



# Propagation de solitons optiques à quelques cycles dans des guides d'ondes couplés

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# Solitons optiques à quelques cycles dans des guides couplés

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and Academy of Romanian Scientists, Bucharest.

<sup>3</sup>Laboratoire MOLTECH-Anjou, CNRS UMR 6200, Université d'Angers.

<sup>4</sup>Laboratoire Electronique Quantique, USTHB, Alger.

## 1 Waveguiding of a few-cycle pulse

- How to model it
- Nonlinear widening of the linear guided modes

## 2 Waveguide coupling in the few-cycle regime

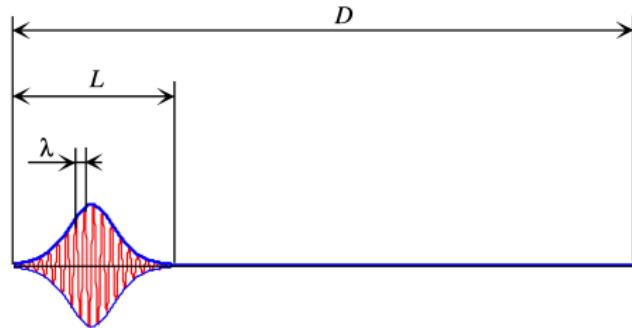
- Derivation of the coupling terms
- Few-cycle optical solitons in linearly coupled waveguides

## 3 Few cycle spatiotemporal solitons in waveguide arrays

- Formation of a solitons from a Gaussian pulse
- Two kind of solitons

# Solitary wave vs envelope solitons

- Envelope soliton: the usual optical soliton in the ps range

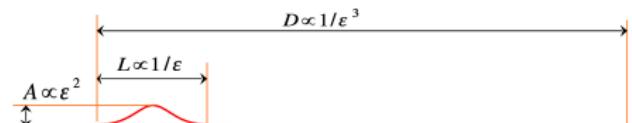


- Pulse duration  $L \gg \lambda$  wavelength
- Typical model: NonLinear Schrödinger equation (NLS)

- It is a soliton if it propagates without deformation on  $D \gg L$ , due to nonlinearity.

In linear regime: spread out by dispersion.

- Solitary wave soliton: the hydrodynamical soliton



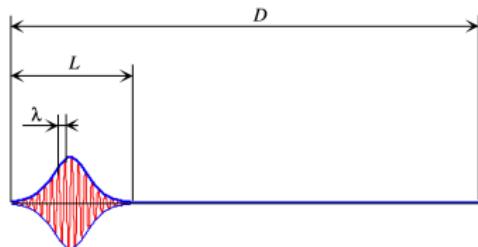
- A single oscillation
- Typical model: Korteweg-de Vries equation (KdV)

- Few-cycle optical solitons:  $L \sim \lambda$

- The slowly varying envelope approximation is not valid

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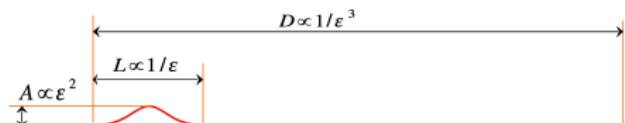


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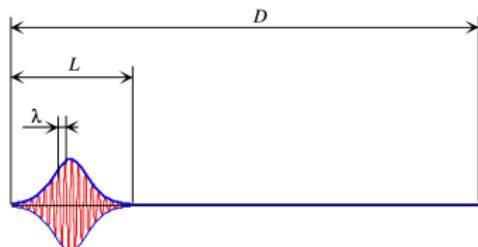
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- Generalized NLS equation
- We seek a different approach based on KdV type models

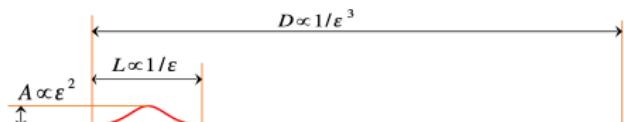
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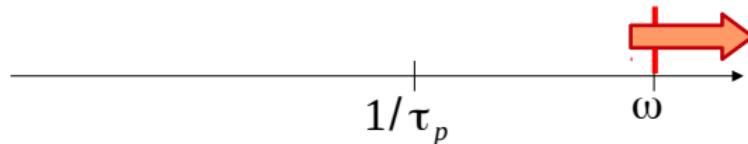
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# The mKdV model

- A two-level model with resonance frequency  $\omega$
- UV transition only, with  $(1/\tau_p) \ll \omega$



⇒ Long-wave approximation  
modified Korteweg-de Vries (mKdV) equation

$$\frac{\partial E}{\partial \zeta} = \frac{1}{6} \frac{d^3 k}{d\omega^3} \left|_{\omega=0} \right. \frac{\partial^3 E}{\partial \tau^3} - \frac{6\pi}{nc} \chi^{(3)}(\omega; \omega, \omega, -\omega) \left|_{\omega=0} \right. \frac{\partial E^3}{\partial \tau}$$

H. Leblond and F. Sanchez, *Phys. Rev. A* **67**, 013804 (2003)

# Waveguide description

- The evolution of the electric field  $E$ :

In (1+1) dimensions:

→ The modified Korteweg-de Vries (mKdV) equation

$$\partial_\zeta E + \beta \partial_\tau^3 E + \gamma \partial_\tau E^3 = 0$$

- Nonlinear coefficient  $\gamma = \frac{1}{2nc} \chi^{(3)}$ ,

Dispersion parameter  $\beta = \frac{(-n'')}{2c}$ ,

# Waveguide description

- The evolution of the electric field  $E$ :

We generalize to (2+1) dimensions:

→ The cubic generalized Kadomtsev-Petviashvili (CGKP) equation

$$\partial_\zeta E + \beta \partial_\tau^3 E + \gamma \partial_\tau E^3 - \frac{V}{2} \int^\tau \partial_\xi^2 E d\tau' = 0$$

- Nonlinear coefficient  $\gamma = \frac{1}{2nc} \chi^{(3)}$ ,

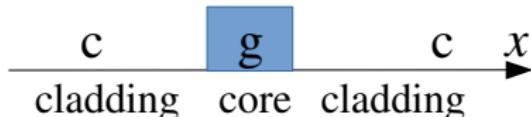
Dispersion parameter  $\beta = \frac{(-n'')}{2c}$ ,

Linear velocity:  $V = \frac{c}{n}$ .

# Waveguide description

- The evolution of the electric field  $E$ :

A waveguide:



→ The cubic generalized Kadomtsev-Petviashvili (CGKP) equation

$$\partial_\zeta E + \beta_\alpha \partial_\tau^3 E + \gamma_\alpha \partial_\tau E^3 - \frac{V_\alpha}{2} \int^\tau \partial_\xi^2 E d\tau' = 0$$

with  $\alpha = g$  in the core and  $\alpha = c$  in the cladding.

- Nonlinear coefficient  $\gamma_\alpha = \frac{1}{2n_\alpha c} \chi_\alpha^{(3)}$ ,

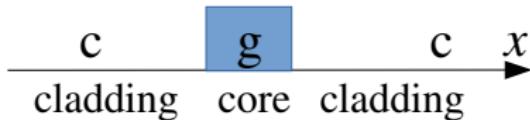
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A waveguide:



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$$\partial_\zeta E + \beta_\alpha \partial_\tau^3 E + \gamma_\alpha \partial_\tau E^3 + \frac{1}{V_\alpha} \partial_\tau E - \frac{V_\alpha}{2} \int^\tau \partial_\xi^2 E d\tau' = 0$$

with  $\alpha = g$  in the core and  $\alpha = c$  in the cladding.

Velocities:  $V_g < V_c$

- Nonlinear coefficient  $\gamma_\alpha = \frac{1}{2n_\alpha c} \chi_\alpha^{(3)}$ ,

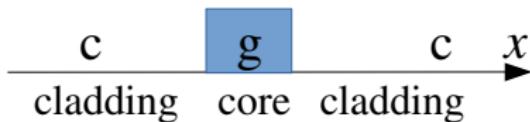
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# Waveguide description

- The evolution of the electric field  $E$ :

A waveguide:



→ The cubic generalized Kadomtsev-Petviashvili (CGKP) equation  
In dimensionless form:

$$\partial_z u = A_\alpha \partial_t^3 u + B_\alpha \partial_t u^3 + v_\alpha \partial_t u + \frac{W_\alpha}{2} \int^t \partial_x^2 u dt'$$

with  $\alpha = g$  in the core and  $\alpha = c$  in the cladding.

Relative inverse velocities:  $v_g > v_c$

- Nonlinear coefficient  $\gamma_\alpha = \frac{1}{2n_\alpha c} \chi_\alpha^{(3)}$ ,

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Linear velocity:  $V_\alpha = \frac{c}{n_\alpha}$ .

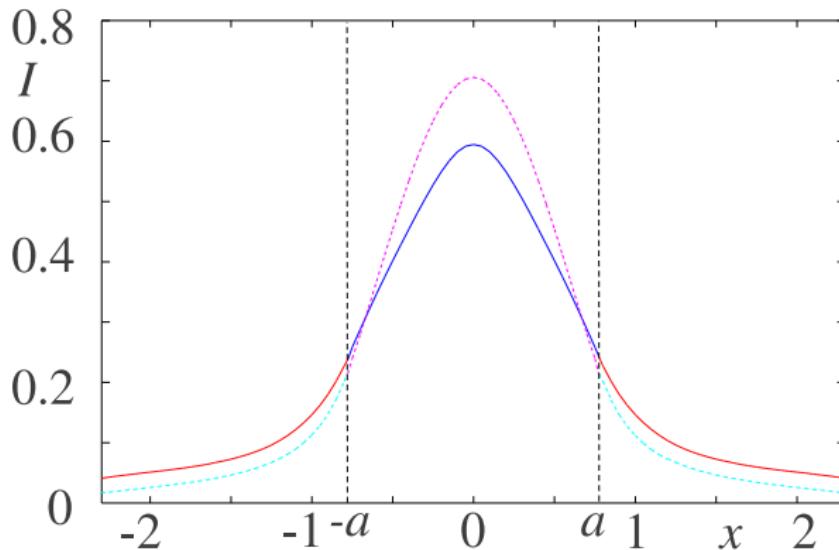
# Nonlinear propagation in linear guide

- We solve the CGKP equation starting from

$$u(x,t,z=0) = A \cos(\omega t) f(x) e^{-t^2/w^2},$$

- $f(x) = \begin{cases} \cos(k_x x), & \text{for } |x| \leq a, \\ Ce^{-\kappa|x|}, & \text{for } |x| > a, \end{cases}$  is a linear mode profile
- Normalized coefficients  $A_1 = A_2 = B_1 = B_2 = W_1 = W_2 = 1$ ,  
→ we assume that
  - Temporal compression occurs
  - Spatial defocusing occurs, (else it collapses!)
  - Dispersion and nonlinearity are identical in core and cladding.

- Guided wave profiles

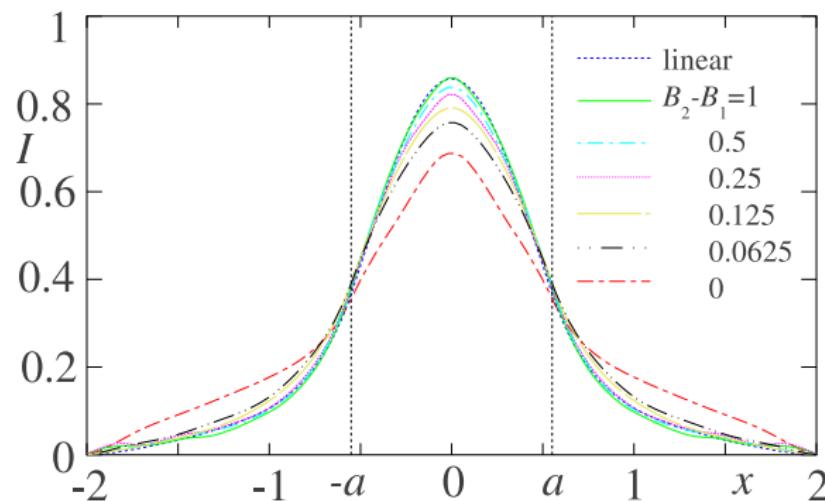


(Normalized so that the total power is 1.  $v_2 = 3$ ,  $w = 2$ .)

- The pulse is less confined in nonlinear (blue, and red) than in linear (pink and cyan) regime.

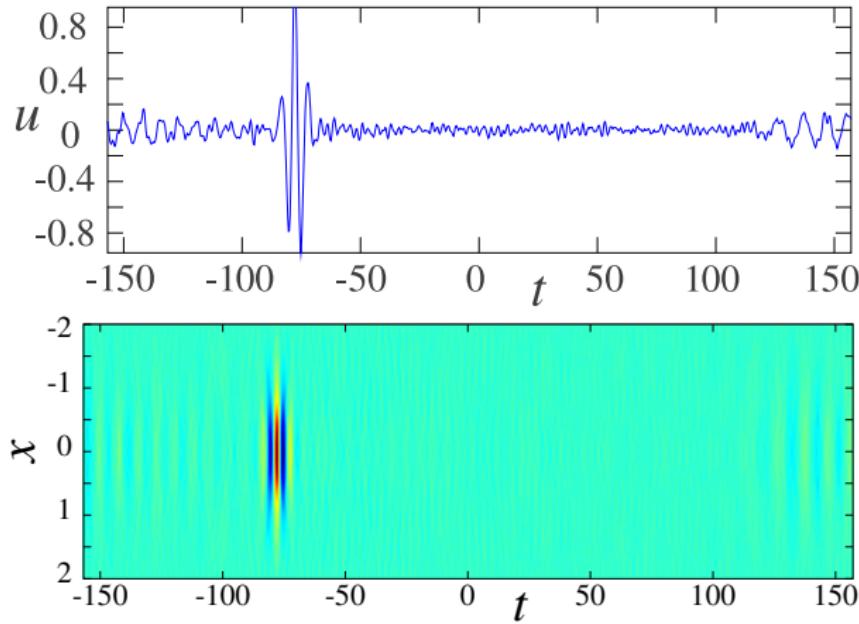
# Nonlinear waveguide

- Wave guided and confined by using nonlinear velocity: a higher nonlinear coefficient in the cladding than that in the core.



Guided profiles of the nonlinear waveguide. Normalized so that the total power is 1.

# Two-cycle soliton of the nonlinear waveguide



$$B_2 - B_1 = 1.$$

H. Leblond and D. Mihalache, *Phys. Rev. A* **88**, 023840 (2013)

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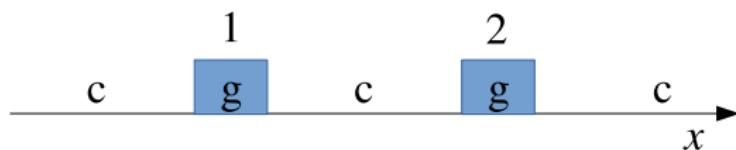
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## 3 Few cycle spatiotemporal solitons in waveguide arrays

- Formation of a solitons from a Gaussian pulse
- Two kind of solitons

- 2D waveguiding structure: two cores 1 and 2 and dielectric cladding



- The generalized Kadomtsev-Petviashvili (GKP) equation (dimensionless)

$$\partial_z u = A_\alpha \partial_t^3 u + B_\alpha \partial_t u^3 + V_\alpha \partial_t u + \frac{w_\alpha}{2} \int^t \partial_x^2 u dt,$$

$\alpha = g$  in the cores 1 and 2,  $\alpha = c$  in the cladding.

- We seek for a solution as

$$u = R(t,z)f_1(x)e^{i\varphi} + S(t,z)f_2(x)e^{i\varphi},$$

i.e., two interacting modes.

- \*  $f_j$ , ( $j = 1, 2$ ): linear mode profiles of individual guides,
- \*  $R(t,z)$ ,  $S(t,z)$ : longitudinal wave profiles,
- \*  $\varphi = \omega t - \beta z$ .

- We report it into the GKP equation and get after averaging on  $x$ :

$$\partial_z R = \partial_z S = \frac{i w_g}{2\omega} \frac{(K_c - K_g) I_2}{1 + I_1} (R + S),$$

- Involve overlap integrals  $I_1 = \int_{-\infty}^{\infty} f_1 f_2 dx$ ,  $I_2 = \int_{g_1} f_1 f_2 dx = \int_{g_2} f_1 f_2 dx$   
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$$\partial_z R = \partial_z S = \frac{i w_g}{2\omega} \frac{(K_c - K_g) I_2}{1 + I_1} (R + S),$$

- The few-cycle pulse is expanded as a Fourier integral of such modes,  
 $u_1 = \int R e^{i\varphi} d\omega$ ,  $u_2 = \int S e^{i\varphi} d\omega$ .
- We report  $\partial_z R$  and  $\partial_z S$  into  $\partial_z u_1$ ,  
and get the linear coupling terms.
- Finally, we get the system of two coupled modified Korteweg-de Vries  
(mKdV) equations

$$\partial_z u_1 = A \partial_t^3 u_1 + B \partial_t u_1^3 + V \partial_t u_1 + C \partial_t u_2 + D \partial_t^3 u_2,$$

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# Nonlinear coupling

- An analogous procedure, treating the nonlinear term as a perturbation, allows to derive the nonlinear coupling terms
- The complete final system is

$$\begin{aligned}\partial_z u_1 &= A\partial_t^3 u_1 + B\partial_t u_1^3 + V\partial_t u_1 \\ &\quad + C\partial_t u_2 + D\partial_t^3 u_2 + E\partial_t (3u_1^2 u_2 + u_2^3) \\ \partial_z u_2 &= A\partial_t^3 u_2 + B\partial_t u_2^3 + V\partial_t u_2 \\ &\quad + C\partial_t u_1 + D\partial_t^3 u_1 + E\partial_t (3u_1 u_2 + u_1^3)\end{aligned}$$

H. Leblond, and S. Terniche, *Phys. Rev. A* **93**, 043839 (2016)

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- We evidence

- a standard linear coupling term,
- a linear coupling term based on dispersion,
- a nonlinear coupling term

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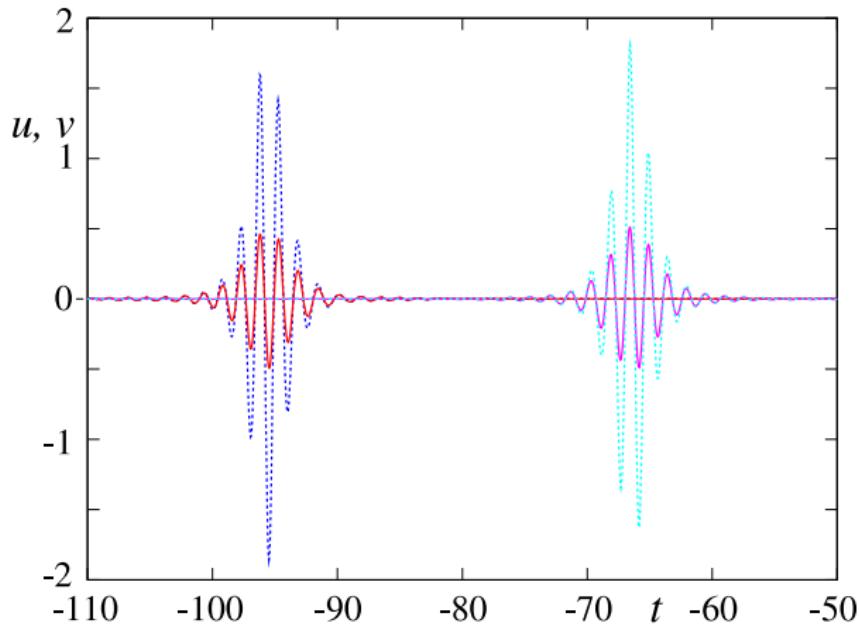
H. Leblond, and S. Terniche, *Phys. Rev. A* **93**, 043839 (2016)

- We assume a purely linear and non-dispersive coupling

$$\begin{aligned}\partial_z u &= -\partial_t(u^3) - \partial_t^3 u - C\partial_t v, \\ \partial_z v &= -\partial_t(v^3) - \partial_t^3 v - C\partial_t u,\end{aligned}$$

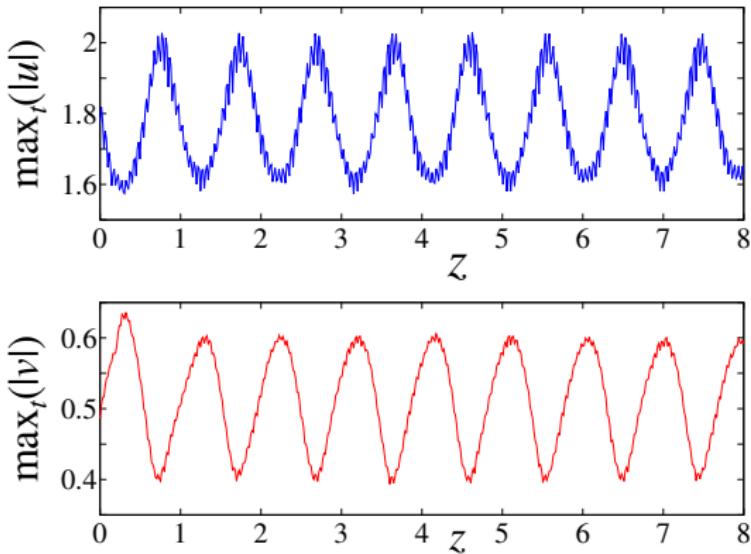
- We look for stationary states (vector solitons) in this model
- The "stationary" states oscillate with  $t$  and  $z$ : .  
few-cycle solitons are **breathers**.

- A typical example of few-cycle vector soliton



(Dotted lines:  $u$ , solid lines:  $v$ . Left: at  $z = 0$ , right: at  $z = 60$ .  $\langle A_u \rangle = 1.837$ ).

- Evolution of soliton's maximum amplitude during propagation.



Soliton with  $\langle A_u \rangle = 1.789$ .

- Two types of oscillations:
  - Fast:** phase - group velocity mismatch
  - Slower:** periodic energy exchange, as in linear regime

- Consider now the coupled equations in the **linearized** case.
- The **monochromatic** solutions are

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} A \\ B \end{pmatrix} e^{-i(\omega t + b\omega^3 z)},$$

- With, due to **coupling**,

$$A = u_0 \cos c\omega z + i v_0 \sin c\omega z,$$

$$B = v_0 \cos c\omega z + i u_0 \sin c\omega z.$$

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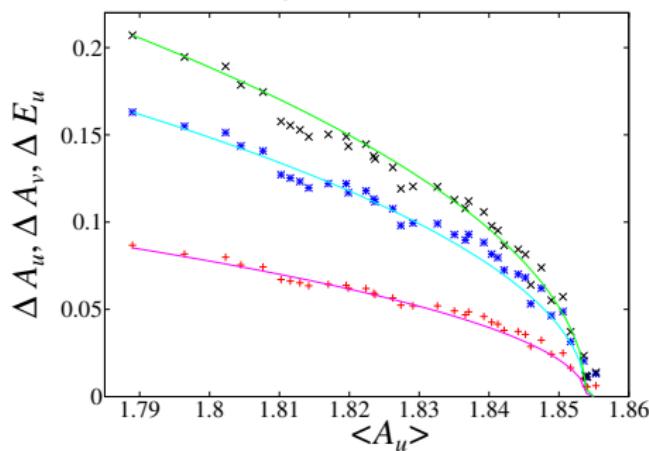
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# Oscillations of the few-cycle vector solitons

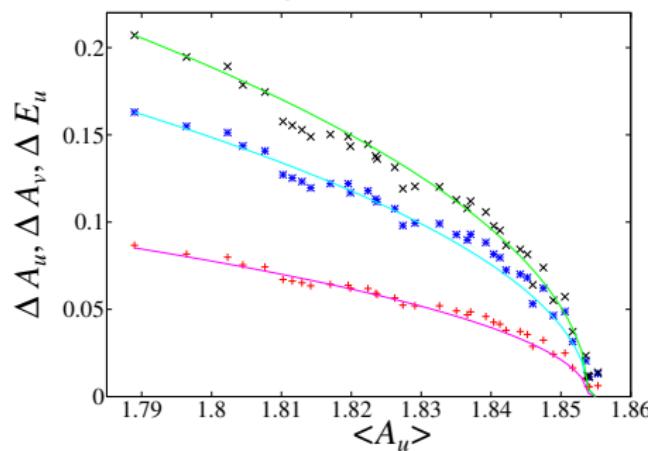
- The energies  $E_u = \int u^2 dt$  and  $E_v = \int v^2 dt$  oscillate almost harmonically, as  $E_u = \langle E_u \rangle + \Delta E_u \sin(2\pi\sigma_a z + \phi_{E,u})$ ,
- The same for  $A_u = \max_t(|u|)$  and  $A_v = \max_t(|v|)$
- Spatial frequency  $\sigma_a \in [1.06, 1.17]$ , increasing with  $\langle A_u \rangle$ .  
(linear:  $\sigma_0 = 1.326$ ).
- Amplitudes of oscillations vs amplitude of field  $u$



black saltires:  $\Delta E_u$ ; blue stars:  $\Delta A_u$ ; red crosses:  $\Delta A_v$

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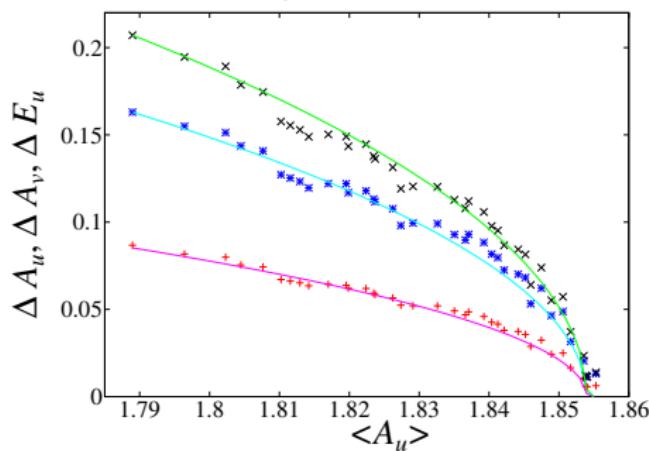
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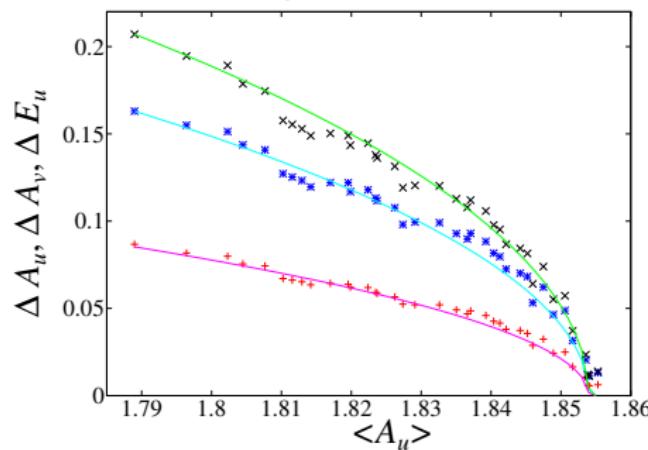
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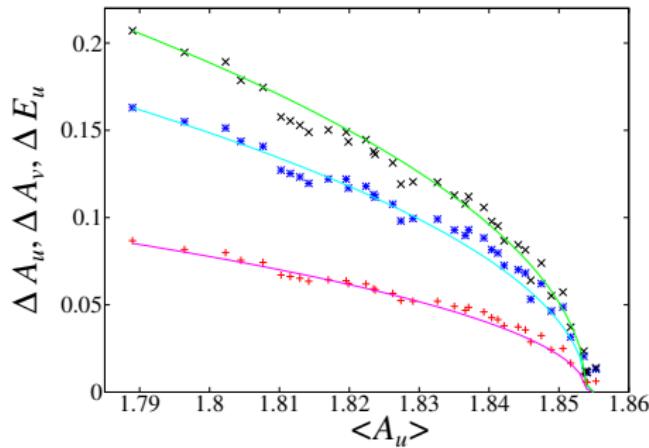
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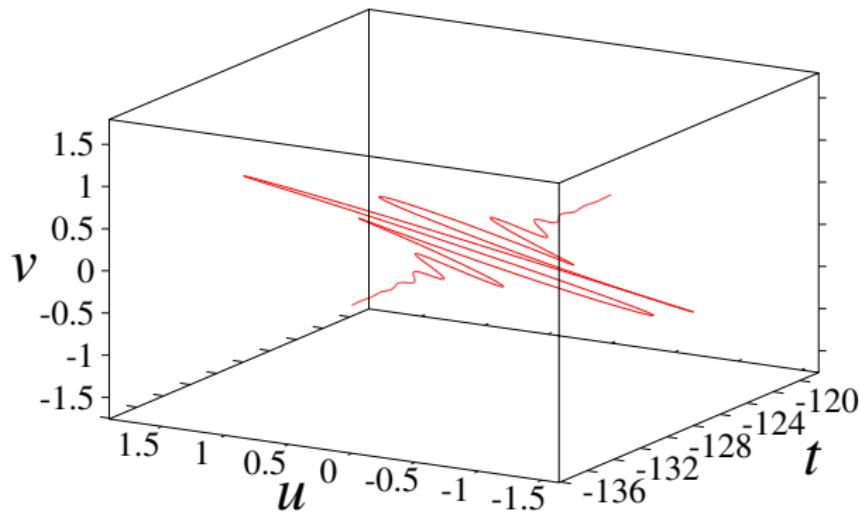


black saltires:  $\Delta E_u$ ; blue stars:  $\Delta A_u$ ; red crosses:  $\Delta A_v$ .

- Well fitted with  $\Delta E_u \simeq R \sqrt{A_0 - \langle A_u \rangle}$ , etc., with  $A_0 = 1.854$ .

## Evolution of the ratio $v/u$

- Almost constant vs  $t$

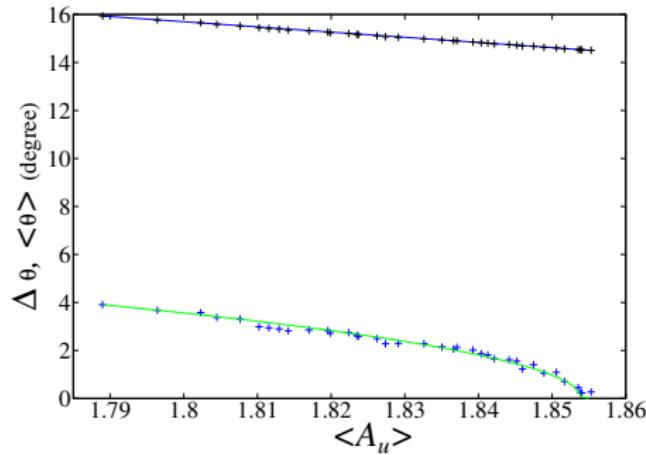


Soliton with  $\langle A_u \rangle = 1.855$ .

# Evolution of the ratio $v/u$

Or  $\theta = \arctan \frac{v}{u}$ .

- Oscillates almost harmonically with  $z$ .
- Amplitudes of oscillations vs field  $u$  amplitude:



Black line: mean value  $< \theta >$ ; green line:  $\Delta\theta$ .

Crosses: raw numerical data; solid lines: linear or parabolic fits.

## 1 Waveguiding of a few-cycle pulse

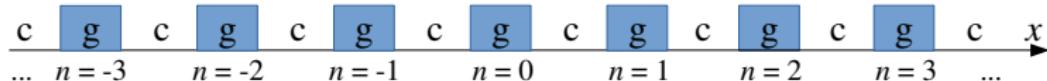
- How to model it
- Nonlinear widening of the linear guided modes

## 2 Waveguide coupling in the few-cycle regime

- Derivation of the coupling terms
- Few-cycle optical solitons in linearly coupled waveguides

## 3 Few cycle spatiotemporal solitons in waveguide arrays

- Formation of a solitons from a Gaussian pulse
- Two kind of solitons



- A set of coupled waveguides within the same model, as:

$$\partial_z u_n = -a \partial_t(u_n^3) - b \partial_t^3 u_n - c \partial_t(u_{n-1} + u_{n+1}),$$

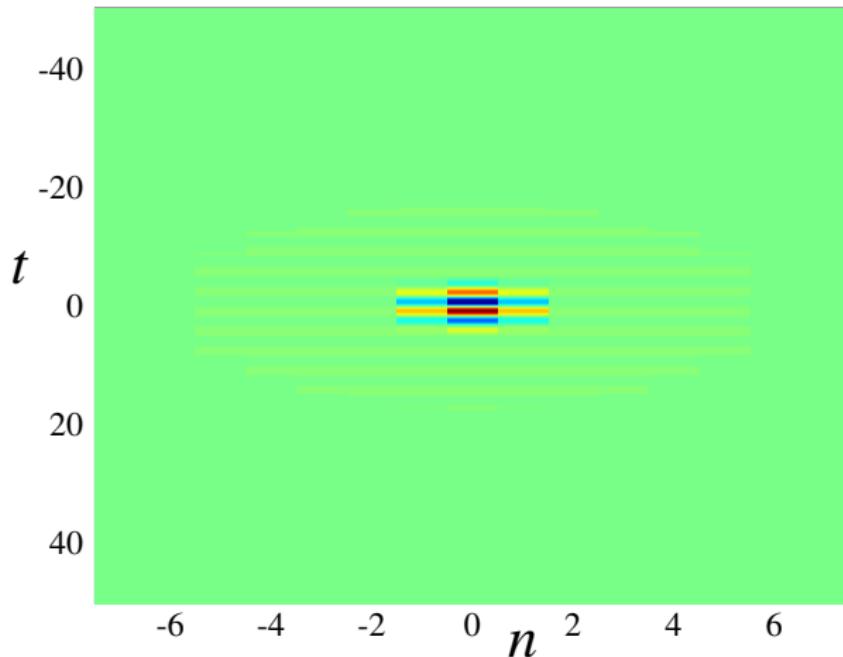
- Initial data

$$u_n(z=0, t) = A_0 \sin(\omega t + \varphi) \exp\left(-\frac{n^2}{x^2} - \frac{t^2}{\tau^2}\right);$$

- We fix  $\varphi_0 = 0$ ,  $x = 1$ ,  $\lambda = 1$ , and we vary  $A_0$  and  $\tau$ .

# Formation of a solitons from a Gaussian pulse

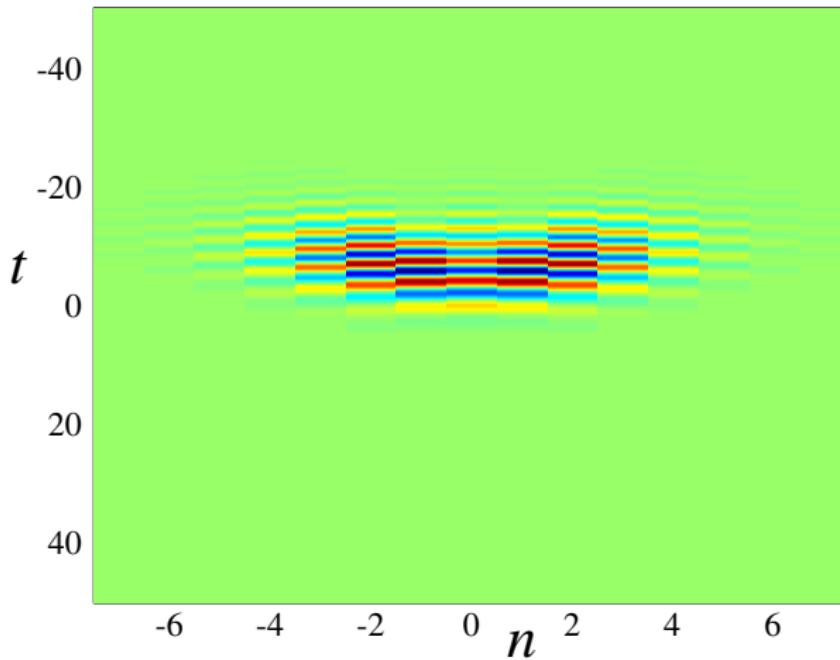
- Input



$z=0, \quad fwhm = 3.5.$

# Formation of a solitons from a Gaussian pulse

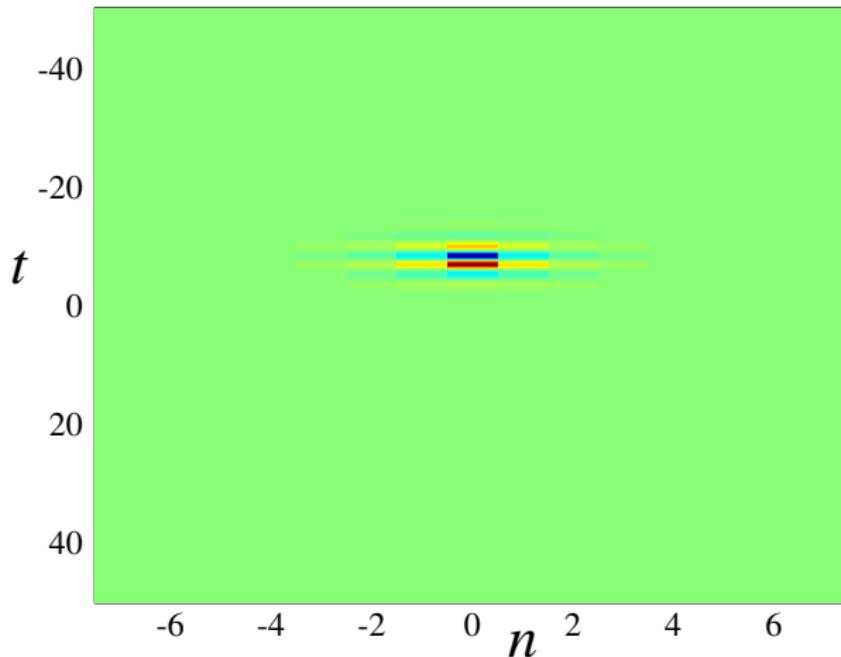
- Low amplitude output: diffraction and dispersion



$$z = 0.72, A_0 = 0.2, \text{ fwhm} = 3.5.$$

# Formation of a solitons from a Gaussian pulse

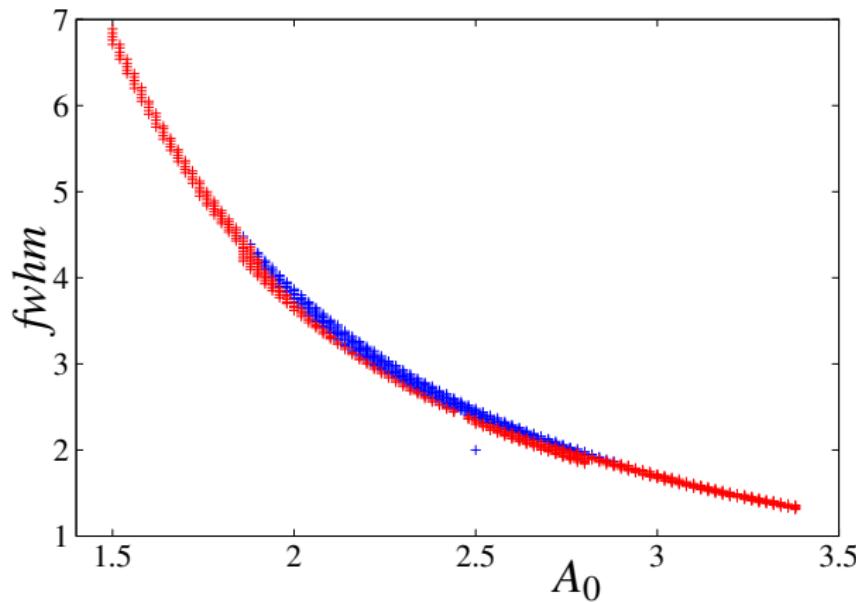
- High amplitude output: space-time localization



$$z = 288, A_0 = 2.06, fwhm = 3.5.$$

# An energy threshold for soliton formation?

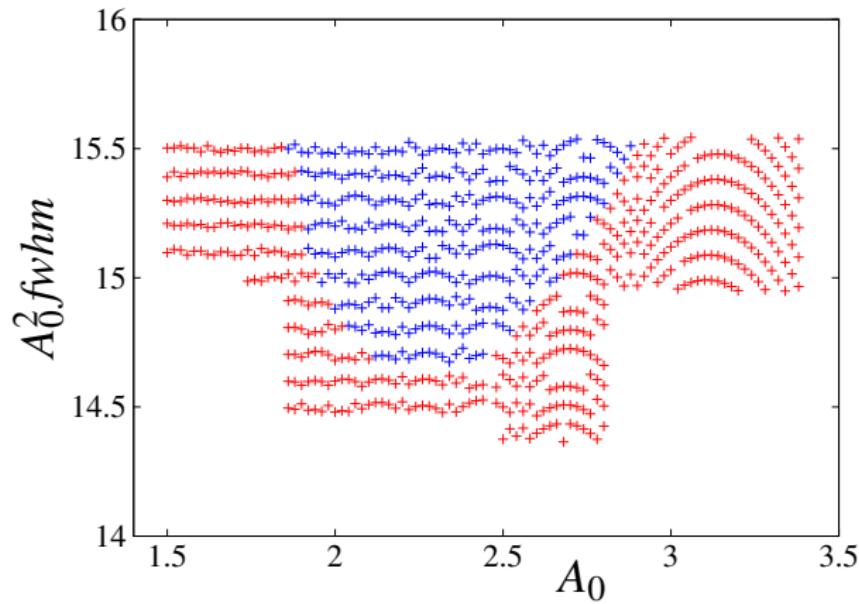
- Domain for soliton formation



Blue: soliton; red: dispersion-diffraction.

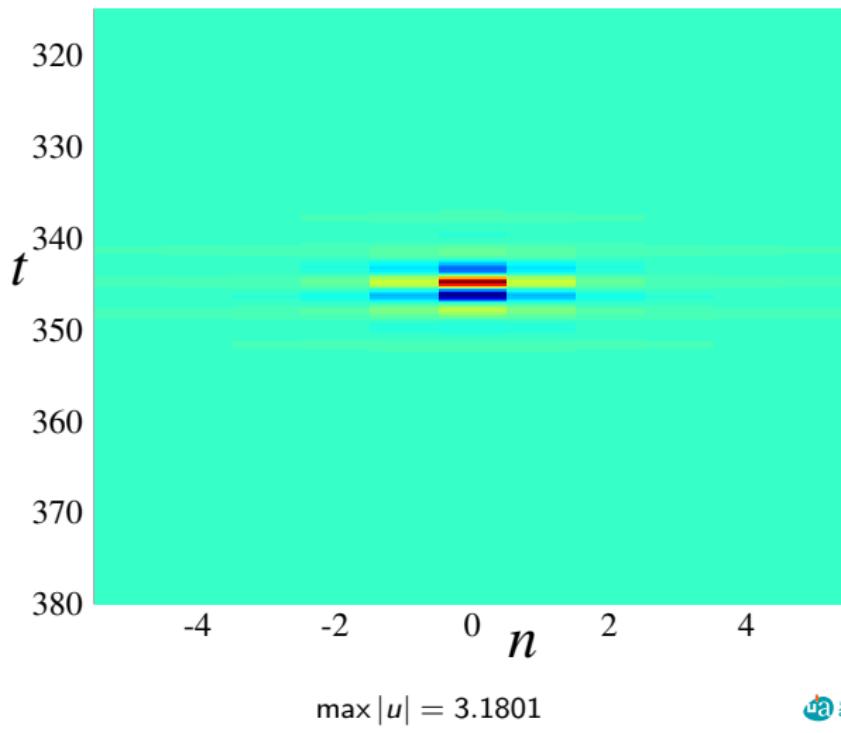
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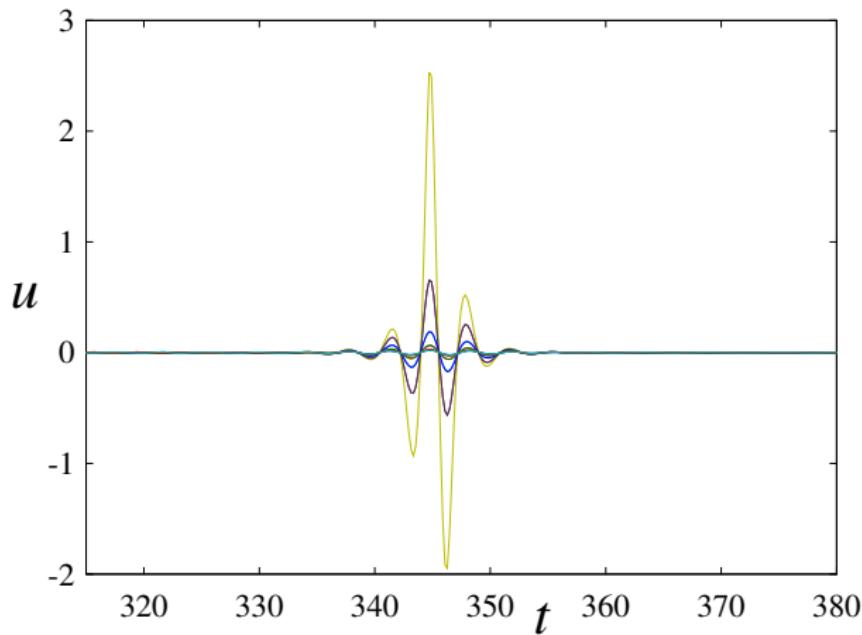


Blue: soliton; red: dispersion-diffraction.

- Two kind of solitons: breathing and fundamental.
  - Breathing soliton: { localized in space and time  
oscillating wave packet

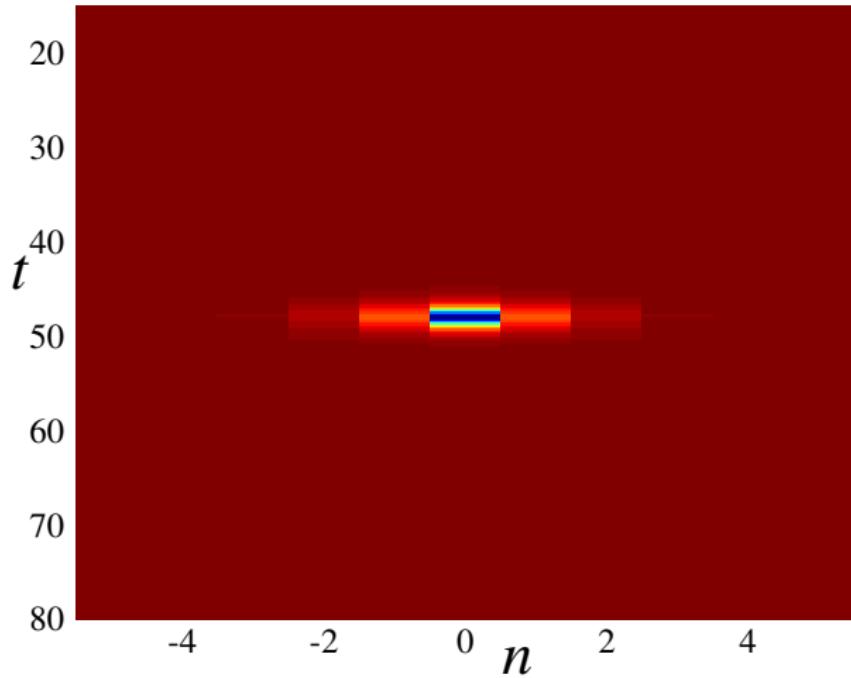


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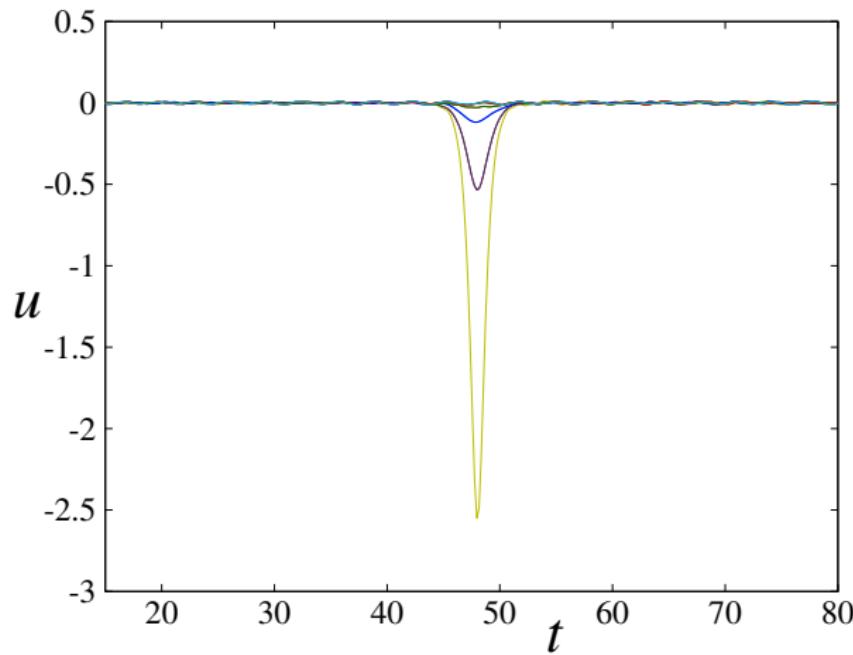
$$\max |u| = 3.1801$$

- Fundamental soliton:  $\left\{ \begin{array}{l} \text{localized in space and time} \\ \text{single humped} \end{array} \right.$



$$\max |u| = 2.5667$$

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$$\max |u| = 2.5667$$

- Thank you for your attention.

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