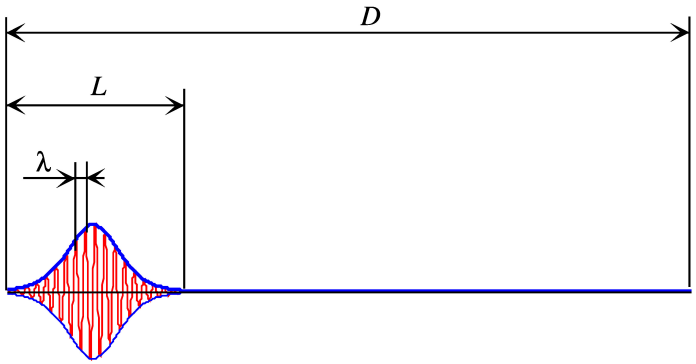

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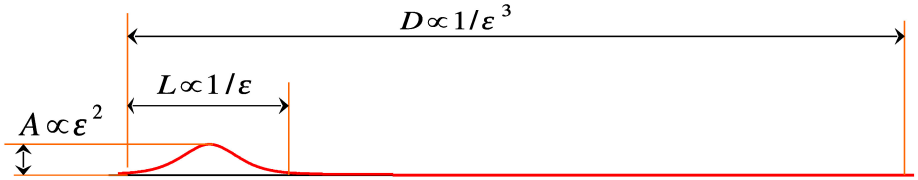

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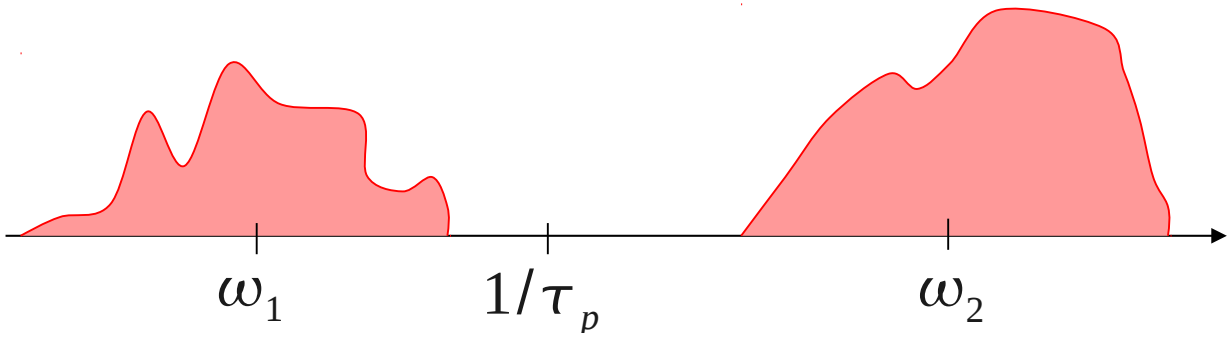
A transparent medium

- The general absorption spectrum of a transparent medium



- A simple model: A two-component medium, each component is described by a two-level model
- We assume that the transparency range is very large:

$$\omega_1 \ll (1/\tau_p) \ll \omega_2$$

(τ_p: FCP duration).

A transparent medium

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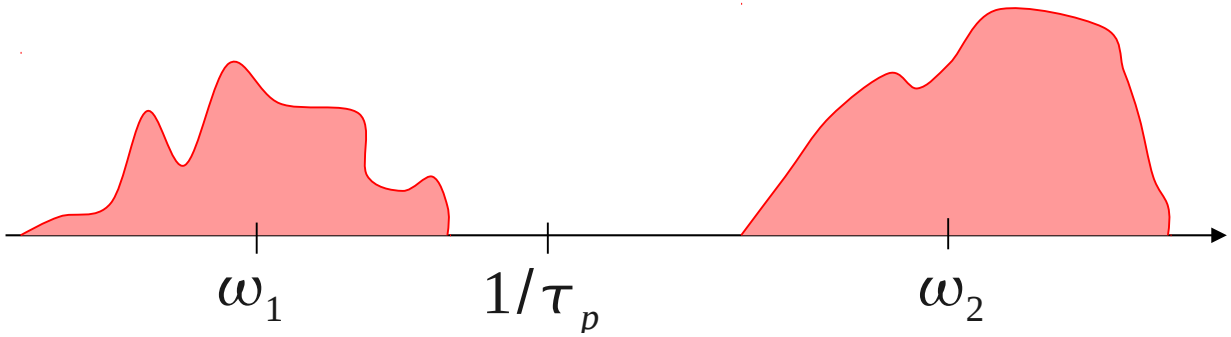

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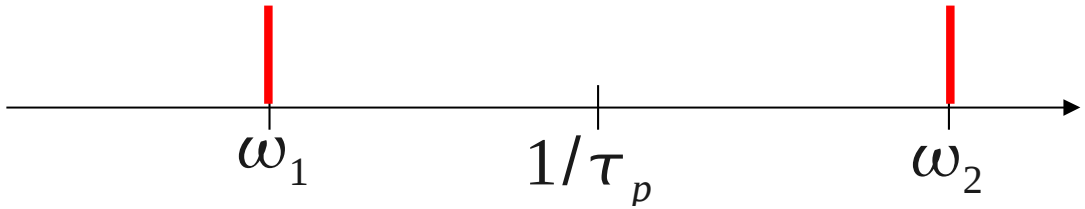

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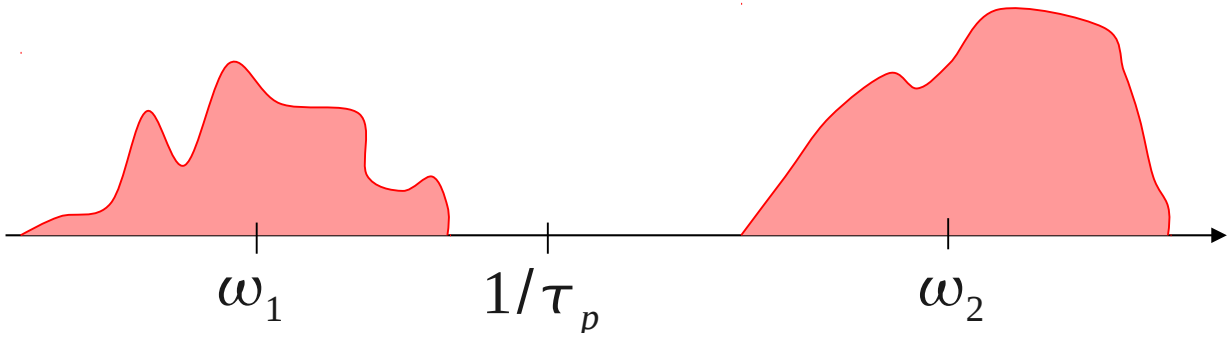

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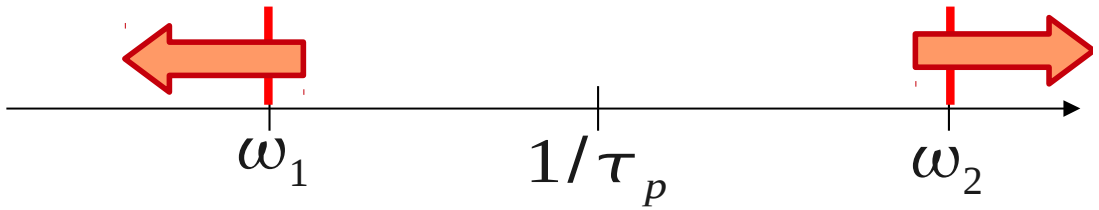

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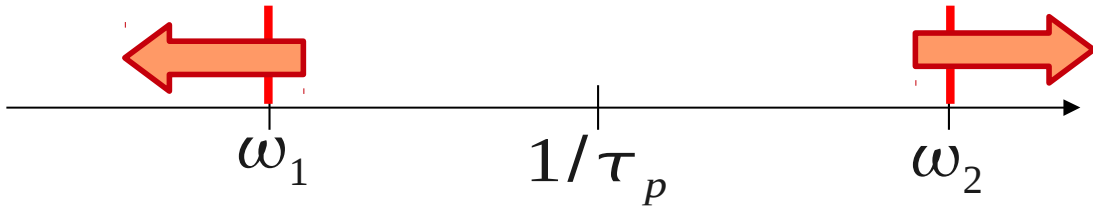

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- In a first stage, the two components are treated separately
- UV transition only, with $(1/\tau_p) \ll \omega_2$

$\Longrightarrow$ Long-wave approximation

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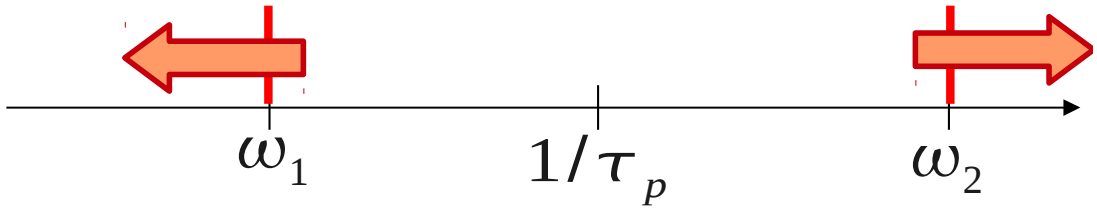

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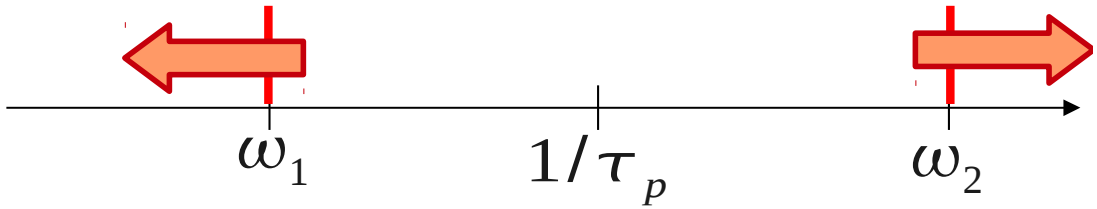

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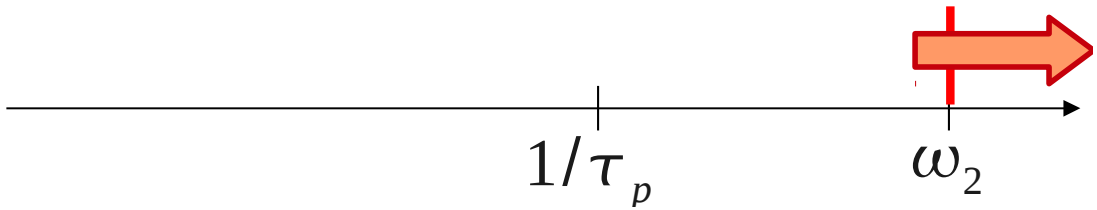

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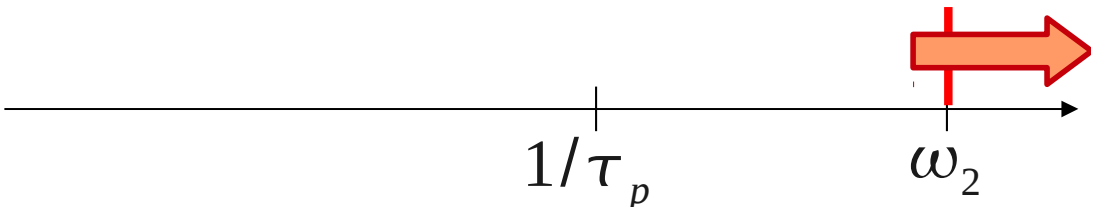

$\Longrightarrow$ Long-wave approximation
modified Korteweg-de Vries (mKdV) equation

$$\frac{\partial E}{\partial \zeta} = \frac{1}{6}\frac{d^3k}{d\omega^3}\bigg|_{\omega=0}\frac{\partial^3 E}{\partial \tau^3} - \frac{6\pi}{nc}\chi^{(3)}(\omega;\omega,\omega,-\omega)\bigg|_{\omega=0}\frac{\partial E^3}{\partial \tau}$$

H. Leblond and F. Sanchez, *Phys. Rev. A* **67**, 013804 (2003)

A transparent medium

- In a first stage, the two components are treated separately
- IR transition only, with $\omega_1 \ll (1/\tau_p)$
  $\Longrightarrow$ Short-wave approximation
- sine-Gordon (sG) equation: $\dfrac{\partial^2\psi}{\partial z \partial t} = c_1 \sin\psi$

  with $c_1 = \dfrac{w_\infty}{w_r}$: normalized initial population difference

  and $\dfrac{\partial\psi}{\partial t} = \dfrac{E}{E_r}$: normalized electric field

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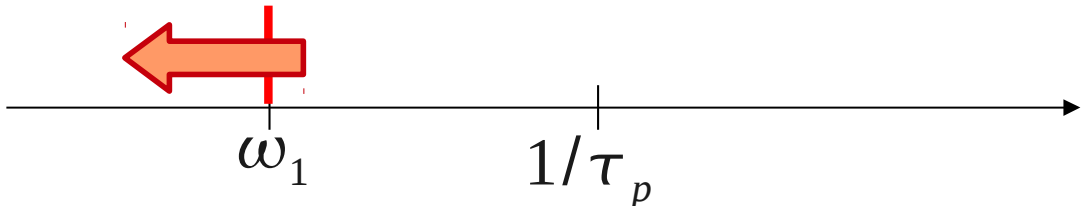

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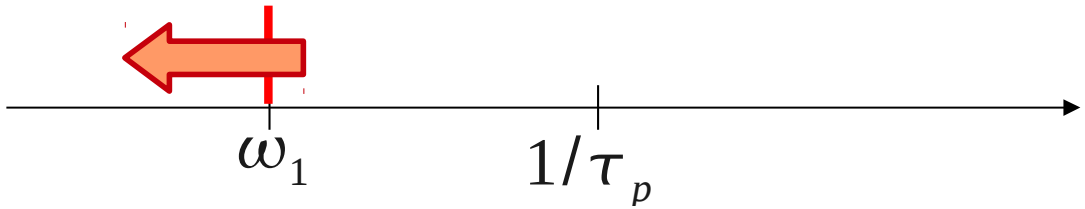

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- Then the two approximations are brought together to yield a general model:

  The mKdV-sG equation

$$\frac{\partial^2\psi}{\partial z\partial t} + c_1 \sin\psi + c_2\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial t}\right)^3 + c_3\frac{\partial^4\psi}{\partial t^4} = 0$$

- $\frac{\partial u}{\partial z} + c_1 \sin\int^t u + c_2\frac{\partial u^3}{\partial t} + c_3\frac{\partial^3 u}{\partial t^3} = 0$

  with $u = \frac{\partial\psi}{\partial t} = \frac{E}{E_r}$: normalized electric field

- Integrable by inverse scattering transform in some cases:

  ,

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A transparent medium

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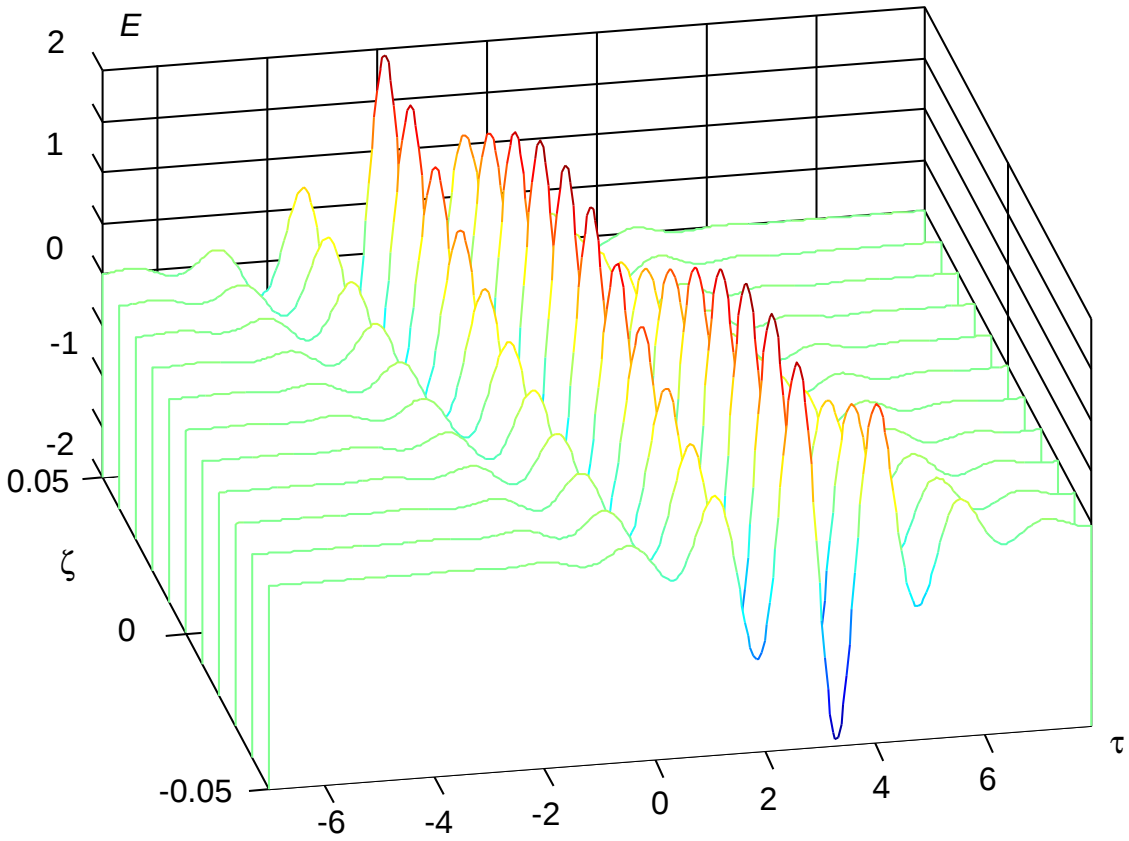
- $$\frac{\partial u}{\partial z} + c_1 \sin \int^t u + c_2 \frac{\partial u^3}{\partial t} + 2c_2 \frac{\partial^3 u}{\partial t^3} = 0$$

  with $u = \dfrac{\partial \psi}{\partial t} = \dfrac{E}{E_r}$: normalized electric field
- Integrable by inverse scattering transform in some cases:

  mKdV, sG, and $c_3 = 2c_2$.

The analytical breather solution



- A few cycle soliton:
  - Not spread out by dispersion
  - Stable
  - However, oscillates (breather)

H. Leblond, S.V. Sazonov, I.V. Mel'nikov, D. Mihalache, and F. Sanchez, *Phys. Rev. A* **74**, 063815 (2006)

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Waveguide description

- The evolution of the electric field E:
  In (1+1) dimensions:
  $\longrightarrow$ The modified Korteweg-de Vries (mKdV) equation

$$\partial_\zeta E + \beta \partial_\tau^3 E + \gamma \partial_\tau E^3 = 0$$

- Nonlinear coefficient $\gamma = \dfrac{1}{2nc}\chi^{(3)}$,
  Dispersion parameter $\beta = \dfrac{(-n'')}{2c}$,

Waveguide description

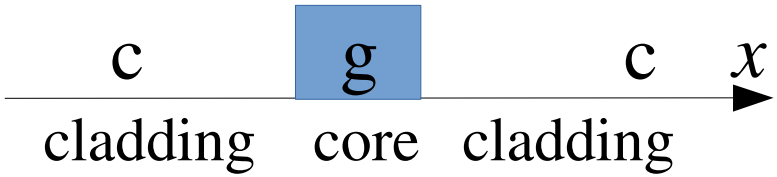
- The evolution of the electric field E:
  We generalize to (2+1) dimensions:
  $\longrightarrow$ The cubic generalized Kadomtsev-Petviashvili (CGKP) equation

$$\partial_\zeta E + \beta \partial_\tau^3 E + \gamma \partial_\tau E^3 - \frac{V}{2} \int^\tau \partial_\xi^2 E d\tau' = 0$$

- Nonlinear coefficient $\gamma = \frac{1}{2nc}\chi^{(3)}$,
  Dispersion parameter $\beta = \frac{(-n'')}{2c}$,
  Linear group velocity: $V = \frac{c}{n}$.

Waveguide description

- The evolution of the electric field E:
  A waveguide:



$\longrightarrow$ The cubic generalized Kadomtsev-Petviashvili (CGKP) equation

$$\partial_\zeta E + \beta_\alpha \partial_\tau^3 E + \gamma_\alpha \partial_\tau E^3 \qquad - \frac{V_\alpha}{2} \int^\tau \partial_\xi^2 E d\tau' = 0$$

with $\alpha = g$ in the core and $\alpha = c$ in the cladding.

- Nonlinear coefficient $\gamma_\alpha = \dfrac{1}{2 n_\alpha c} \chi_\alpha^{(3)}$,

  Dispersion parameter $\beta_\alpha = \dfrac{(-n_\alpha'')}{2c}$,

  Linear group velocity: $V_\alpha = \dfrac{c}{n_\alpha}$.

Waveguide description

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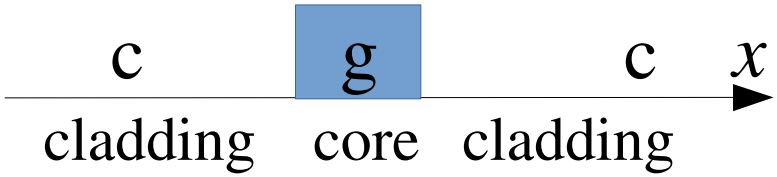

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$$\partial_\zeta E + \beta_\alpha \partial_\tau^3 E + \gamma_\alpha \partial_\tau E^3 + \frac{1}{V_\alpha}\partial_\tau E - \frac{V_\alpha}{2}\int^\tau \partial_\xi^2 E d\tau' = 0$$

with $\alpha = g$ in the core and $\alpha = c$ in the cladding.
Velocities : $V_g < V_c$

- Nonlinear coefficient $\gamma_\alpha = \dfrac{1}{2n_\alpha c}\chi_\alpha^{(3)}$,

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Waveguide description

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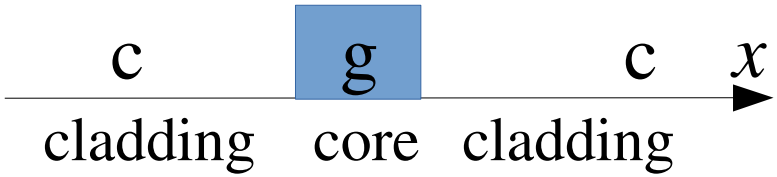

  $\longrightarrow$ The cubic generalized Kadomtsev-Petviashvili (CGKP) equation
  In dimensionless form:

$$\partial_z u = A_\alpha \partial_t^3 u + B_\alpha \partial_t u^3 + v_\alpha \partial_t u + \frac{W_\alpha}{2} \int^t \partial_x^2 u dt'$$

  with $\alpha = g$ in the core and $\alpha = c$ in the cladding.
  Relative velocities : $v_g > v_c$

- Nonlinear coefficient $\gamma_\alpha = \dfrac{1}{2n_\alpha c}\chi_\alpha^{(3)}$,

  Dispersion parameter $\beta_\alpha = \dfrac{(-n''_\alpha)}{2c}$,

  Linear group velocity: $V_\alpha = \dfrac{c}{n_\alpha}$.

Linear guided modes

- The dimensionless CGKP equation

$$\partial_z u = A_\alpha \partial_t^3 u + B_\alpha \partial_t u^3 + v_\alpha \partial_t u + \frac{W_\alpha}{2} \int^t \partial_x^2 u dt'$$

- We linearize and seek for solutions of the form $u = f(x) \exp[i(\omega t - k_z z)]$.
- Waveguide dispersion relation: $\tan(k_x a) = \dfrac{\kappa}{k_x}$
- Mode profiles of the form $\begin{cases} f = R\cos(k_x x) & \text{in the core} \\ f = Ce^{-\kappa|x|} & \text{in the cladding} \end{cases}$

Linear guided modes

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- Material dispersion relation: $k_{x,\alpha}^2 = \frac{2\omega}{V_\alpha} \left(A_\alpha \omega^3 - k_z - \omega v_\alpha \right)$
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- Waveguide dispersion relation: $\tan(k_x a) = \dfrac{\kappa}{k_x}$
- Mode profiles of the form $\begin{cases} f = R\cos(k_x x) & \text{in the core} \\ f = Ce^{-\kappa|x|} & \text{in the cladding} \end{cases}$
- Guiding condition: $A_2\omega^3 - \omega v_2 < k_z < A_1\omega^3$

As in the usual symmetric dielectric planar waveguide.

Nonlinear propagation in linear guide

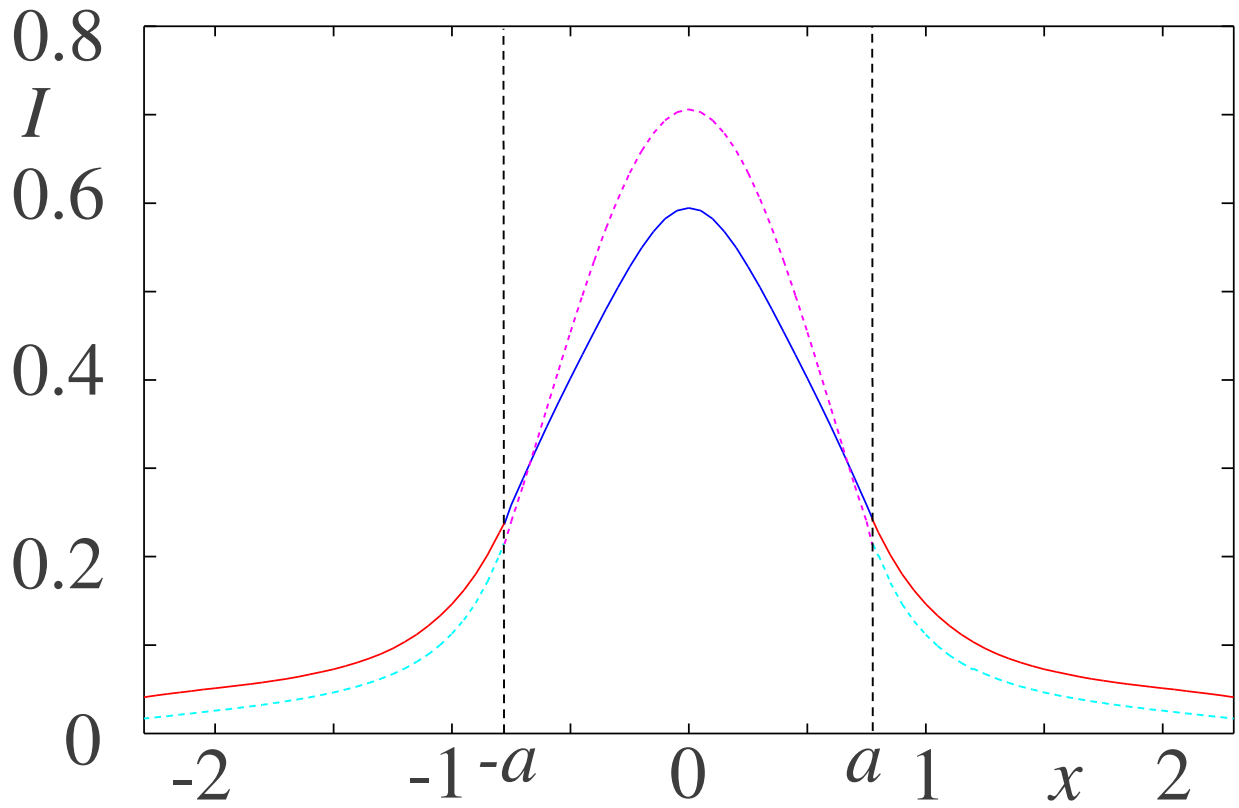
- We solve the CGKP equation starting from

$$u(x,t,z=0) = A\cos(\omega t) f(x) e^{-t^2/w^2},$$

- $f(x) = \begin{cases} \cos(k_x x), & \text{for } |x| \leq a, \\ Ce^{-\kappa|x|}, & \text{for } |x| > a, \end{cases}$ is a linear mode profile
- Normalized coefficients $A_1 = A_2 = B_1 = B_2 = W_1 = W_2 = 1$,

  $\longrightarrow$ we assume that
  - Temporal compression occurs
  - Spatial defocusing occurs, (else it collapses!)
  - Dispersion and nonlinearity are identical in core and cladding.

- Guided wave profiles



(Normalized so that the total power is 1. $v_2 = 3$, $w = 2$.)

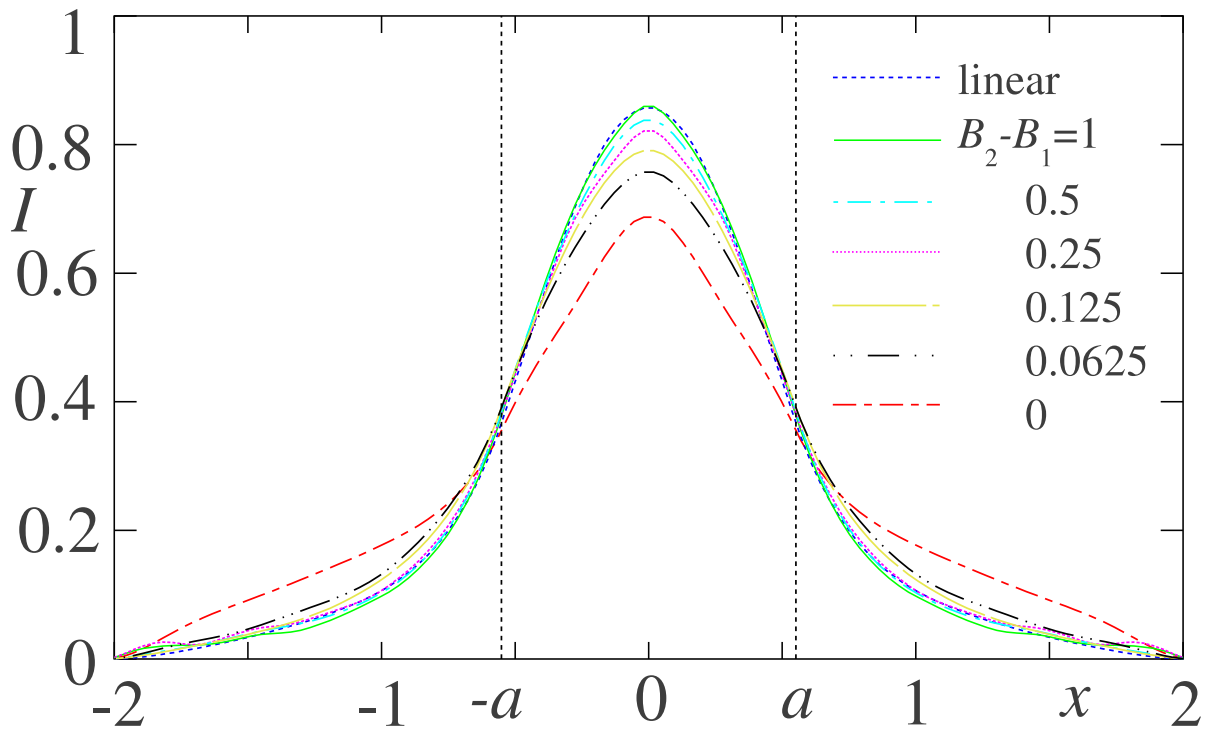
- The pulse is less confined in nonlinear (blue, and red) than in linear (pink and cyan) regime.

Nonlineary may reduce confinement. Indeed:

- Waveguiding is due to total internal reflection.
  $\longrightarrow$ when the wave propagates too fast in the cladding,
  it cannot match the field oscillations in the core.
- In the few-cycle regime,
  not only linear, but also nonlinear velocity (large and positive here).
- The nonlinear velocity is larger in the core
  and creates a nonlinear variation of the index.
- It may compensate the linear part, and reduce the confinement.
- Important in very narrow guides,
  where linear confinement itself is low.

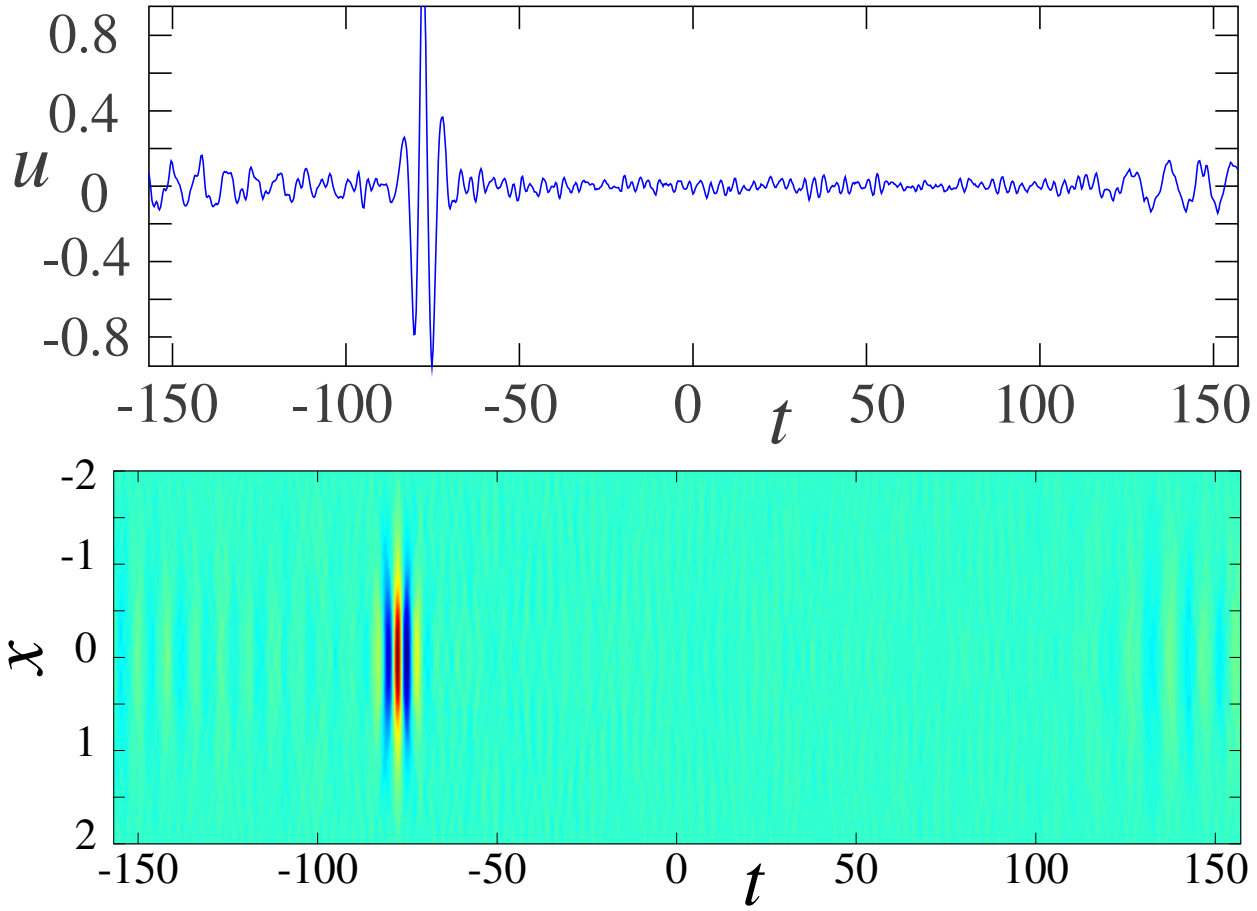
Nonlinear waveguide

- Wave guided and confined
  by using nonlinear velocity:
  a higher nonlinear coefficient in the cladding than that in the core.



Guided profiles of the nonlinear waveguide. Normalized so that the total power is 1.

Two-cycle soliton of the nonlinear waveguide



$B_2 - B_1 = 1.$

H. Leblond and D. Mihalache, *Phys. Rev. A* **88**, 023840 (2013)

1. Models for few-cycle solitons
   - The mKdV-sG equation
   - Nonlinear widening of the linear guided modes

2. Waveguide coupling in the few-cycle regime
   - Derivation of the coupling terms
   - Examples of behavior of the linear non-dispersive coupling
   - Few-cycle optical solitons in linearly coupled waveguides

3. Few cycle spatiotemporal solitons in waveguide arrays
   - Formation of a solitons from a Gaussian pulse
   - Two kind of solitons

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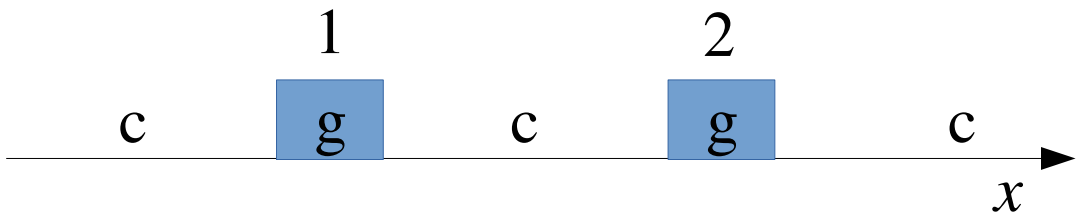
- 2D waveguiding structure: two cores 1 and 2 and dielectric cladding



- The generalized Kadomtsev-Petviashvili (GKP) equation (dimensionless)

$$\partial_z u = A_\alpha \partial_t^3 u + B_\alpha \partial_t u^3 + V_\alpha \partial_t u + \frac{w_\alpha}{2} \int^t \partial_x^2 u dt,$$

$\alpha = g$ in the cores 1 and 2, $\alpha = c$ in the cladding.

- Individual waveguides, in the linearized model ($B_\alpha = 0$):
  The field is $u = Rf_j(x)e^{i(\omega t - \beta z)} = Rf_j(x)e^{i\varphi}$,
  f_j, ($j = 1, 2$): guided mode profile, R: amplitude.
- We seek for a solution as

$$u = R(z)f_1(x)e^{i\varphi} + S(z)f_2(x)e^{i\varphi},$$

  i.e., two interacting modes.
- We report it into the GKP equation and get
  – in the cladding $\partial_z R f_1 + \partial_z S f_2 = 0$
  – in guide 1 $\partial_z R f_1 + \partial_z S f_2 = \frac{i w_g}{2\omega}(K_c - K_g) S f_2$
  and so on.
- Multiplying this by f_1 and integrating over all x
  removes the x-dependency of f_1, f_2 and we get

$$\partial_z R = \partial_z S = \frac{i w_g}{2\omega} \frac{(K_c - K_g) I_2}{1 + I_1} (R + S),$$

- Involve overlap integrals $I_1 = \int_{-\infty}^{\infty} f_1 f_2 dx$, $I_2 = \int_{g_1} f_1 f_2 dx = \int_{g_2} f_1 f_2 dx$
  ("$\int_{g_j} \cdot dx$" is the integral over the core $j = 1$ or 2.)

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- The few-cycle pulse is expanded as a Fourier integral of such modes, $u_1 = \int R e^{i\varphi} d\omega$, $u_2 = \int S e^{i\varphi} d\omega$.
- We report $\partial_z R$ and $\partial_z S$ into $\partial_z u_1$, and get the linear coupling terms.
- Finally, we get the system of two coupled modified Korteweg-de Vries (mKdV) equations

$$\partial_z u_1 = A \partial_t^3 u_1 + B \partial_t u_1^3 + V \partial_t u_1 + C \partial_t u_2 + D \partial_t^3 u_2,$$

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Nonlinear coupling

- An analogous procedure, treating the nonlinear term as a perturbation, yields a evolution equation

$$\partial_z u_1 = \mathcal{L}(u_1) + \partial_t \left[I_3 u_1^3 + I_4 \left(3u_1^2 u_2 + u_2^3 \right) \right].$$

- The integrals involved are $I_3 = \int_{-\infty}^{\infty} B_\alpha f_1^4 dx$, $I_4 = \int_{-\infty}^{\infty} B_\alpha f_1^3 f_2 dx$.
- The complete final system is

$$\begin{aligned}
\partial_z u_1 &= A\partial_t^3 u_1 + B\partial_t u_1^3 + V\partial_t u_1 \\
&\quad + C\partial_t u_2 + D\partial_t^3 u_2 + E\partial_t \left(3u_1^2 u_2 + u_2^3 \right) \\
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H. Leblond, and S. Terniche, *Phys. Rev. A* **93**, 043839 (2016)

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- We evidence
  - a standard linear coupling term,
  - a linear coupling term based on dispersion,
  - a nonlinear coupling term

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H. Leblond, and S. Terniche, *Phys. Rev. A* **93**, 043839 (2016)

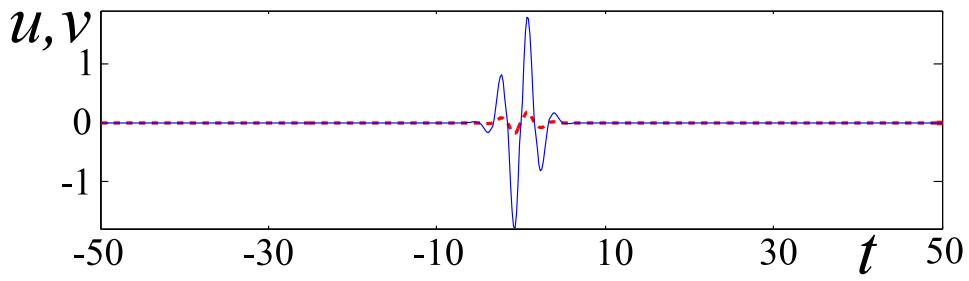
- We assume a purely linear and non-dispersive coupling

$$\begin{aligned} \partial_z u &= -\partial_t(u^3) - \partial_t^3 u - C\partial_t v, \\ \partial_z v &= -\partial_t(v^3) - \partial_t^3 v - C\partial_t u, \end{aligned}$$

- The initial data is

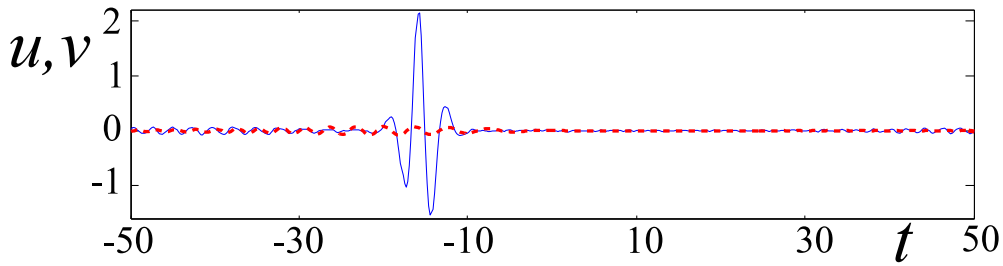
$$\begin{aligned} u &= A_u \sin\left(\omega_u t + \varphi_u\right) e^{-(t-t_u)^2/\tau_u^2}, \\ v &= A_v \sin\left(\omega_v t + \varphi_v\right) e^{-(t-t_v)^2/\tau_v}. \end{aligned}$$

- A soliton is launched in channel u and a smaller input in channel v.
  Initial pulses:

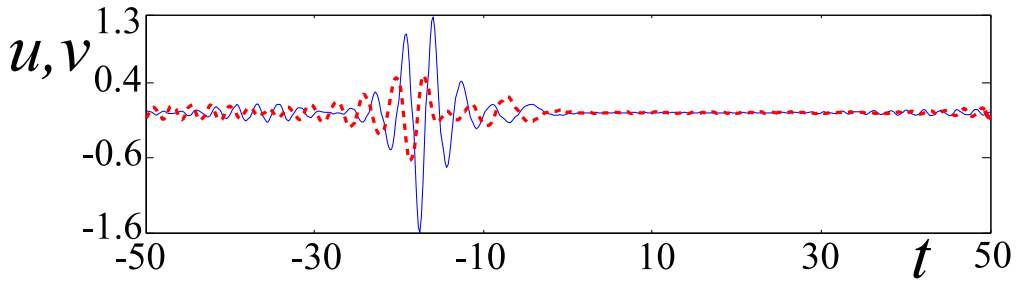


  u forms a soliton, v is spread out.

- Output, uncoupled:



- Output, coupled:

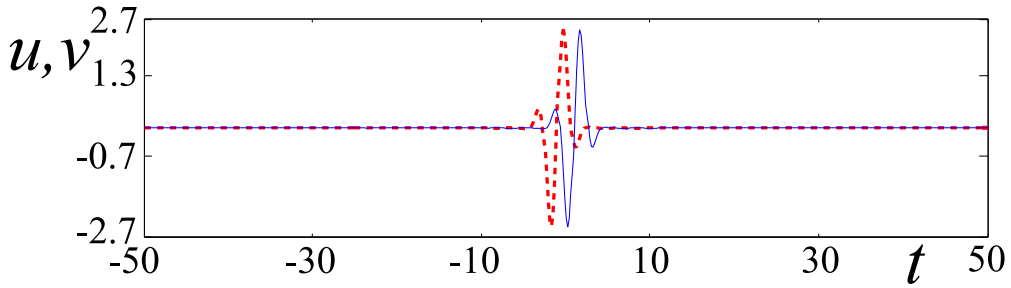


  A vector soliton forms.

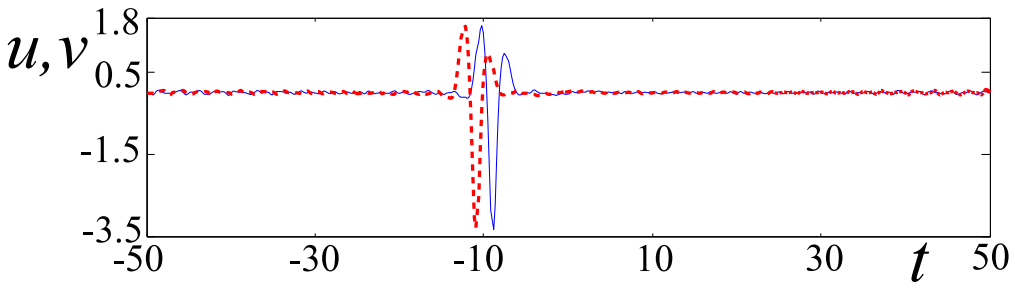
  (Blue line: u, red line: v. Output at $z = 2$,coupling $C = -1$ or 0. $A_u = 2$, $A_v = 0.2$, $\lambda_u = \lambda_v = 1$, $FWHM_u = FWHM_v = 3$, $\varphi_u = \varphi_v = 0$, $t_u = t_v = 0$.

- Some energy is transferred form one channel to the other.

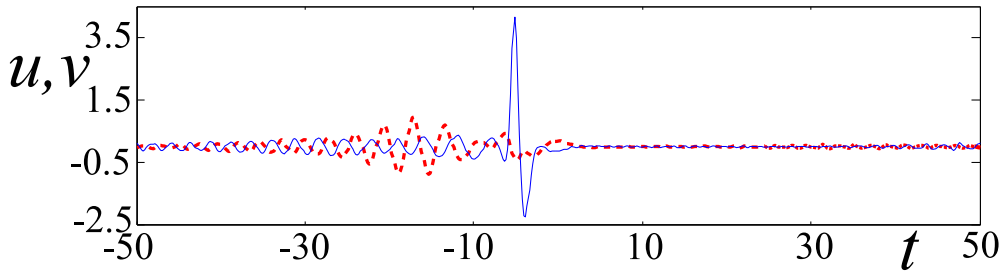
- Mutual trapping of solitons:
  It can occur if their center don't coincide initially
  Initial pulses:



- Output, uncoupled:

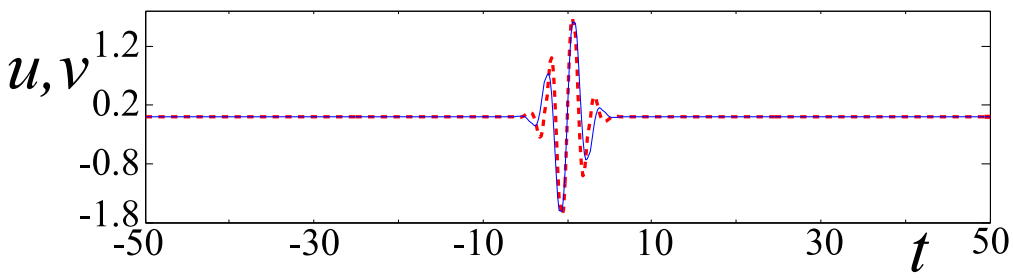


- Output, coupled:

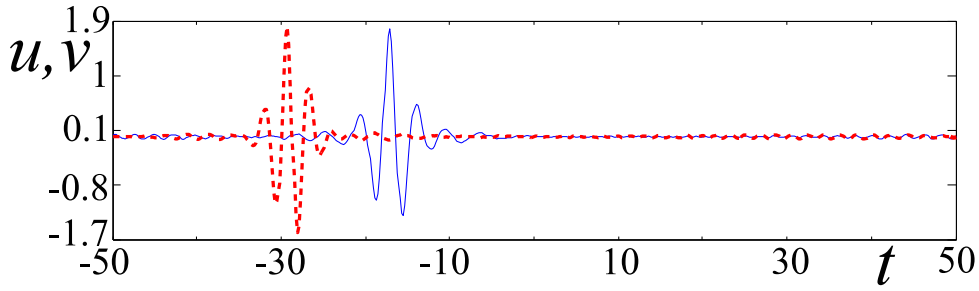


Blue line: u, red line: v. Output at $z = 2$, coupling $C = -1$ or 0. $A_u = A_v = 3$, $\lambda_u = \lambda_v = 1$, $FWHM_u = FWHM_v = 2$, $\varphi_u = \varphi_v = 0$, $t_u = 1$, $t_v = -1$).

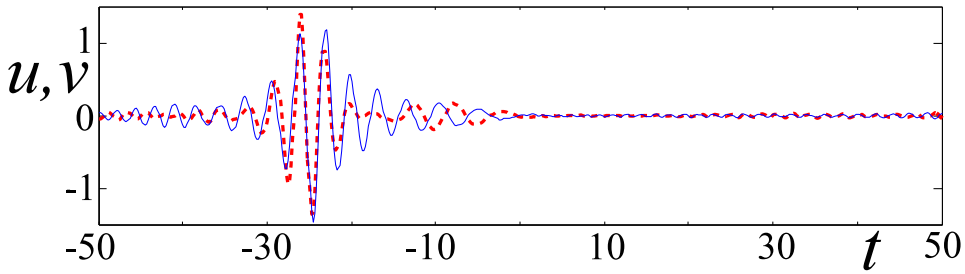
- Two solitons with different frequencies can lock together.
  Initial pulses:



- Output, uncoupled:



- Output, coupled:



Blue line: u, red line: v. Output at $z = 2$, coupling $C = -1$ or 0. $A_u = A_v = 1.8$, $\lambda_u = 1$, $\lambda_v = 0.8$, $FWHM_u = FWHM_v = 3$, $\varphi_u = \varphi_v = 0$, $t_u = t_v = 0$).

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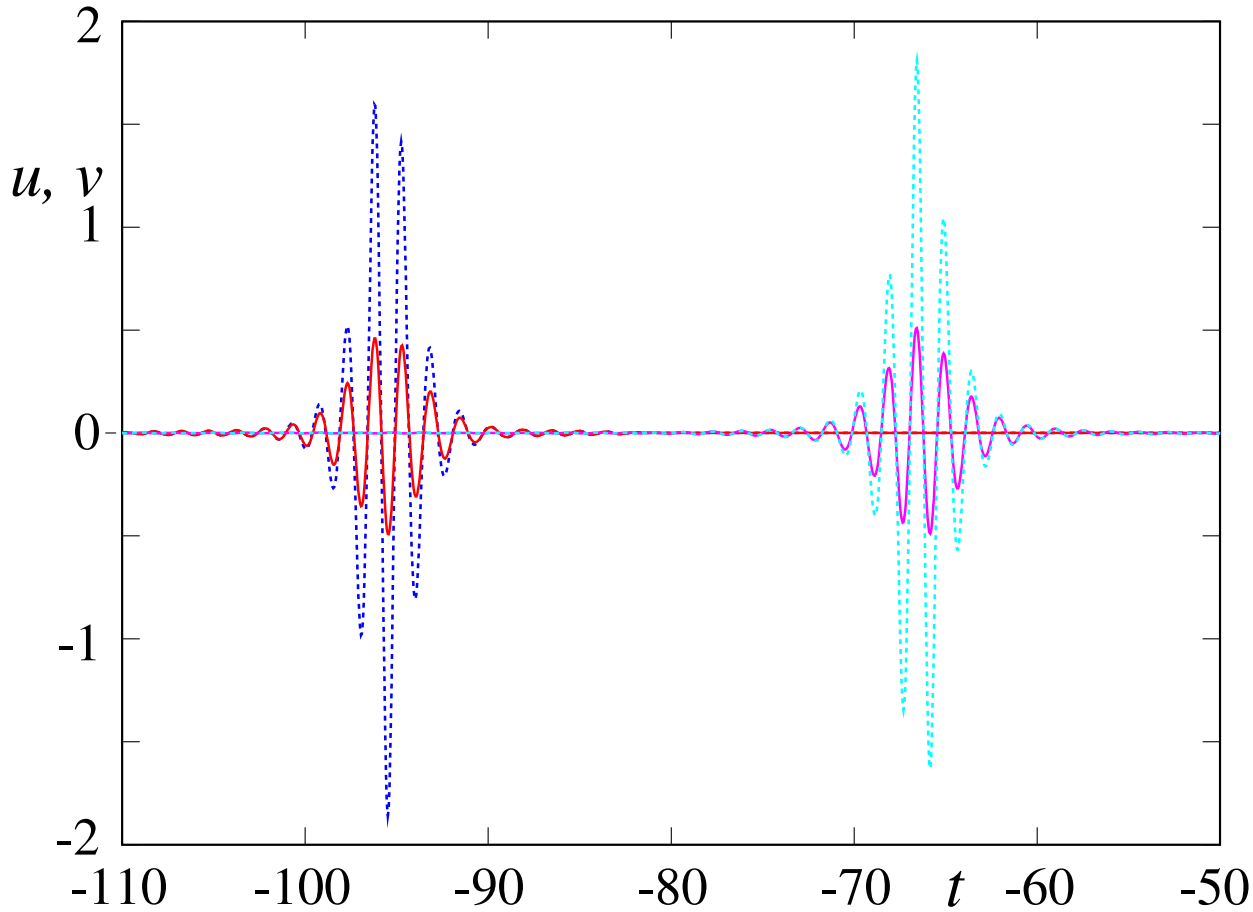
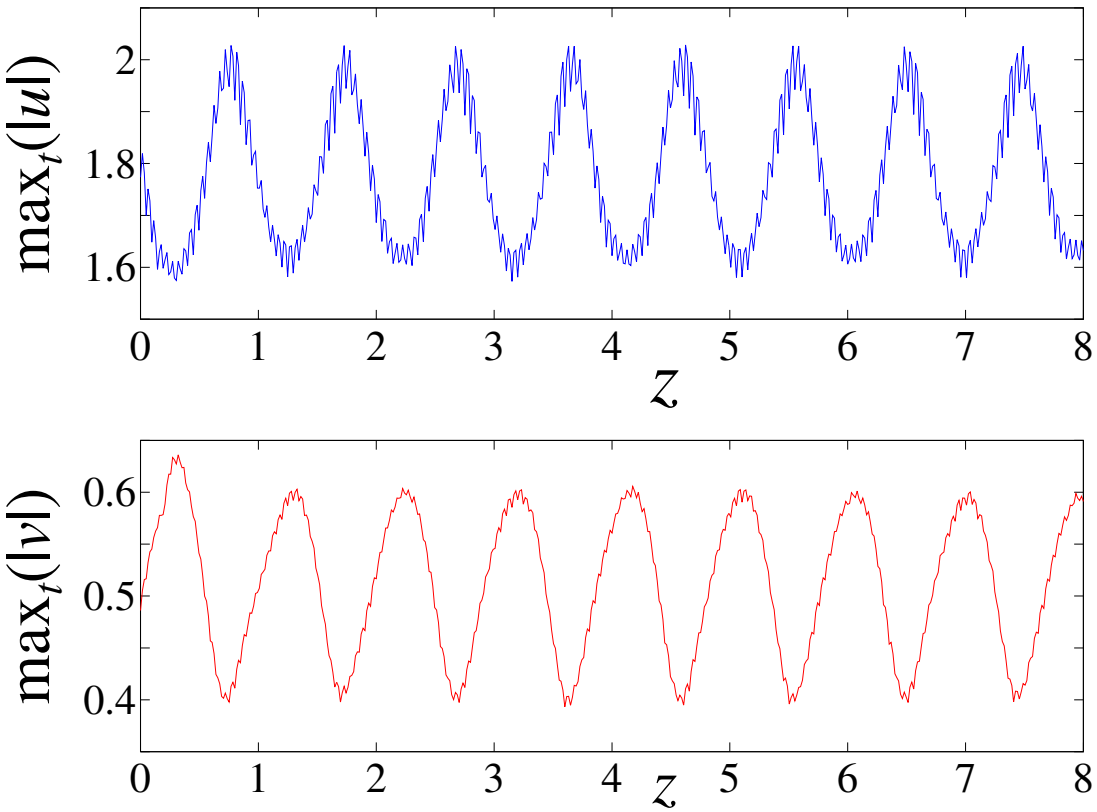
- We look for stationary states (vector solitons) in this model
- The "stationary" states oscillate with t and z: .
  few-cycle solitons are breathers.
- A typical example of few-cycle vector soliton



(Dotted lines: u, solid lines: v. Left: at $z = 0$, right: at $z = 60$. $< A_u > = 1.8377$).

- Evolution of soliton's maximum amplitude during propagation.



Soliton with $< A_u >= 1.789$.

- Two types of oscillations:
  - Fast: phase - group velocity mismatch
  - Slower: periodic energy exchange, as in linear regime

- Consider now the coupled equations in the linearized case.
- The monochromatic solutions are

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix} e^{-i(\omega t + b\omega^3 z)},$$

- With, due to coupling,

$$A = u_0 \cos c\omega z + i v_0 \sin c\omega z,$$
$$B = v_0 \cos c\omega z + i u_0 \sin c\omega z.$$

- The maximum amplitude and the power density of the wave oscillate with spatial frequency $c\omega/\pi = \sigma_0 = 1.326$.

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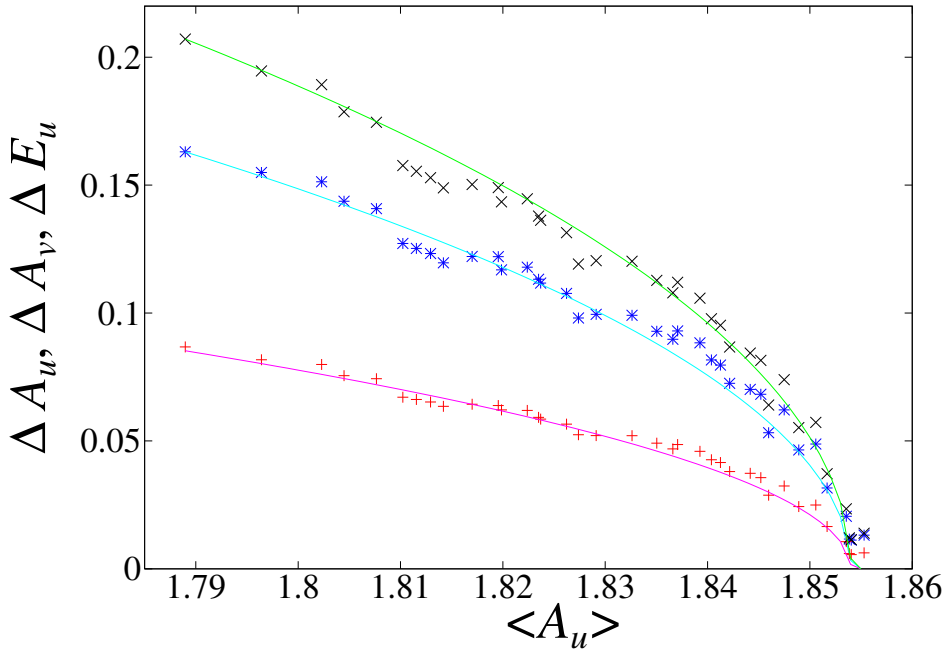
Oscillations of the few-cycle vector solitons

- The energies $E_u = \int u^2 dt$ and $E_v = \int v^2 dt$ oscillate almost harmonically, as $E_u =< E_u > +\Delta E_u \sin(2\pi\sigma_a z + \phi_{E,u})$,
- The same for $A_u = \max_t(|u|)$ and $A_v = \max_t(|v|)$
- Spatial frequency $\sigma_a \in [1.06,1.17]$, increasing with $< A_u >$.

  (linear: $\sigma_0 = 1.326$).
- Amplitudes of oscillations vs amplitude of field u



black saltires: ΔE_u; blue stars: ΔA_u; red crosses: ΔA_v

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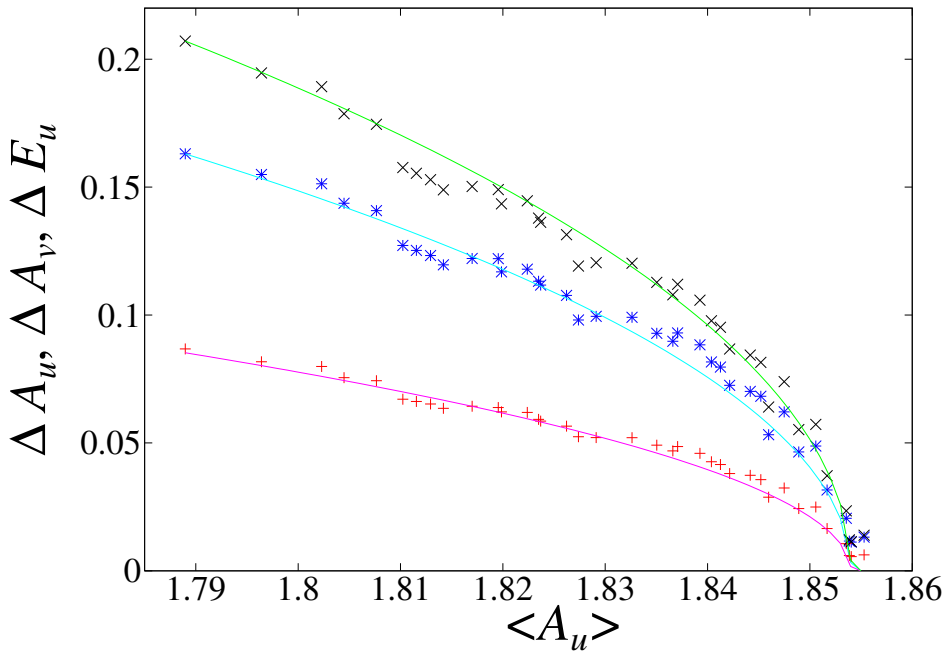

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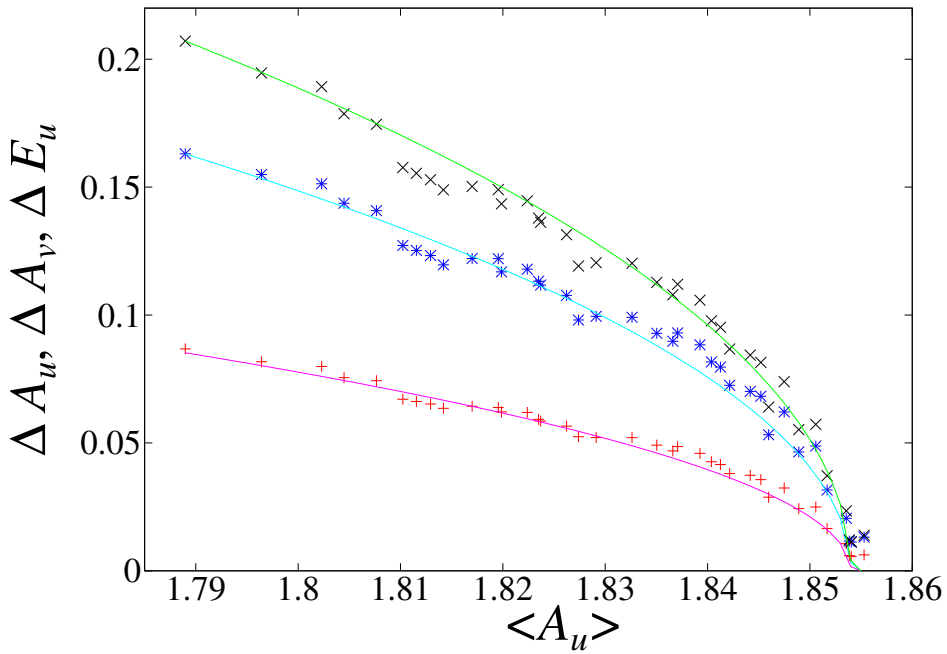

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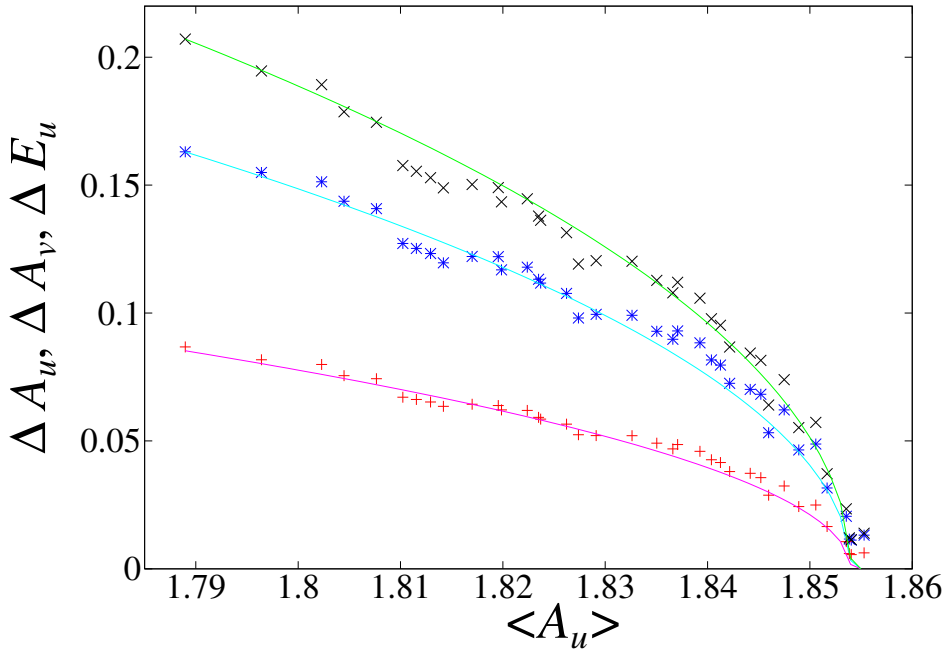

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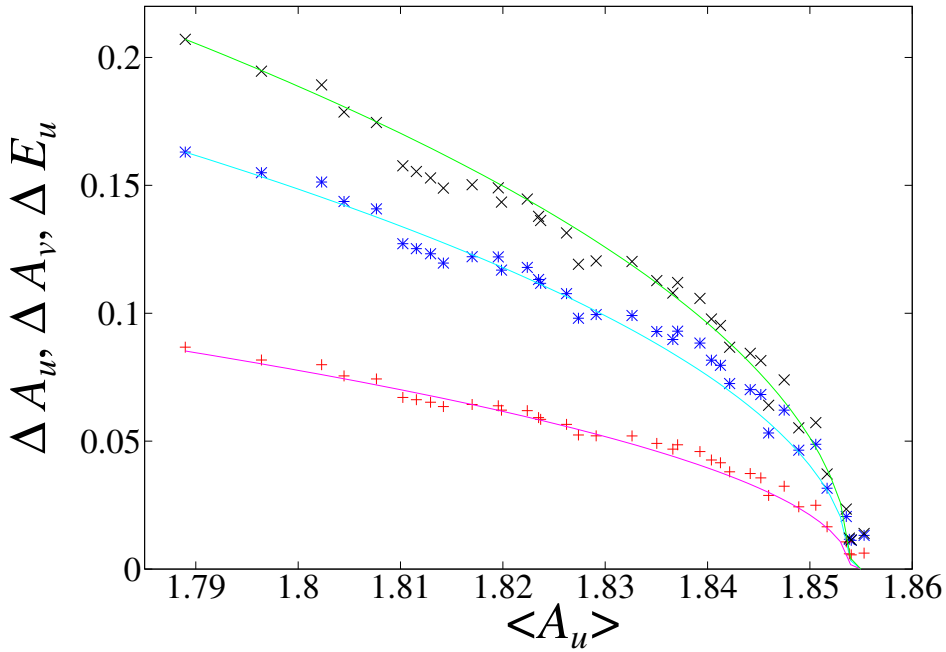

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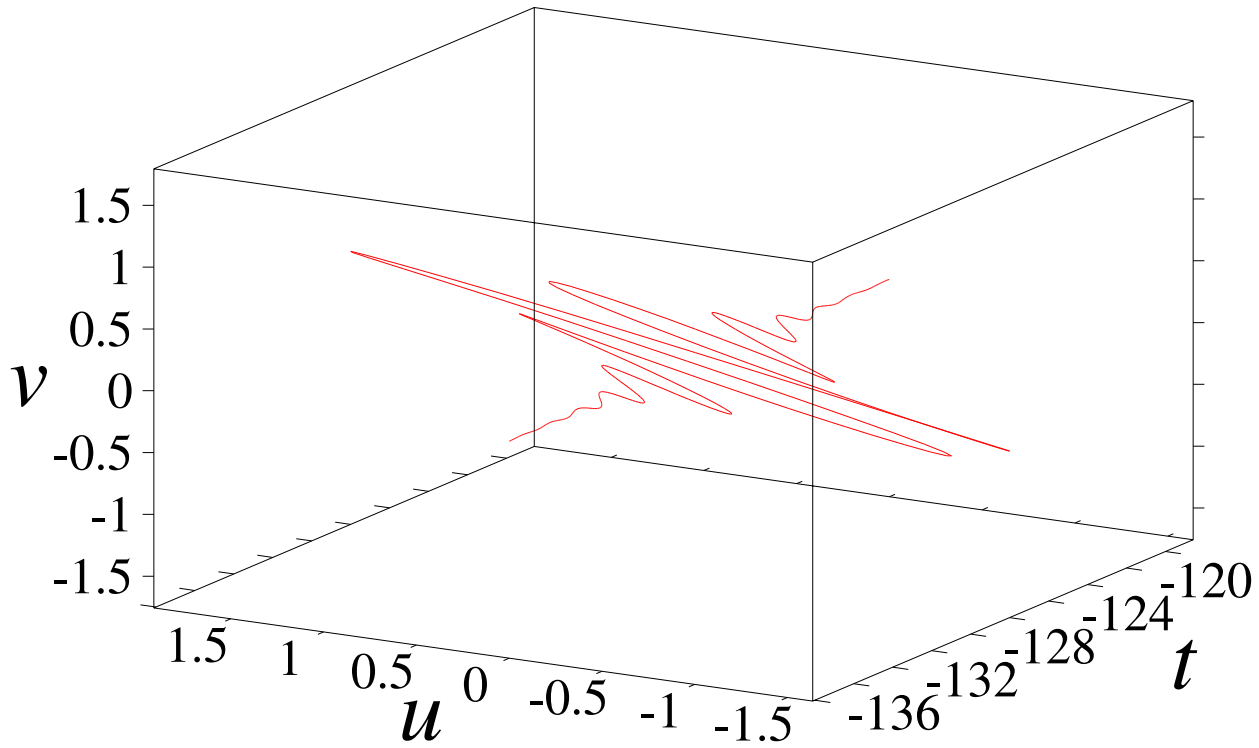
- Well fitted with $\Delta E_u \simeq R\sqrt{A_0 - < A_u >}$, etc., with $A_0 = 1.854$.

Evolution of the ratio v/u

- Almost constant vs t

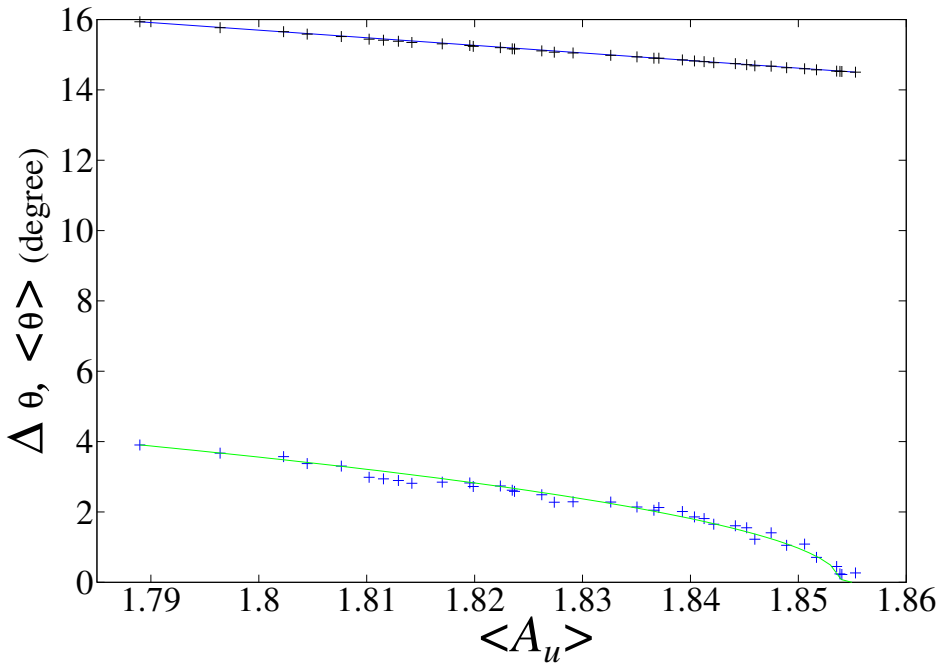


Soliton with $< A_u >= 1.855$.

Evolution of the ratio v/u

Or $\theta = \arctan \frac{v}{u}$.

- Oscillates almost harmonically with z.
- Amplitudes of oscillations vs field u amplitude:



Black line: mean value $< \theta >$; green line: $\Delta\theta$.

Crosses: raw numerical data; solid lines: linear or parabolic fits.

S. Terniche, H. Leblond, D. Mihalache, and A. Kellou, submitted to *Phys.Rev. A*

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- A set of coupled waveguides within the same model, as:

$$\partial_z u_n = -a\partial_t(u_n^3) - b\partial_t^3 u_n - c\partial_t\left(u_{n-1} + u_{n+1}\right),$$

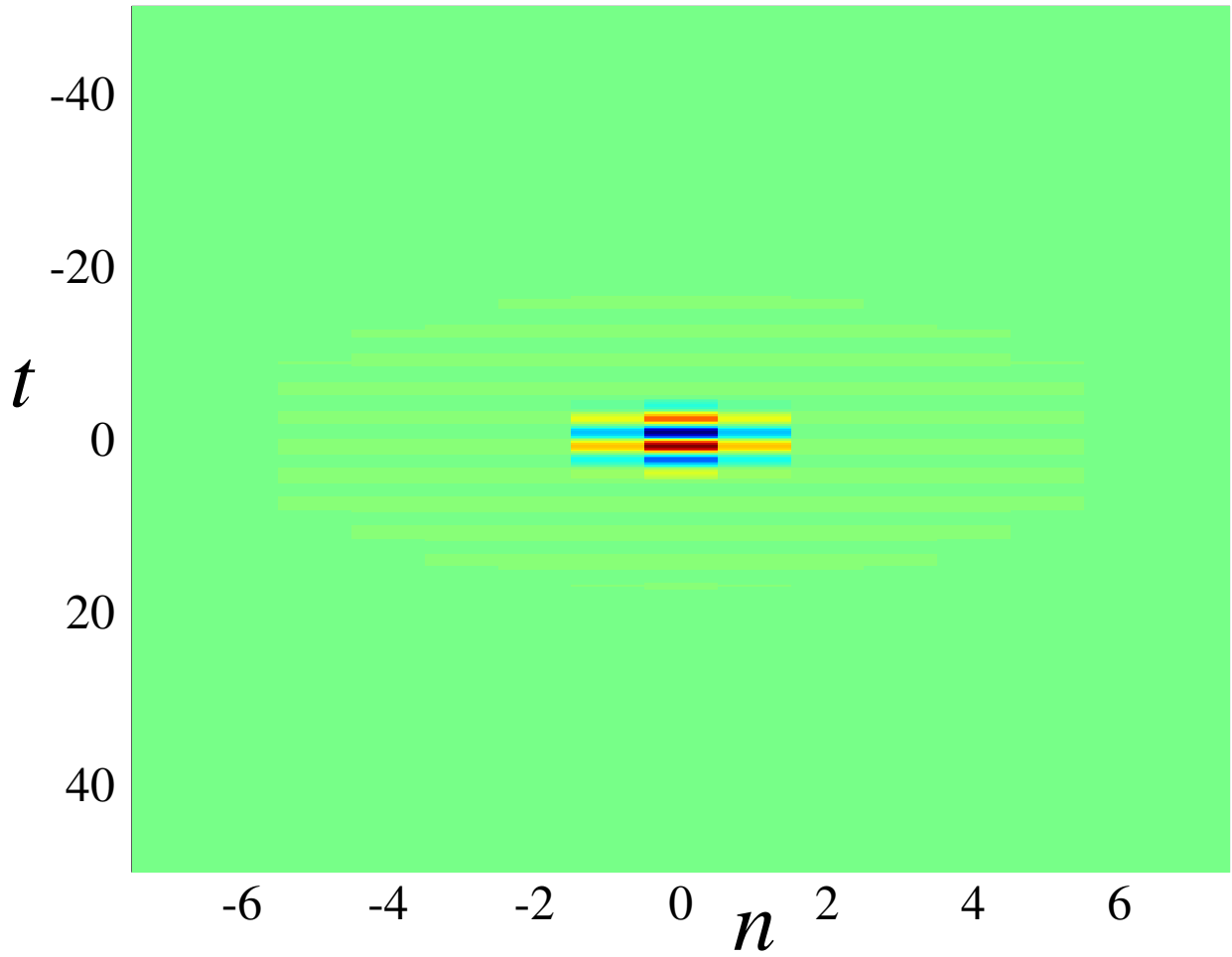
- Initial data

$$u_n(z=0,t) = A_0 \sin(\omega t + \varphi) \exp\left(-\frac{n^2}{x^2} - \frac{t^2}{\tau^2}\right);$$

- We fix $\varphi_0 = 0$, $x = 1$, $\lambda = 1$, and we vary A_0 and τ.

Formation of a solitons from a Gaussian pulse

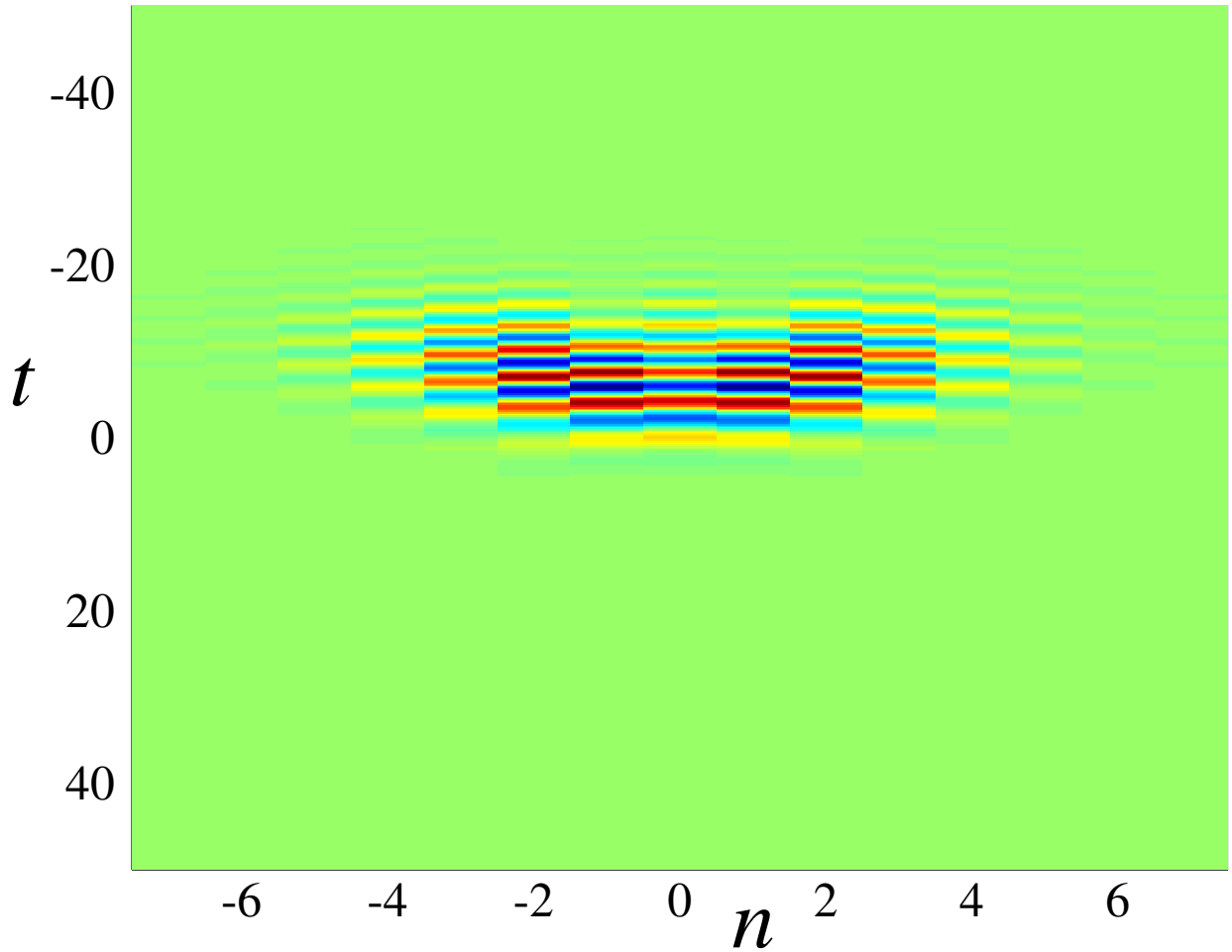
- Input



z=0, $fwhm = 3.5.$

Formation of a solitons from a Gaussian pulse

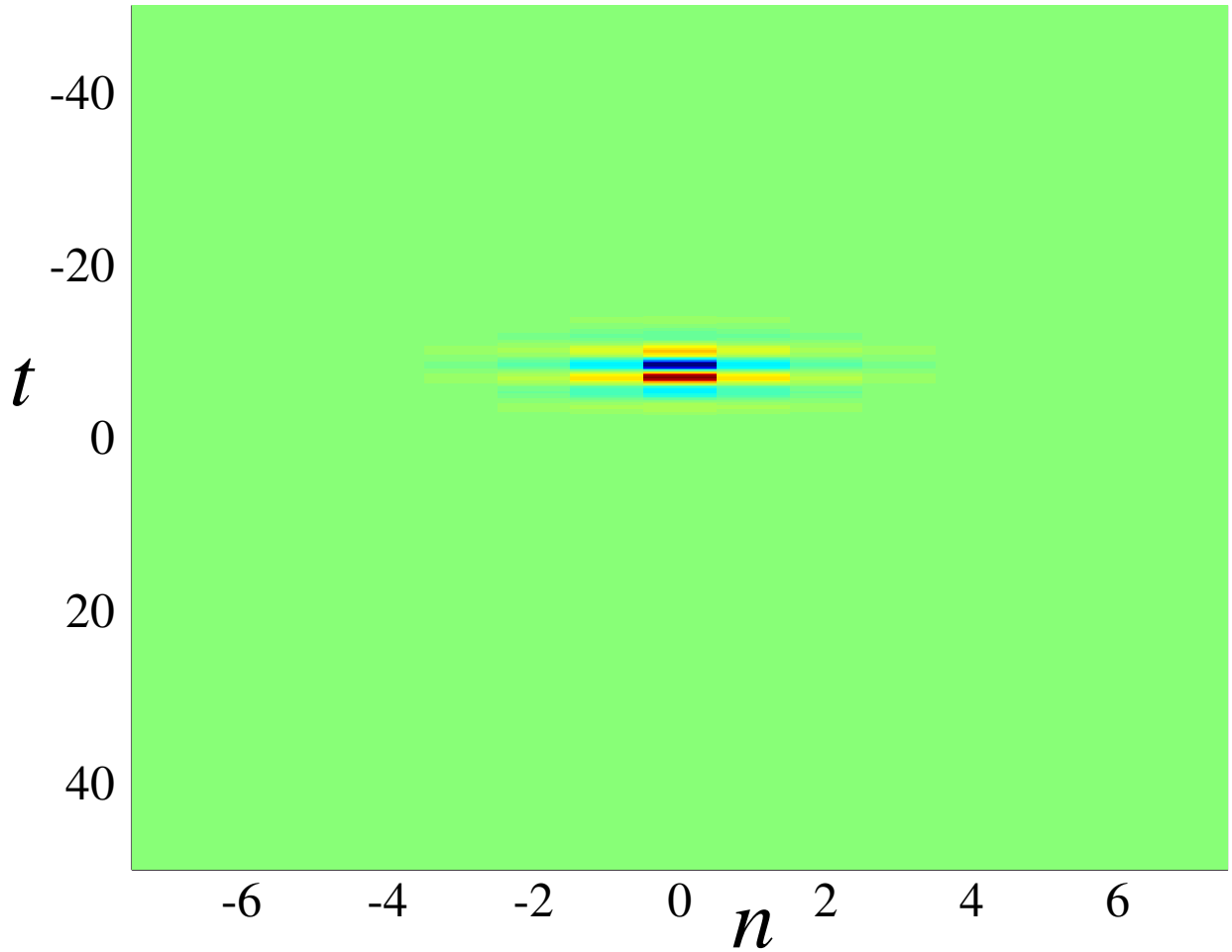
- Low amplitude output: diffraction and dispersion



$z = 0.72,\ A_0 = 0.2,\quad fwhm = 3.5.$

Formation of a solitons from a Gaussian pulse

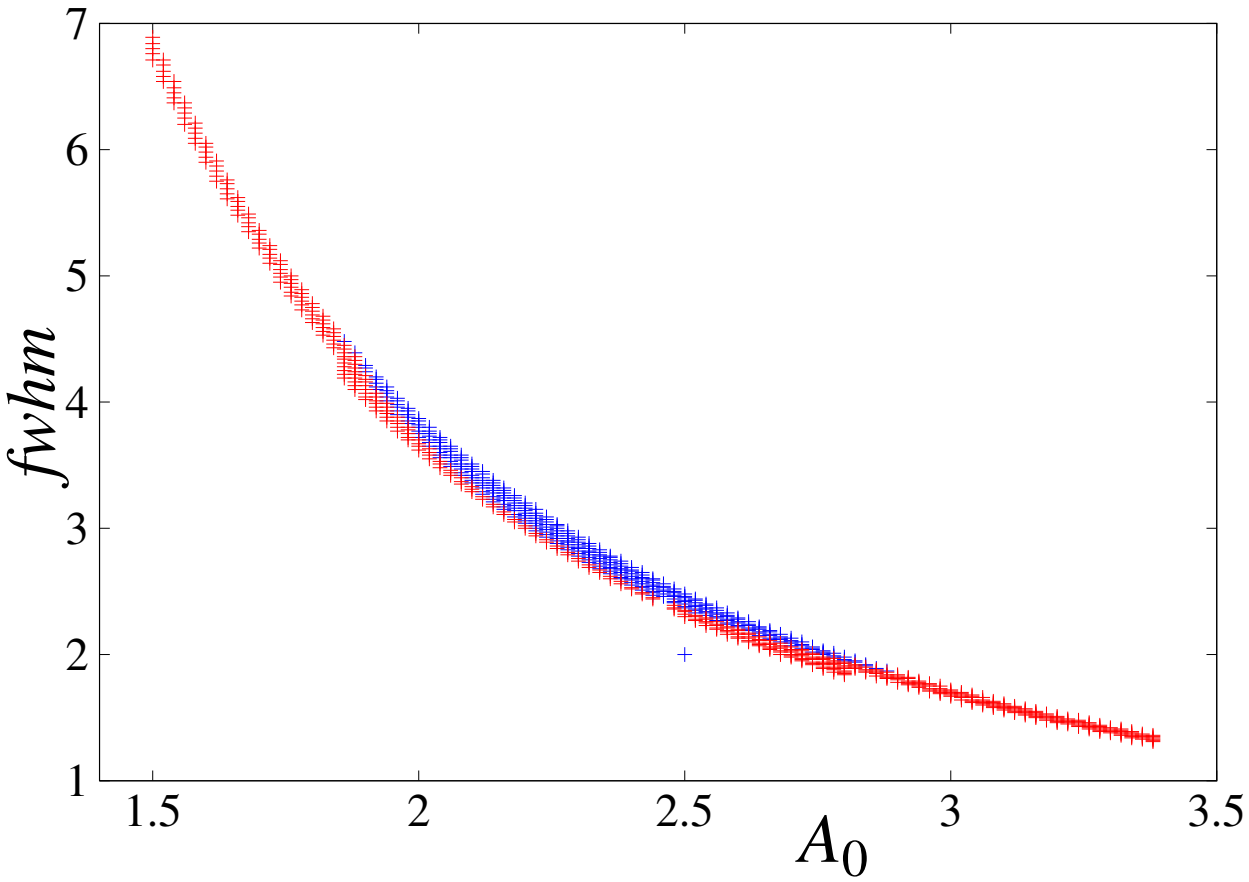
- High amplitude output: space-time localization



$z = 288,\ A_0 = 2.06,\ fwhm = 3.5.$

An energy threshold for soliton formation?

- Domain for soliton formation



Blue: soliton; red: dispersion-diffraction.

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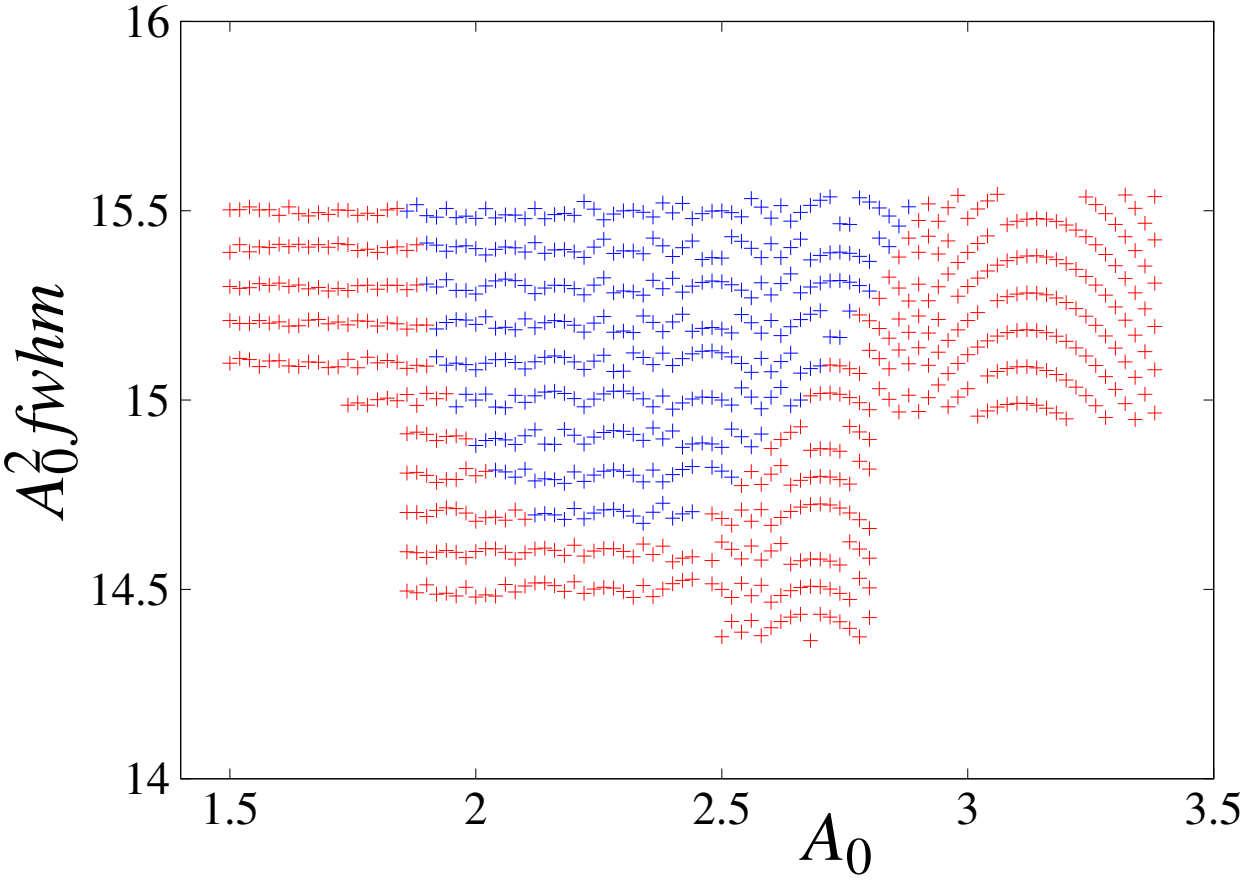

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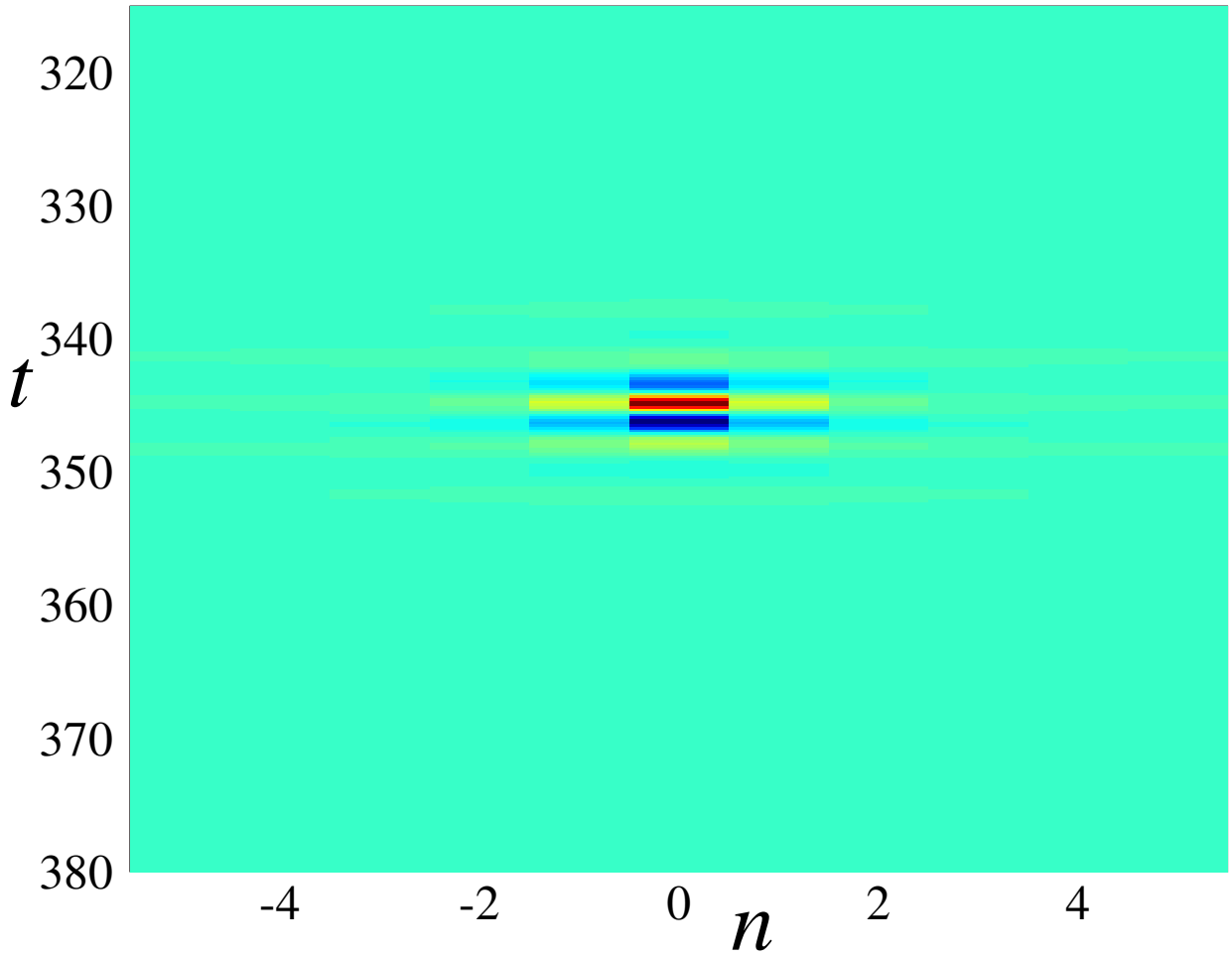
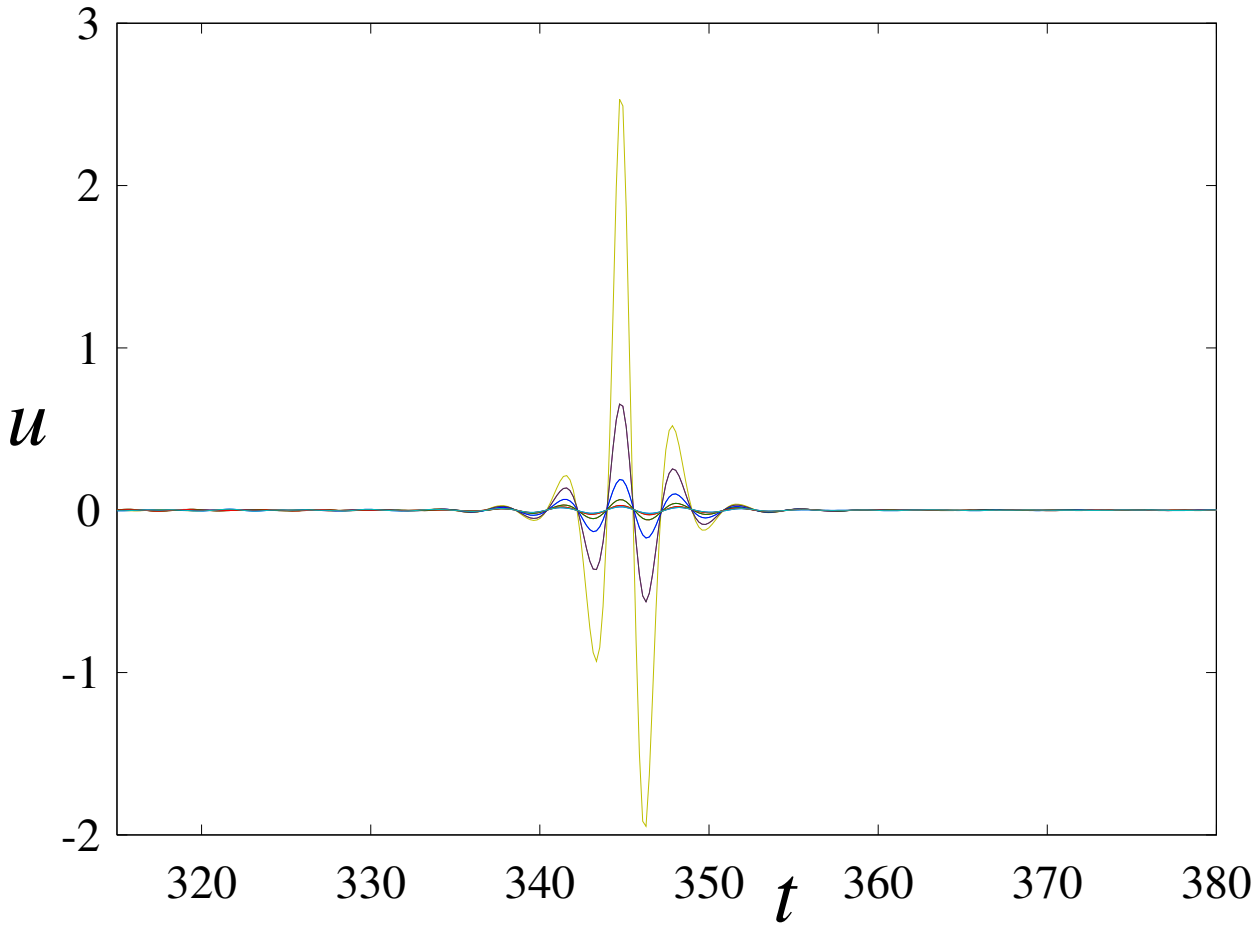
- Two kind of solitons: breathing and fundamental.
- Breathing soliton: { localized in space and time / oscillating wave packet



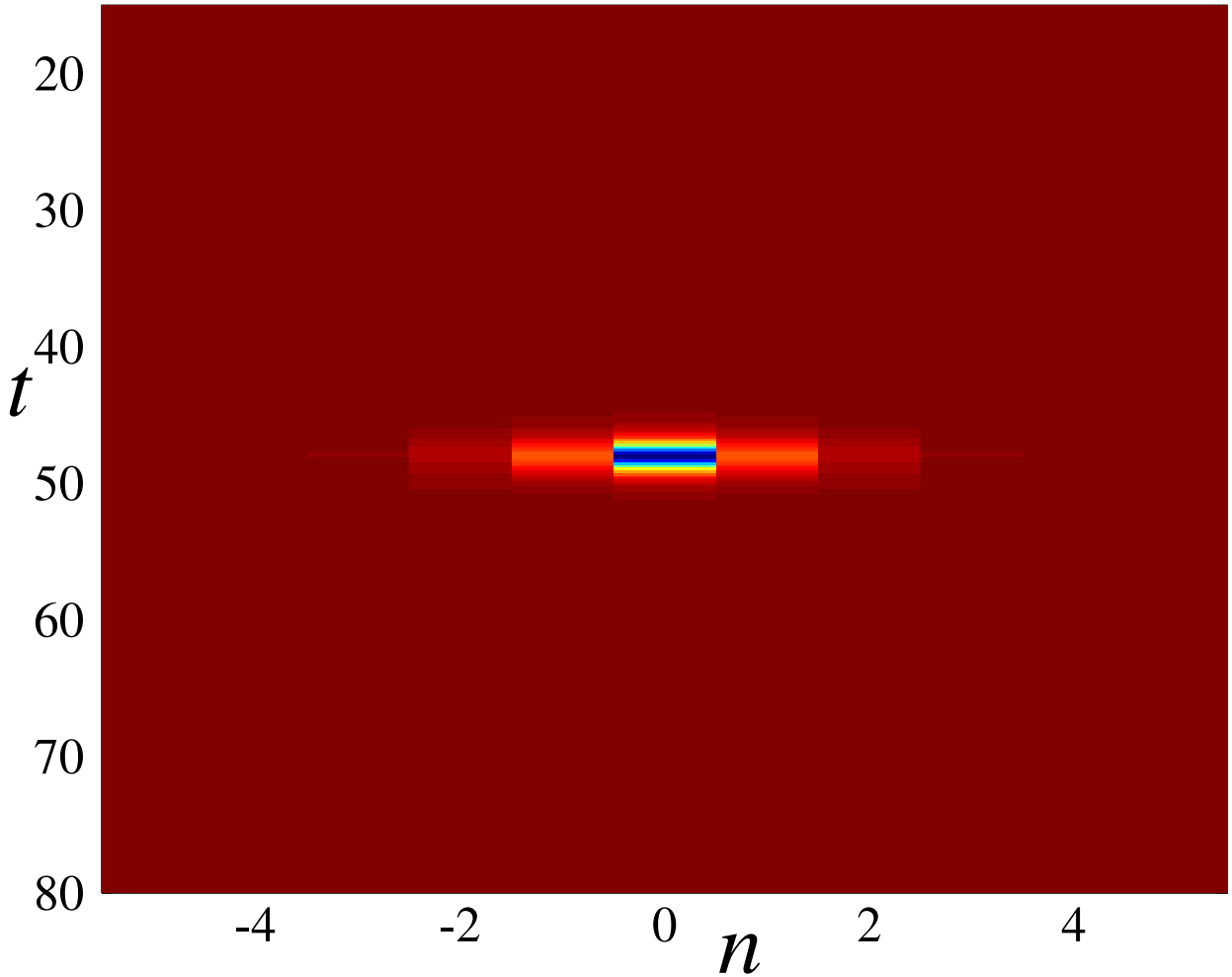
$\max |u| = 3.1801$

- Two kind of solitons: breathing and fundamental.
- Breathing soliton: $\left\{ \begin{array}{l} \text{localized in space and time} \\ \text{oscillating wave packet} \end{array} \right.$



$\max |u| = 3.1801$

- Fundamental soliton: $\begin{cases} \text{localized in space and time} \\ \text{single humped} \end{cases}$



$\max |u| = 2.5667$

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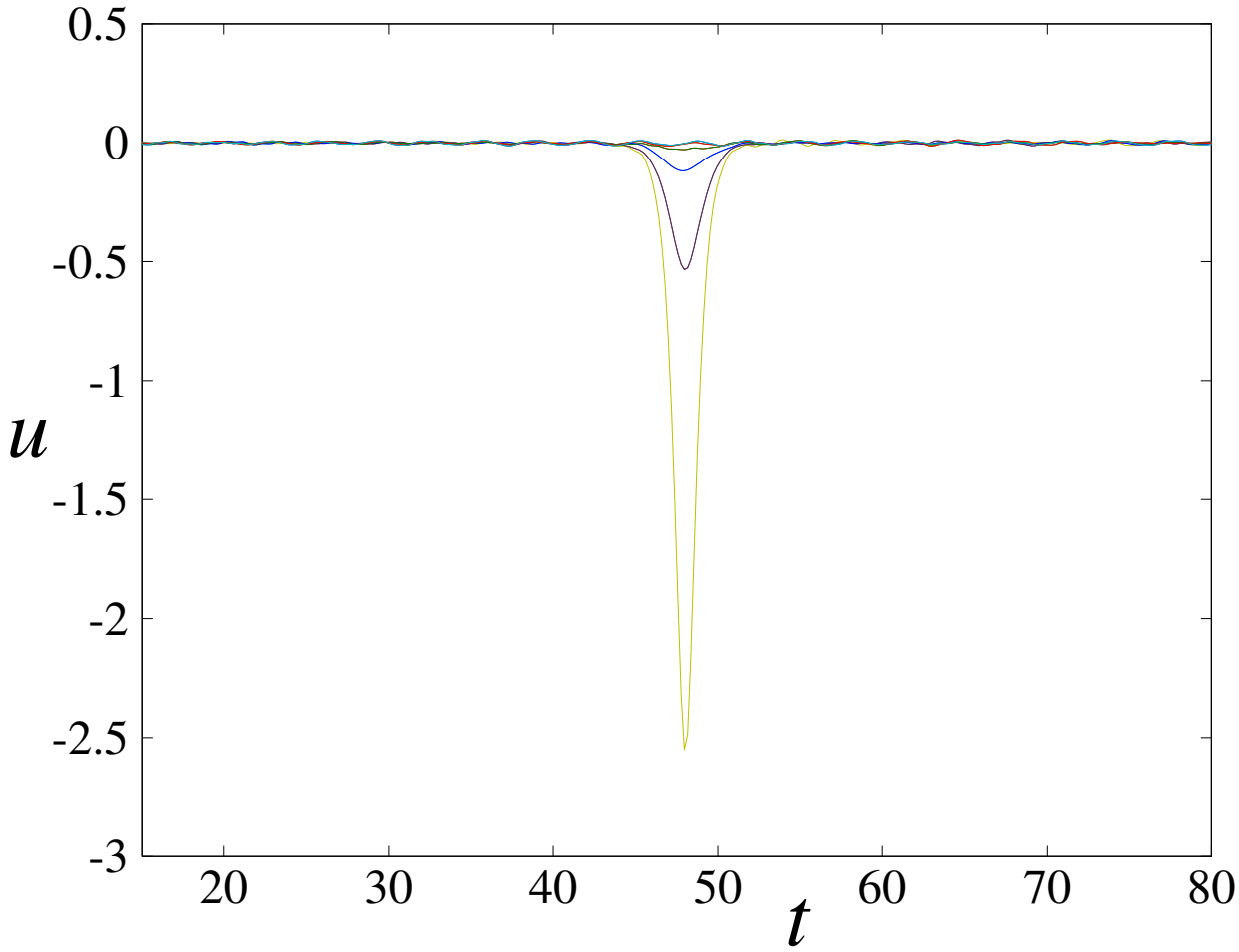

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